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Relative effectiveness of plasmaspheric hiss and VLF hiss in the inner belt energetic electron precipitation

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Relative effectiveness of plasmaspheric hiss and VLF hiss in the inner belt energetic electron precipitation has been studied. The results show that plasmaspheric hiss can induce significant precipitation of electrons by causing small changes in their energy whereas in case of VLF hiss, nearly ten times higher energy changes are required for similar precipitation. The minimum pitch angle α_m corresponding to peak energy change varies with *L*-value in the same fashion as the minimum wave length (λ_m) corresponding to peak energy emission varies with temperature *T* in case of black body radiation. The results, further, hint that the plasmaspheric hiss is more effective in energetic electron precipitation towards the inner edge ($L \simeq 1.2$) of inner radiation belt, which is in good agreement with the findings of earlier studies.

Keywords: Plasmaspheric hiss, VLF hiss, Energetic electron precipitation

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1 Introduction

The radiation belt electrons undergo gyro-resonant interaction with a variety of waves inside the plasmasphere[1,2]. The whistler-mode waves, involved in wave particle interactions occurring in the magnetosphere, play an important role in the loss of the trapped particles and in limiting the trapped flux levels[3,4]. In fact, the waves perturb the adiabatic invariant motion of the particles and the wave induced pitch angle and energy changes result in lowering of the mirror altitude and hence, their precipitation into the lower ionosphere[5]. The precipitated particles give rise to some interesting phenomena such as secondary ionization, conductivity enhancement, optical and X-ray emissions, heating and transient increment of the electron population in the lower ionosphere and cause auroras and trimpi events.

The pitch angle scattering of resonant electrons by incoherent wide-band whistler mode plasmaspheric hiss ($f \leq 1$ kHz) and VLF hiss ($f \geq 3$ kHz) has proven to be a major loss mechanism for radiation belt electrons[6]. Such wave particle interactions may result in the formation of a electron slot between inner and outer radiation belts[7,8]. When the inner belt region is intensified with these emissions, the possibility of wave-particle interactions and energetic electron precipitation is increased considerably[9,10]. The association of plasmaspheric hiss with the low latitude precipitation zone has been well established[11].

The pitch angle diffusion and precipitation of energetic electrons by plasmaspheric hiss and VLF hiss in the inner radiation belt have been frequently studied[7,8,12,13]. In contrast, the relative effectiveness of these two types of waves in pitch angle diffusion and precipitation of energetic electrons in the inner radiation zone is not evaluated so far and therefore, this is the main objective of the present study.

2 Theoretical background

The general first order gyro-resonance condition for the interaction of energetic electrons and whistler-mode waves is expressed as[14]:

$$\omega(1+\beta\mu\cos\alpha) = \Omega_e \left(1-\beta^2\right)^{1/2} \quad (1)$$

where, α, is the electron pitch angle; $\beta = v/c$ (v, is electron velocity; c, velocity of light in vacuum); μ, the whistler mode refractive index; Ω_e, the angular gyro-frequency of the electron; and ω, the angular wave frequency.

The phase velocity of the wave ($v_p = c/\mu$) is assumed to be equal to the electron's parallel velocity and the wave normal is taken to be zero as usual. In this case, the electron experiences no force and its energy remains fixed in a frame of reference moving with a velocity equal to the wave phase velocity. For energy changes in a stable frame

$$dW_\perp / dW = 1 - (dW_\parallel / dW) = 1 - (v_\parallel / v_P) \qquad (2)$$

where, $dW_\perp$, $dW_\parallel$, and dW represent changes in perpendicular, parallel and total electron energy, respectively.

The ratio of relative changes in pitch angle and energy of an electron can be expressed in terms of a parameter[15,16] :

$$K = (d\alpha / \alpha) / (dW / W) = \frac{(\Omega_e / \omega) - \sin^2 \alpha}{\alpha \sin 2\alpha} \qquad (3)$$

where, $d\alpha$, is deviation from half loss cone angle α_o.

