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MHD free convective flow through a porous medium between two vertical parallel plates

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Free convective flow of a viscous, incompressible, electrically conducting fluid through a porous medium between two vertical parallel plates which are heated or cooled uniformly, under a pressure gradient in presence of uniform transverse magnetic field has been studied. The governing equations are solved to obtain approximate analytical solutions for velocity field and temperature distribution for small values of Rayleigh number, using regular perturbation technique. Two physical situations, namely, steady heating of ascending cold fluid (SHACF) and steady cooling of ascending hot fluid (SCAHF) have been investigated. The effect of various parameters on velocity field, temperature distribution, mass flow rate ratio, friction factor ratio and heat transfer rate has been discussed.

1 Introduction

The flow of fluids through porous medium in channels open at both the ends is a common occurrence in both geophysical and industrial environments. In addition, such flows are of common interest due to their engineering applications such as, cooling of electric equipment, thermal energy storage system, solar collector with a porous absorber and so on. In such flows, the variation of temperature governs the fluid motion. This type of fluid motion under the action of uniform magnetic field has applications in astrophysical, geophysical and engineering problems such as, cores of nuclear reactors, power transformation structures for high power density electric machines, etc.

Lehnert[1] considered natural and mixed convection flow of a viscous incompressible fluid between two non-conducting vertical plates. Chang & Yen[2] and Yen & Chang[3] discussed the effects of electrically conducting walls on the velocity field in a channel and the Couette flow, respectively. Heat transfer aspect of the channel flow under the action of transverse magnetic field was presented by Soundalgekar[4]. Soundalgekar & Dhavale[5] also discussed fully developed steady flow between two slightly, electrically conducting walls under the influence of uniform transverse magnetic field. The free convection in MHD flow through a vertical channel, under different physical situations, has also been studied by several authors[6-11]. Recently, Singh[12] has discussed MHD effects on convective flow of a viscous fluid between two electrically conducting, parallel plates.

In the present investigation, the author proposes to study the fully developed free convective MHD flow of an incompressible, viscous, electrically conducting fluid between two infinite, vertical, parallel porous plates for two physical situations namely, SHACF and SCAHF. Approximate solutions for velocity field, temperature distribution and other results obtained in the study have been discussed numerically.

2. Mathematical Formulation

Consider laminar convective flow of incompressible, electrically conducting, viscous fluid through a vertical channel consisting of two parallel porous plates open at both the ends. Let x'-axis be taken along vertically upward direction through the central line of the channel and y'-axis perpendicular to the x'-axis and passing through lower end of channel. The plates of the channel are at $y' = \pm h$. A uniform magnetic field B_0 is applied parallel to y'-axis. In order to derive the governing equations of the flow problem, the following assumptions are made :

(i) The walls of the channel are maintained at a uniform temperature gradient $\left(\frac{\Gamma}{h}\right)$ in a vertical direction; (ii) Wall temperature T'_w is given by

$T_w' = T_0' + \left(\frac{\Gamma}{h}\right)x'$, where x' is the distance in vertical direction, T_0' is the temperature of the wall at the origin and h is the half width of the channel; (iii) Flow field and temperature field are symmetrical about the central line of the channel at $y' = 0$; (iv) Flow is steady and fully developed and velocity is a function of y' only; (v) Influence of variation in density is included in the body force term; (vi) Viscosity, thermal conductivity and specific heat are independent of temperature; (vii) Porous medium is homogeneous and isotropic and permeability is constant; (viii) Convective fluid and porous medium are in local thermodynamic equilibrium everywhere, in the flow region; (ix) Hall effect, polarization effect and induced magnetic field are negligible and; (x) For positive Rayleigh number, pressure and buoyancy oppose each-other whereas for negative Rayleigh number, they reinforce each-other.

Under the above assumptions, the dimensionless form of momentum and energy equations are:

$$\frac{d^2u}{dy^2} - M_1^2 u = -P + R_a T \qquad \ldots(1)$$

$$\frac{d^2T}{dy^2} = -u \qquad \ldots(2)$$

The relevant boundary conditions in non-dimensional form are:

$$\frac{du}{dy} = 0,\ \frac{dT}{dy} = 0 \quad \text{at} \quad y = 0$$

$$u = 0,\ \ T = 0 \quad \text{at} \quad y = \pm 1 \qquad \ldots(3)$$

The non-dimensional quantities introduced in obtaining above equations and boundary conditions are defined as:

$$x = \frac{x'}{h},\quad y = \frac{y'}{h},\quad u = \frac{u'h}{\lambda},\quad T = \frac{T_w' - T_0'}{\Gamma},$$

$$M_1^2 = M^2 + K^2,$$

$P = \frac{-h^3}{\lambda\vartheta}\left|\frac{1}{\rho_0}\frac{dp}{dx} + g\right|$ (dimensionless pressure gradient), $K^2 = \frac{h}{\sqrt{k}}$ (permeability parameter)

