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Optimal preventive maintenance model of complex degraded systems: A real life case study

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This study presents a comparison of two models to find optimum maintenance policy of complex degraded systems. Using Markov processes, optimal value of mean time to preventive maintenance is determined by maximizing availability of complete systems. Models include a stochastic degradation process, random failures and a set of maintenance actions and their effects. A real life case study is presented to illustrate implementation of models. A two-component transportation system for collecting waste utilized by Public utility company, Mediana Niš, Serbia, is analyzed.

Keywords: Markov processes, Preventive maintenance, Transport system

Introduction

Increase of system reliability and availability, as well as maintenance cost reduction, present priority tasks of modern engineering system maintenance[1]. Optimal maintenance policies are reported[2,3] for repairable components. Problem of replacement or overhaul of single-unit systems (SUSs), which degrade with usage, is one of the standard applications of Markov processes. Sim & Endreny[4] obtained optimal value of mean time to minimal preventive maintenance (PM) by minimizing unavailability of device. In a similar model of multi-state[5], repairing action, after a partial failure, could not bring system to as good as new state, but could only bring it to same state as before the failure. Chan & Asgarpoor[6] obtained optimal value of mean time to PM by maximizing availability of single components with respect to mean time to minimal PM. In recent years, an increasing interest in multi-component maintenance models has been shown[7-9]. Maintenance of a multi-component system (MCS) is different from that of a SUS because economic and failure dependence exist in MCSs. Each subsystem, as a part of some complex system, may not be considered as a SUS individually, because applying existing optimum maintenance models of a SUS to each of such subsystems may not lead to a global optimal maintenance policy for the system as a whole. Usual criteria on optimization of maintenance policies are based on maintenance cost measures[10] (expect maintenance costs per unit of time, total discounted costs, gain,...) or on reliability measures (availability, average up time, or average down time)[2].

This study presents a model for predicting availability of a multi-state degraded system (MSDS) that comprises two dependent subsystems. Optimal value of mean time to PM is determined by maximizing availability of complete system with respect to mean time to minimal PM of both subsystems.

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Experimental Section

Implementation of developed model is shown on the example of a two-component transportation system for collecting, transport and disposal (CTD) of waste utilized by Public utility company, Mediana Niš. Many of the utility companies in Serbia, and especially those in smaller towns, have a problem with diverse structure of vehicle fleets, as well as with vehicles that are at the end of their life cycle. This fact makes application of a unique maintenance strategy based on the recomendations of manufacturers of Garbage Collection Trucks (GCT) inadequate. Approach used in this study is to consider modeling stochastic networks as multidimensional Markov processes. Typical examples of stochastic networks are[11] computer & telecommunications networks, the Internet, manufacturing networks, logistics and supply-chain networks, and in many other areas (biology, physics, and economics). A similar idea is applied in analysis of multi-component degradation systems.

Model Description

Markov Processes

Three major categories of approaches[12] (age based approach, Markovian approach and optimal stopping approach) are widely used in maintenance modeling. Markovian approach, which includes modeling and optimization with Markov processes, has tremendous modeling power. This study discusses properties of Markov process. Most technical systems are subject to deterioration as a result of usage and age. Many studies proposed that deterioration and maintenance can be illustrated by a sequence of states[13]. If system deteriorating process is defined by discrete system states then a discrete-time Markov process can be represented by a transition probabilities matrix $\mathbf{A}$ and vector of state probabilities $P(t)=[P_i(t)]$ in the form of Chapman-Kolmogorov equation[14] as

$$\frac{dP(t)}{dt}=P(t)\cdot\mathbf{A} \qquad \ldots(1)$$

If $t\rightarrow\infty$ or if system works for a long time, then probabilities $P_i(t)$ tend to the constant value P_i^*, which, in that case, represents probabilities of stationary system states (steady state probabilities) and do not depend on time. In this case, $dP(t)/dt$ is zero. Steady-state probability of a discrete-time Markov Process can be calculated by solving linear equations as

$$\begin{cases} 0=P\cdot\mathbf{A} \\ \sum_{i=1}^{n} P_i=1 \end{cases} \qquad \ldots(2)$$

Continuous-parameter Markov process has been applied most extensively to model reliability and maintenance problems. Optimal value of mean time to PM can be obtained by maximizing availability of the system[15]. This study uses concept of steady state availability, or long run equipment availability (probability that the system will be in operational state over a stated period of time).