As the maximum losses by pitch angle diffusion occur for the electrons in resonance with the whistler–mode spectrum exhibiting maximum intensity, the bounce-averaged diffusion coefficient $D(\alpha)$ for the first order gyro-resonance interaction may be approximated by the relation[17].

$$D(\alpha) = \frac{2\Omega_e^2 \left(B_w / B_o\right)^2}{E_n \left(E_n^2 - 1\right)^{1/2} \mu \Delta\omega} \qquad (4)$$

where, B_w, is the whistler-mode wave amplitude near resonance; B_o, the equatorial geomagnetic field; $\Delta\omega$, the frequency range about the resonance frequency for which pitch angle scattering occurs; and E_n, the relativistic energy of resonant electrons normalized by the factor $m_o c^2 = 511.88$ keV. In the above relation, the resonance pitch angle is taken to be near $\cos \alpha = 1/2$ (ref. 17).

The relative effectiveness of two types of the waves under consideration can be evaluated by calculating the ratio:

$$R_w = \frac{D(\alpha) \text{ for plasmaspheric hiss}}{D(\alpha) \text{ for VLF hiss}} \qquad (5)$$

Since the electron life-time is inversely proportional to $D(\alpha)$, the ratio T_R of life-times of the electrons resonant with the plasmaspheric hiss and VLF hiss, respectively may be written as

$$T_R = 1/R_w \qquad (6)$$

3 Wave magnetic field intensity data

Tsurutani *et al.*[9] have reported the observation of intense plasmaspheric hiss emissions aboard OGO-6 satellite in the inner radiation zone at ~450 km altitude during disturbed periods, which have been found to have an average peak power spectral density of $4 \times 10^{-7}\ \gamma^2 \mathrm{Hz}^{-1}$ at 550 Hz and a bandwidth of 300 Hz. Assuming an enhancement of 5 dB, this intensity is extrapolated to the equatorial plane to yield an intensity of $1.26 \times 10^{-6}\ \gamma^2\ \mathrm{Hz}^{-1}$ there. This gives a wave field of 19.44 mγ at 550 Hz. Nishino & Tanaka[18] have reported intense storm–time VLF hiss emissions at Japanese ground stations with an intensity of ~ $10^{-16}\ \mathrm{W\ m^{-2}\ Hz^{-1}}$, frequency 5 kHz and a bandwidth of about 2 kHz. This intensity is also extrapolated to the inner zone ($L \leq 1.7$) equatorial plane by considering an enhancement of 15 dB. This yields an intensity of $2.14 \times 10^{-10}\ \gamma^2\ \mathrm{Hz}^{-1}$ there, which in turn, gives a wave field of 0.66 mγ

Here a question arises. Why do we consider a three times higher enhancement for 5 kHz waves as compared to 550 Hz waves? The answer lies in three basic facts, viz.: (i) the most probable generation region of ELF/VLF whistler-mode waves is the equatorial region from where they propagate downwards suffering propagation losses; (ii) the propagation loss is proportional to the wave frequency; and (iii) the ground observed waves of 5 kHz suffer an additional propagation loss of about 5 dB in the altitude range of 50-500 km of the ionosphere[19].

4 Results and discussion

Calculations have been carried out at different L values in the range 1.2 – 1.7 in the inner belt equatorial plane. The values of K-parameter calculated from Eq. (3) for different frequencies of 550 Hz, 1 kHz and 5 kHz at the equatorial location of $L = 1.7$ are plotted in Fig. 1 as a function of pitch angle α. Figure 1 shows that the calculated K-values for plasmaspheric hiss (f = 550 Hz and 1 kHz) are higher than those for the VLF hiss (f = 5 kHz). For a particular wave frequency, K-parameter shows higher values for lower values of α, a minimum around $\alpha \cong 60°$ and an increasing trend with further increase of α reaching infinite value at $\alpha = 90°$. At $\alpha = 90°$, nearly a common tangent to the curves (dotted line in Fig. 1) can be drawn. Thus an asymptote is obtained at $\alpha = 90°$ which precludes the possibility of wave-particle interaction at $\alpha \to 90°$. However, $K \gg 1$ in all the cases, thereby indicating that small energy transfer may induce considerable pitch angle changes and even a weak wave may produce significant pitch angle perturbations at lower L-shells in the inner radiation zone. Further, for a given wave frequency, the calculated K-value increases with decreasing L-value showing that the pitch angle scattering and