$R_a = \frac{g\beta\, a^3\Gamma}{\lambda\vartheta}$ (Rayleigh number) and $M = B_0 h\sqrt{\frac{\rho}{\sigma}}$ (magnetic parameter)

where k is the permeability of the porous medium, σ is the electrical conductivity, ρ is the density of the fluid, Rayleigh number (R_a) is positive when $T_w'>T_0'$ (steady heating of ascending cold fluid) and Rayleigh number (R_a) is negative when $T_w'<T_0'$ (steady cooling of ascending hot fluid) and the other symbols have their usual meaning.

3 Solution of the Problem

For small values of Rayleigh number, (R_a) the velocity field (u) and temperature distribution (T) can be expressed as:

$$u = u_0 + R_a u_1 + O(R_a)^2$$

$$T = T_0 + R_a T_1 + O(R_a)^2 \qquad \ldots(4)$$

Substituting Eq. (4) in Eqs (1) and (2), the coupled equations in, u_0, u_1, T_0 and T_1 have been obtained. The solutions of these coupled equations under corresponding boundary conditions, after substituting in Eq. (4), are:

$$u = \frac{P}{M_1^2}\left(1 - \frac{\cosh\ M_1 y}{\cosh\ M_1}\right) + \frac{R_a P}{M_1^4}\left[\frac{2}{M_1^2} + \frac{1}{2}(y^2 - 1) - \frac{2\cosh\ M_1 y}{M_1^2 \cosh\ M_1}\right] \qquad \ldots(5)$$

$$T = \frac{P}{M_1^2}\left[\frac{1}{2}(1 - y^2) - \frac{1}{M_1^2}\left(1 - \frac{\cosh\ M_1 y}{\cosh\ M_1}\right)\right] + \frac{R_a P}{M_1^4}\left[\ \frac{1}{24}\left(1 - y^4\right) - \frac{1}{4}\left(1 - y^2\right) + \frac{1}{M_1^2}\left(1 - y^2\right) - \frac{2}{M_1^4}\left(1 - \frac{\cosh M_1 y}{\cosh M_1}\right)\ \right] \qquad \ldots(6)$$

If M_p denotes mass flow rate per unit width of the channel in presence of uniform transverse magnetic field and porous medium and M_a is the mass flow rate in absence of magnetic field and porous medium. Following Chandrasekhara & Narayanan[7], the mass flow rate ratio $\frac{M_p}{M_a}$ is given by:

Table 1 —Temperature distribution for various values of M (P=10.0 and K=10.0)

	SHACF (R_a =5.0)			SCAHF (R_a = −5.0)		
y/T	M=1.0	M=1.5	M=2.0	M=1.0	M=1.5	M=2.0
0.0	0.047551	0.046993	0.046233	0.049498	0.048894	0.048072
0.2	0.045818	0.045083	0.044354	0.047472	0.046892	0.046105
0.4	0.039816	0.039351	0.038715	0.041397	0.040803	0.040209
0.6	0.030144	0.029793	0.029316	0.031297	0.030919	0.030405
0.8	0.016669	0.016477	0.016216	0.017277	0.017071	0.016789
1.0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000

Table 2 —Temperature distribution for various values of K (P=10.0 and M=1.0)

	SHACF (R_a =5.0)			SCAHF (R_a = −5.0)		
y/T	K=10.0	K=25.0	K=50.0	K=10.0	K=25.0	K=50.0
0.0	0.047551	0.007935	0.001996	0.049498	0.007988	0.001999
0.2	0.045618	0.007617	0.001916	0.047472	0.007667	0.001819
0.4	0.039816	0.006662	0.001676	0.041397	0.006705	0.001679
0.6	0.030144	0.005071	0.001277	0.031297	0.005102	0.001279
0.8	0.016689	0.002842	0.000718	0.017277	0.002858	0.000719
1.0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000

Table 3 —Temperature distribution for various values of P (K=10.0 and M=1.0)