Maintenance Models with Degradation and Full Repair after Random Failure

Corrective maintenance[16,17] (CM) is carried out after system break down and intended to put equipment into a state to perform a required function. It is performed as an unpredictable event in time since exact moment of component failure is unknown. PM is carried out at predetermined intervals or in keeping with prescribed criteria and intended to reduce probability of failure or degradation of equipment function[16,17]. Applying PM activities too frequently certainly maintains desired system reliability and availability but it also creates significant maintenance cost. On the other hand, if PM frequency is too low, that certainly implies lower maintenance cost but also causes reduction in system availability and an increase in system failure occurrence risk. It is of utmost importance to make right decisions in efficiency-safety-availability area to achieve maximum availability with lowest maintenance cost[18]. This study introduces maintenance model of a single component - Model-1 (Fig. 1) and maintenance model with two dependent subsystems - Model-2 (Fig. 2).

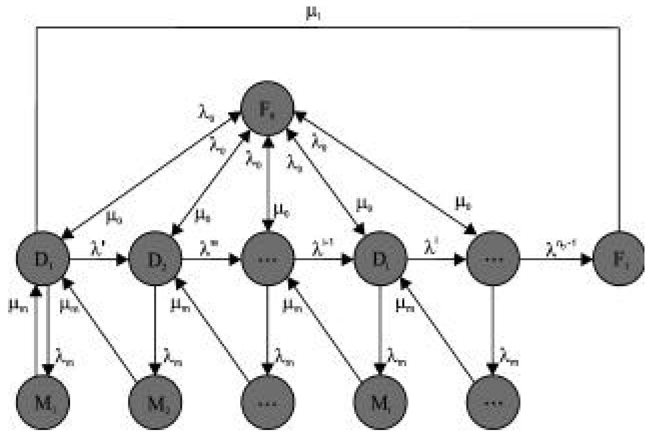

Fig. 1—Maintenance model with degradation and full repair after random failure (Model-1)

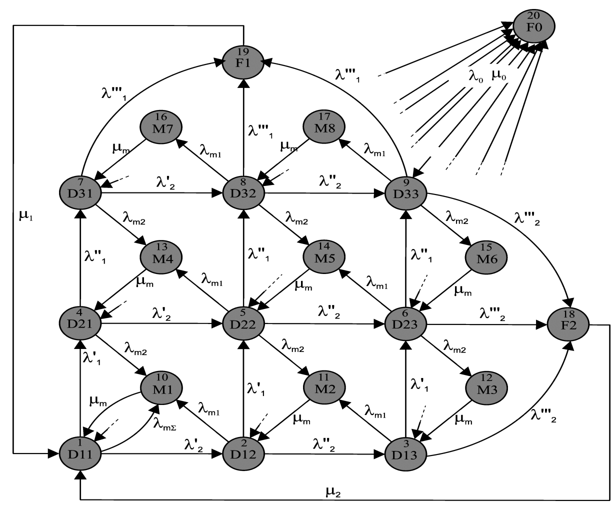

Fig. 2—Maintenance model of a system composed of two components (Model-2)

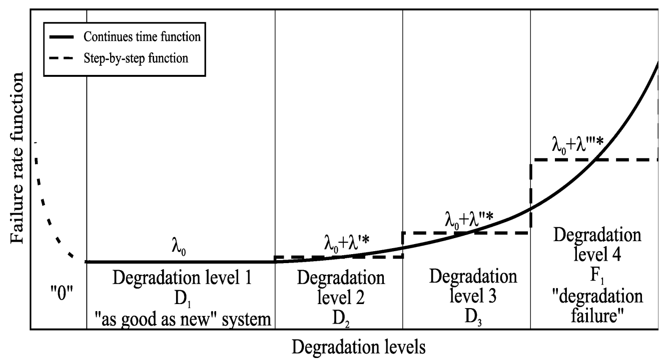

Fig. 3—Degradation levels (states) as areas in continuous time failure rate function

In many systems[5], components might not always fail fully, but can degrade, and there can be multiple states of degradation [D_i ($i=1\div n_D$), where n_D is total number of degradation states]. If degradation is modeled as occurring in a limited number of discrete steps, then minimal PM M_i sets back the process by one step. This improves component from state i to state $i-1$ of degradation. If component is in state one of degradation, it remains in that state on completion of minimal PM. In this model[4,6], both random failure F_0 and failure due to degradation F_1 can occur. Failure-maintenance model (Fig. 1) is state space model, not Markov model, as no assumptions made on time-distributions of individual transitions.

This study presents a model for predicting availability of MSDSs with two dependent subsystems. Assume that a system is subject to a variety of three-failure processes, in which two are degradation processes and third is a random failure process. Similar to SCM, system is considered to be initially in a good state $D_{1,1}$ (Fig. 2). As time passes, it can either go to first degraded state ($D_{1,2}$ or $D_{2,1}$) upon degradation of a certain subsystem, go to minimal PM state (M_1) that brings back subsystem in as good as new state ($D_{1,1}$), or it can go to a random failed state (F_0) subject to random failure. When subsystem 1 reaches first degraded state $D_{1,2}$, system can remain in that state, go to second degraded state ($D_{1,3}$) upon degradation of subsystem 1, go to first degraded state ($D_{2,2}$) upon degradation of subsystem2, go to minimal PM state (M_1) that improves subsystem1 from state $D_{1,2}$ to state $D_{1,1}$ of degradation, or it can go to a random failed state (F_0). Same process will continue for all states of degradation except the last state ($D_{1,3}$). If system reaches last degraded state, its performance is considered to have reached an unacceptable limit, and treated as a degradation failure (F_1). Same principle is applied for subsystem 2.