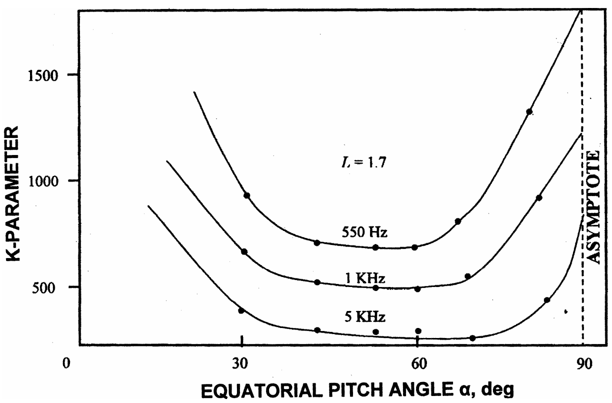

Fig. 1 — Plots showing the variation of K-parameter with equatorial pitch angle α at L=1.7 for the wave frequencies of 550 Hz, 1 kHz and 5 kHz

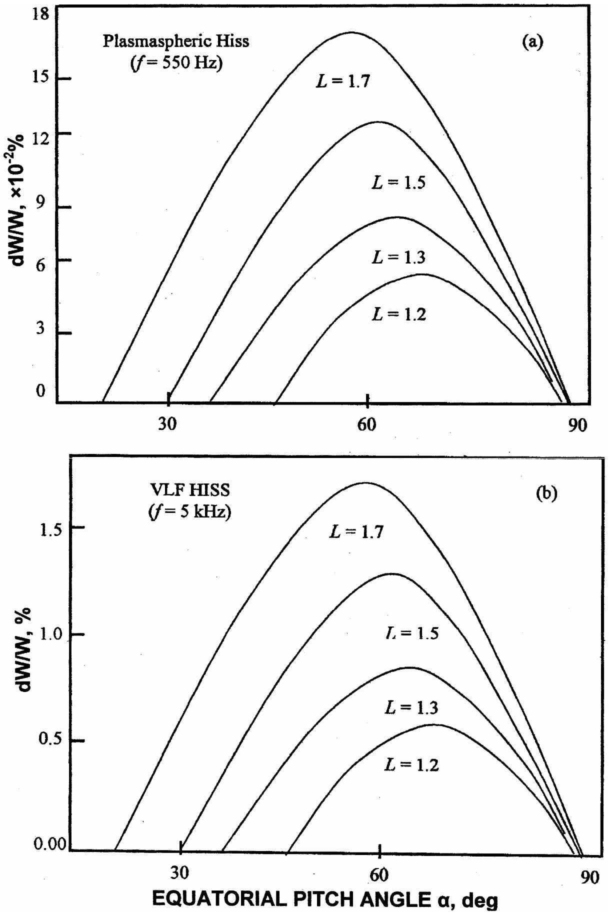

Fig. 2 — Plots showing the variation of the percentage energy change with equatorial pitch angle α at L = 1.2, 1.3, 1.5 and 1.7 for: (a) plasmaspheric hiss; and (b) VLF hiss

hence, diffusion must be higher towards lower L-shells.

The energy change is calculated using Eq. (3) in the form

$$\frac{dW}{W} = (d\alpha/\alpha) / K \qquad (7)$$

in which $d\alpha/\alpha\cdot$[α_o being the loss cone angle given by $\alpha_o = \sin^{-1} (B_o/B_{100})$, where B_o and B_{100} are the geomagnetic field values at the equator and at an attitude of 100 km along the same field line]. The percentage energy change is obtained by multiplying dW/W by 100. Figure 2 presents the percentage energy changes required for energetic electron precipitation by plasmaspheric hiss [Fig. 2(a)] and VLF hiss [Fig. 2(b)] for the L-values between 1.2 and 1.7. Figure 2 clearly indicates that the plasmaspheric hiss can induce significant precipitation of electrons by causing small changes in their energy, whereas in the case of VLF hiss nearly ten times higher energy changes are required for similar precipitation. The electrons, assuming pitch angles $\alpha \le \alpha_o$, easily fall into the loss cone without requiring any energy change in both the cases. The electrons lose maximum energy in their precipitation when their pitch angles are in the range ≈ 60-70°. The values of (dW/W) are not shown for α=90° because the wave-particle interaction is precluded at this value of α (ref. 15).