	SHACF (R_a =5.0)			SCAHF (R_a = −5.0)		
y/T	P=1.0	P=5.0	P=10.0	P=1.0	P=5.0	P=10.0
0.0	0.004755	0.023776	0.047551	0.004951	0.024749	0.049498
0.2	0.004582	0.022809	0.045618	0.004747	0.023738	0.047472
0.4	0.003982	0.019908	0.039816	0.004141	0.020698	0.041397
0.6	0.003014	0.015072	0.030144	0.003131	0.015648	0.031297
0.8	0.001667	0.008334	0.016669	0.001728	0.008638	0.017277
1.0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000

Table 4 —Temperature distribution for various values of R_a (M=1.0, K=10.0, P=1.0)

	SHACF			SCAHF		
y/T	R_a=5.0	R_a =10.0	R_a =15.0	R_a =−5.0	R_a =−10.0	R_a = −15.0
0.0	0.047551	0.038789	0.029053	0.049498	0.050472	0.051445
0.2	0.045818	0.037277	0.028008	0.047472	0.048398	0.049325
0.4	0.039816	0.032701	0.024796	0.041397	0.042187	0.04297[illegible]
0.6	0.030144	0.024957	0.019193	0.031297	0.031873	0.0324[illegible]
0.8	0.016669	0.013934	0.010894	0.017277	0.017581	0.0175[illegible]
1.0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000

$$\frac{M_p}{M_o} = \frac{\frac{1}{M_1^2}\left(1-\frac{\tanh M_1}{M_1}\right)+\frac{R_a}{M_1^4}\left(\frac{2}{M_1^2}-\frac{1}{3}-\frac{2\tanh M_1}{M_1^3}\right)}{\frac{1}{3}+R_a\left(-\frac{272}{5040}\right)} \quad \ldots(7)$$

The non-dimensional friction factor ratio τ/τ following Morton[13] is obtained as

$$\frac{\tau_p}{\tau_o} = \frac{-M_1^5\left(\frac{1}{3}-\frac{17}{315}R_a\right)}{M_1^4 \tanh M_1 + R_a(2\tanh M_1 - [illegible])} \quad \ldots(8)$$

where τ_p is the friction factor in presence of uniform transverse magnetic field and τ_a porous medium and is the friction factor in absence of magnetic field and porous medium.

Table 5 — Effects of M, K and R on mass flow rate ratio, friction factor ratio and heat transfer rate in SHACF

M	K	R_a	M_p/M_a	τ_p/τ_a	N_u
1.0	10.0	5.0	0.13799	−0.64061	−2.78142
1.5	10.0	5.0	0.13642	−0.64452	−2.78236
2.0	10.0	5.0	0.13429	−0.64996	−2.78367
1.0	15.0	5.0	0.06456	−0.95571	−2.83862
1.0	20.0	5.0	0.03716	−1.27214	−2.87246
1.0	10.0	10.0	−0.04171	2.09028	−2.78733
1.0	10.0	15.0	−0.01775	4.84299	−2.79348

Table 6 — Effects of M, K and R_a on mass flow rate ratio, friction factor ratio and heat transfer rate in SCAHF

M	K	R_a	M_p/M_a	τ_p/τ_a	N_u
1.0	10.0	−5.0	0.015038	−6.03801	−2.77027
1.5	10.0	−5.0	0.014881	−6.07566	−2.77131
2.0	10.0	−5.0	0.014621	−6.12797	−2.77275
1.0	15.0	−5.0	0.006901	−9.05615	−2.83261
1.0	20.0	−5.0	0.003945	−12.07181	−2.86874
1.0	10.0	−10.0	0.010567	−8.70501	−2.76510
1.0	10.0	−15.0	0.008207	−11.35121	−2.75993

Following Morton[13], the dimensionless heat transfer rate in terms of Nusselt number (N_u) is:

$$N_u = \frac{\left.\frac{dT}{dy}\right|_{y=1}}{\int_0^1 T dy} = \frac{\left(1-\frac{\tanh M_1}{M_1}\right)+\frac{R_1}{M_1^2}\left[\frac{2}{M_1^2}\left(1-\frac{\tanh M_1}{M_1^3}\right)-\frac{1}{3}\right]}{\frac{1}{M_1^2}\left(1-\frac{\tanh M_1}{M_1}\right)-\frac{1}{3}+\frac{R_a}{M_1^2}\left[\frac{1}{M_1^4}\left(1-\frac{\tanh M_1}{M_1}\right)-\frac{2}{3}\left(\frac{1}{M_1^2}-\frac{1}{5}\right)\right]} \quad \ldots(9)$$