Model Parameters

Because of restriction of Markov theory, it is assumed that duration of each state of degradation as well as times for repairing a failed subsystem and maintenance time are exponentially distributed. Thus models with time-independent transition rates will not be realistic approaches for maintenance planning[19]. To overcome these restrictions, continuous time dependent failure rate function (bath tub curve) can be substituted by discrete step-by-step function only for the purposes of easier modeling[20] (Fig. 3). Also, degradation is a continuous process in time, and generally, it can be modeled by a continuous probabilistic function. However, in practice, description of the system operating (technical) condition is accomplished through a finite number of system states, and hence, continuous degradation process is simplified by dividing it into a number of different discrete levels[9,21]. Therefore, a degradation level is introduced as a system state with relevant technical conditions at similar level of operating ability. In a specific degradation level, system failure rate is assumed to have a constant value. On the other hand, a discrete state space is a necessary condition in modeling by means of discrete-state Markov process, which adequately characterizes process of system deterioration[22], an approach already reported[4,6,13,23].

Number of degradation levels (degradation levels in the function of intensity of failures correspond to degradation states in the model) may vary and can be selected from performance data or by judgment based on experience. In this study, sojourn times (mean times between states) are reciprocal values of failure rate in a specific level. There are 4 degradation levels (Fig. 3) corresponding to 4 different failure rate areas. First area (area 0) represents burn-in period or infant mortality, which is attributable to that equipment manufacture occurs either because manufacturer's quality standards are too loose, or because parts concerned have been badly installed[1]. These early problems can only be eliminated by testing them out during manufacturing. Second area (degradation level 1) represents useful life period or state of system, which is as good as new (Fig. 2, state $D_{1,1}$). Characteristic of this period is occurrence of only random failures with failure rate l_0.

Third and fourth area (degradation levels 2 and 3) represent periods, in which failure rate function have an increasing trend. Failure rates for each area are l''^{*}, l'''^{*} respectively. Final area (degradation level 4) represents state of the system when failure occurs F_1. A strong increase in failure rate function approximated with value l''''^{*} is characteristic for this area. In proposed model, failure rate values l''^{*}, l'''^{*} and l''''^{*} represent transition probabilities between specific degradation states. From each degradation state D_i $(i = 1 \div 3)$, system can go to random failure state F_0 with transition probability (random failure rate) l_0 (Fig. 2). Regardless of the number of degradation states in models (n_D), following condition must be met[21]:

$$1/\,l' + 1/\,l'' + \cdots + 1/\,l^{n_D} = MTBF \qquad \ldots(3)$$

Thus, sum of expected sojourn times of system in degradation states, if no maintenance is carried out, must be equal to mean time between failures (MTBF). This means that values l^{i*} $(i = 1 \div n_D)$ obtained by approximation of continuous time dependent failure rate function cannot be directly entered into model, but it is necessary to perform their recalculation in order to satisfy the condition defined by Eq. (3). Transition rates used in the model (l^i) can be determined as

$$l^i = l^{i*} / k_{n_D} \quad \text{where}$$

$$k_{n_D} = MTBF \,/ \left(1/\,l'^{*} + 1/\,l''^{*} + \cdots + 1/\,l^{n_D^{*}}\right) \qquad \ldots(4)$$

Maintenance is modeled as a Poisson process with subsystem 1 l_{m1} and subsystem 2 l_{m2}. Values l_{m1} and l_{m2} represent frequency of transition to maintenance state and that are a parameter whose optimal value must be determined. Also, maintenance times are exponentially distributed with a mean of $1/m_m$. After failure, system can be repaired. Difference between repairs after random failures and repairs after degrading failures is made in the model. Repairs after random failures return system to a previous state and repairs after degrading failures return the system to as good as new state ($D_{1,1}$). Time of repair is exponentially distributed and depends on type of failure. Repairs after failures due to degradation are modeled with means of $1/m_1$ for subsystem 1 and $1/m_2$ for subsystem 2 repairs after random failures with a mean of $1/m_0$. State space diagram with defined transition rates now becomes a Markov maintenance model. State transition matrix for Model-1 of a single unit component (Fig. 1) has a form given in Eq. (5) and for Model-2 of system with two dependent subsystems (Fig. 2) has a form given in Eq. (6) as