It is quite interesting to note that the curves presented in Fig. 2 are similar to the energy distribution curves for black body radiation. The value of minimum pitch angle, α_m, corresponding to the peak energy change decreases with L-value in the same fashion as the minimum wave length λ_m corresponding to peak energy emission decreases with temperature T in case of the black body radiation. The required energy changes for energetic electron precipitation are greater at higher L-values than those required at the lower L-values. For example, the calculated value of energy change at L=1.7 for the plasmaspheric hiss at 550 Hz for α=60° is found to be 0.18% only, which is considerably higher than that found at L = 1.2 (0.025% only). Similarly, the values of calculated energy changes in the case of VLF hiss (5 kHz) have been found to be 1.64% (L=1.7) and 0.22% (L=1.2), respectively. This implies that the energetic electrons easily fall into the loss cone to be precipitated into the lower region of the ionosphere due to pitch angle and energy changes induced by the plasmaspheric hiss towards the lower edge ($L \simeq 1.2$) of the inner radiation belt[20,21]. Thus, diffusion and precipitation rate should be higher towards the lower edge of the inner belt. This may be probably due to decreasing narrowness of the loss cone towards lower L-values.

Next, the bounce–averaged diffusion coefficients are calculated for both the plasmaspheric hiss and VLF hiss at 550 Hz and 5 kHz, respectively at different L-values using Eq. (4), wave-magnetic field intensities and the method followed by Prakash *et al.*[13] The pitch angles of the resonant electrons are considered to be outside the loss cone, i.e. those having values greater than α_o, which has the values 45.1°, 37.5°, 28.1° and 22.3° at L-values 1.2, 1.3, 1.5 and 1.7, respectively. The various parameters used in the present calculations are shown in Table 1.

The electron number densities (n_o) shown in Table 1 correspond to the low latitude diffusive equilibrium model used earlier by a number of researchers[13,22,23]. The values of n_o used in the present calculations are similar to those used by Sonwalker *et al.*[24] in their ray tracing simulation studies at lower L-shells ($L \leq 1.7$). The values of angular electron gyrofrequency (Ω_e) and the angular plasma frequency (ω_p) have been determined by the following relations:

$$\frac{\Omega_e}{2\pi} = 873.6x10^3 / L^3 \tag{8a}$$

$$\frac{\omega_p}{2\pi} = 8.98\, n_0^{1/2} \tag{8b}$$

For the sake of brevity in the paper, $D(\alpha)$ values are not shown graphically or in tabular form. Instead, the ratio R_w [as defined in Eq. (5)] of D (α) values are plotted for the plasmaspheric hiss and VLF hiss at different L-values in the inner zone. Figure 3 shows these plots as a function of pitch angle α, which clearly depicts that R_w has positive values in all cases from 22.5 to 160. So, it is inferred that plasmaspheric hiss is more effective than VLF hiss in the inner zone energetic electron precipitation. The ratio R_w is roughly constant upto $\alpha \approx 70°$ and beyond this R_w decreases rapidly acquiring a value as low as ~ 23 at 90°. This indicates that the electrons are stably trapped at pitch angles close to 90°. Further, as indicated in Fig. 3, R_w is found to increase with decreasing L-value and acquire a maximum value at the inner edge ($L \simeq 1.2$) of the inner radiation belt. This hints that the plasmaspheric hiss is rather more effective in energetic electron precipitation towards the inner edge ($L \simeq 1.2$) of inner radiation belt.

Table 1 — Different parameters used in D (α) calculations

L-value	Equatorial electron density n_o, m^{-3}	Angular plasma frequency ω_p, rad s^{-1}	Angular electro gyrofrequency Ω_e, rad s^{-1}
1.2	$8.13x10^9$	$5.0875x10^6$	$3.1765x10^6$
1.3	$4.61x10^9$	$3.8310x10^6$	$2.4984x10^6$
1.5	$2.71x10^9$	$2.9372x10^6$	$1.6264x10^6$
1.7	$1.98x10^9$	$2.5107x10^6$	$1.1172x10^6$

Figure 4 presents the variation of T_R, ratio of the life-times of energetic electrons resonantly interacting with the plasmaspheric hiss and VLF hiss respectively, [as defined in Eq. (6)] with equatorial pitch angle α. For $\alpha \leq 70°$, T_R is positive but small (lower than 10^{-2}), which further confirms that the plasmaspheric hiss is more effective than VLF hiss in inner belt energetic electron precipitation. T_R decreases as L-value decreases, reaching a minimum at the inner edge ($L \simeq 1.2$), which is in agreement with the findings of Pinto & Gonzalez[20] and Prakash *et al.*[21] that the plasmaspheric hiss shows its maximum effectiveness in the energetic electron