4 Discussion

In order to get the physical insight into the problem, the velocity (u), temperature distribution (T), mass flow rate ratio (M_p/M_a) skin-friction ratio (τ_p/τ_a) and heat transfer rate in terms of Nusselt number (N_u) obtained in Eqs (5)-(9) have been discussed numerically for various parameters encountered into the problem. To be realistic, the numerical values of the parameters are chosen following Morton[13] and Chandrasekhara & Narayanan[7]. The discussion has been made for two physical situations namely steady heating of the ascending cold fluid (SHACF) and steady cooling of the ascending hot fluid (SCAHF). Due to symmetry, the values of y have been chosen from y = 0.0 to y = 1.0 throughout the discussion.

The effects of dimensionless magnetic parameter (M), permeability parameter (K), Rayleigh number (R_a) and pressure (P) on velocity field shown in Eq. (5) have been numerically represented in Figs 1 to 4, respectively for SHACF and SCAHF. From Fig. 1 it is observed that, the velocity in SHACF is less than the velocity in SCAHF for chosen value of the parameters and that an increase in M reduces the velocity field in SHACF and SCAHF. It is also notable that as y increases from y = 0.0 to y = 0.5, the nature of velocity is somewhat stationary and thereafter decreases. The velocity decreases very rapidly from y = 0.8 to y = 1.0. Fig. 2 represents that, the effect of K on velocity field is similar to that of M in both SHACF and SCAHF. But, in SCAHF the velocity decreases much rapidly in comparison to SHACF. This implies that, K may be used as a measure to reduce the velocity, which can be an important application to engineering problems. The effect of Rayleigh number (R_a) on velocity field in SHACF and SCAHF is shown in Fig. 3. Obviously, as R_a increases the physical situation SCAHF changes to SHACF and the velocity decreases very rapidly at y = 0.0 i.e. the central line of the channel and reverse flow is noted when R_a = 10.0. This implies that, R_a can be an important measure to reduce the velocity at the central line of the channel or to get reverse flow needed in astrophysical problems. The effect of P is represented in Fig. 4. It is observed that, an increase in P results in an increase in velocity in SHACF while reverse effect is observed in SCAHF.

The effects of dimensionless magnetic parameter (M), porosity parameter (K), pressure (P) and Rayleigh number (R_a) on temperature distribution for SHACF (R_a = 5.0) and SCAHF (R_a = −5.0) have been numerically represented in Tables 1 to 4, respectively, for chosen values of the parameters encountered in Eq. (6). It is observed from Tables 1 and 2 that, an increase in M or K results in a decrease in temperature distribution in SHACF and SCAHF for a given value of y and viceversa. However, the distribution of temperature in SCAHF is greater than in SHACF for all values of y and M. An increase in pressure, increases the temperature distribution in SHACF and SCAHF (Table 3). However, the temperature distribution is more in SCAHF in comparison to SHACF. The effect of R_a is shown in Table 4. It is observed that an increase in R_a decreases temperature distribution in SHACF while increases in SCAHF.

The effects of M, K and R_a on mass flow rate ratio (M_r/M_a), friction factor ratio (τ_r/τ_a) and heat transfer rate (N_u) for M = 1.0, 1.5, 2.0 and K = 10.0, 15.0, 20.0 in SHACF and SCAHF are shown in Tables 5 and 6, respectively. An increase in M or K or R_a reduces mass flow rate ratio (M_r/M_a), skin-friction ratio (τ_r/τ_a) and heat transfer rate, in terms of Nusselt number (N_u) in both the situations SHACF and SCAHF (Tables 5 and 6).

5 Conclusions

The conclusions of the present study are as follows:

(i) An increase in M or K results in a decrease in velocity field in SHACF and SCAHF; (ii) as R_a increases from $R_a < 0$ to $R_a > 0$, the physical situation SCAHF changes to physical situation SHACF and the velocity decreases very rapidly so that, reverse flow is observed; (iii) an increase in P results in an increase in velocity field in SHACF while reverse effect is observed in SCAHF; (iv) an increase in M or K leads to a decrease in temperature distribution in SHACF and SCAHF, while reverse effect is observed for an increase in P; (v) an increase in R_a increases temperature distribution in SCAHF while reverse effect is observed in SHACF while reverse effect is observed in SHACF; (vi) an increase in M or K or R_a results in a decrease in M_r/M_a, τ_r/τ_a, and N_u for SCAHF and SHACF.

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