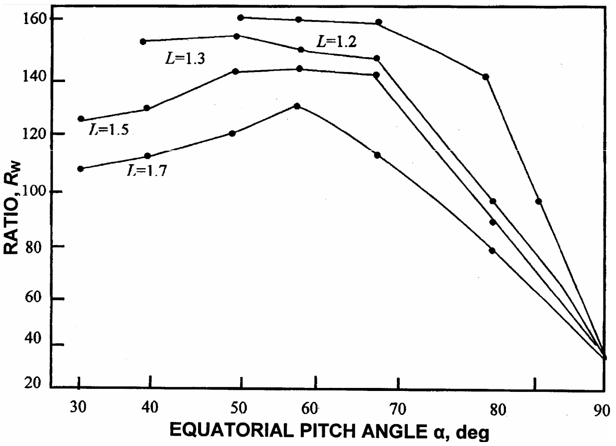

Fig. 3 — Plots showing the variation of the ratio R_w with pitch angle α at L = 1.2, 1.3, 1.5 and 1.7

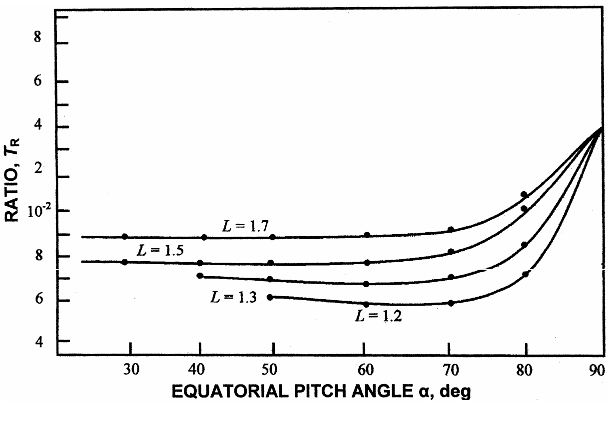

Fig. 4 — Plots showing the variation of the ratio T_R with pitch angle α at L = 1.2, 1.3, 1.5 and 1.7

precipitation at the inner edge ($L \simeq 1.2$) of inner radiation belt. Further, T_R increases rapidly beyond $\alpha \approx 70°$ acquiring a value 4.4×10^{-2} at $\alpha=90°$ showing that near $\alpha \approx 90°$, whistler mode waves are not effective in electron precipitation and hence the electrons are stably trapped near $\alpha=90°$.

It may be pointed out here that in the inner belt around $L \leq 1.2$, the energetic electrons are mostly of MeV range. During quiet periods, precipitation of such electrons by ELF and VLF waves may be quite difficult and coulomb collision may be a dominant process in the precipitation of electrons. But during storm and sub-storm periods, the lower inner belt region ($1.08 \leq L \leq 1.2$) is intensified with ELF hiss (so called inner zone hiss) emissions[9,25] and precipitation of ~ MeV electron by such waves (and VLF waves too) is possible[9,26,27]. Indian and Japanese researchers have observed incoherent VLF his emissions which can also cause loss of trapped electrons[6,26]. Further, the loss cone angle at $L = 1.2$ is 45.1° which is not small and may help in the loss process of ~ MeV electrons.

5 Conclusions

Relative effectiveness of plasmaspheric hiss and VLF hiss in the inner belt energetic electron precipitation has been evaluated. In the light of the above results, it is concluded that

- The plasmaspheric hiss can induce significant precipitation of electrons by causing small changes in their energy during gyro-resonance interaction, whereas nearly ten times higher energy changes are required in case of VLF hiss.
- The minimum pitch angle corresponding to peak energy change is found to vary with L-value in the same fashion as the minimum wave length corresponding to peak energy emission in case of a black body radiation.
- The plasmaspheric hiss emerges to be rather more effective in energetic electron precipitation towards the lower edge of inner radiation belt.
- Near $\alpha=90°$, the whistler-mode waves are not effective in electron precipitation and hence the electrons are stably trapped.

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