$$\mathbf{A}_{single} = \begin{bmatrix} -\Sigma_1 & l' & 0 & l_m & 0 & 0 & 0 & l_0 \\ 0 & -\Sigma_2 & l'' & 0 & l_m & 0 & 0 & l_0 \\ 0 & 0 & -\Sigma_3 & 0 & 0 & l_m & l''' & l_0 \\ m_m & 0 & 0 & -\Sigma_4 & 0 & 0 & 0 & 0 \\ m_m & 0 & 0 & 0 & -\Sigma_5 & 0 & 0 & 0 \\ 0 & m_m & 0 & 0 & 0 & -\Sigma_6 & 0 & 0 \\ m_{1/2} & 0 & 0 & 0 & 0 & 0 & -\Sigma_7 & 0 \\ m_0 & m_0 & m_0 & 0 & 0 & 0 & 0 & -\Sigma_8 \end{bmatrix} \qquad \ldots(5)$$

$$\mathbf{A}_{double} = \begin{bmatrix}
-\Sigma_1 & l_2' & 0 & l_1' & 0 & 0 & 0 & 0 & 0 & l_{m\Sigma} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & l_0 \\
0 & -\Sigma_2 & l_2'' & 0 & l_1' & 0 & 0 & 0 & 0 & l_{m1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & l_0 \\
0 & 0 & -\Sigma_3 & 0 & 0 & l_1' & 0 & 0 & 0 & 0 & l_{m1} & 0 & 0 & 0 & 0 & 0 & 0 & l_2''' & 0 & l_0 \\
0 & 0 & 0 & -\Sigma_4 & l_2' & 0 & l_1'' & 0 & 0 & l_{m2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & l_0 \\
0 & 0 & 0 & 0 & -\Sigma_5 & l_2'' & 0 & l_1'' & 0 & 0 & l_{m2} & 0 & l_{m1} & 0 & 0 & 0 & 0 & 0 & 0 & l_0 \\
0 & 0 & 0 & 0 & 0 & -\Sigma_6 & 0 & 0 & l_1'' & 0 & 0 & l_{m2} & 0 & l_{m1} & 0 & 0 & 0 & l_2''' & 0 & l_0 \\
0 & 0 & 0 & 0 & 0 & 0 & -\Sigma_7 & l_2' & 0 & 0 & 0 & 0 & l_{m2} & 0 & 0 & 0 & 0 & 0 & l_1''' & l_0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -\Sigma_8 & l_2'' & 0 & 0 & 0 & 0 & l_{m2} & 0 & l_{m1} & 0 & 0 & l_1''' & l_0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\Sigma_9 & 0 & 0 & 0 & 0 & 0 & l_{m2} & 0 & l_{m1} & l_2''' & l_1''' & l_0 \\
m_m & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\Sigma_{10} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & m_m & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\Sigma_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & m_m & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\Sigma_{12} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & m_m & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\Sigma_{13} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & m_m & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\Sigma_{14} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & m_m & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\Sigma_{15} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & m_m & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\Sigma_{16} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & m_m & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\Sigma_{17} & 0 & 0 & 0 \\
m_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\Sigma_{18} & 0 & 0 \\
m_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\Sigma_{19} & 0 \\
m_0 & m_0 & m_0 & m_0 & m_0 & m_0 & m_0 & m_0 & m_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\Sigma_{20}
\end{bmatrix} \qquad \ldots(6)$$

where $-\Sigma_i$ represents negative of sum of all the remaining elements in row i, and $l_{m\Sigma}$ represents half of the sum of values l_{m1} and l_{m2} [$l_{m\Sigma} = (l_{m1} + l_{m2})/2$].

State $D_{1,1}$ of Model-2 (Fig. 2) presents a state identical to the state of new system; it is not possible to entirely realistically model exits from that state into maintenance state M_1 for each system separately. If subsystem1 enters maintenance state M_1 with intensity l_{m1}, after returning to state $D_{1,1}$, entire system is in the state identical to state of new system, which is incorrect from the aspect of subsystem 2. Situation is similar in the case of subsystem 2. If subsystem2 enters state $D_{1,1}$ into maintenance state M_1 with intensity l_{m2}, after returning to state $D_{1,1}$, entire system is in the state identical to state of new system, which is an incorrect assumption in the case of subsystem$_1$. This means that intensity of entering maintenance state M_1 cannot be a sum of individual intensities of both subsystems. In order to keep Model-2 as close to real system as possible, assumption that intensity of entering maintenance state M_1 from state $D_{1,1}$ is equal to half of the sum of intensities l_{m1} and l_{m2} is introduced.

Using Matlab software, system of Eq. (2) can be solved. Solution is a steady state probability row vector $P^*_{single} = \left[P_1^* \; P_2^* \; P_3^* \; \ldots \; P_i^* \; \ldots \; P_8^*\right]$ for Model-1 and $P^*_{double} = \left[P_1^* \; P_2^* \; P_3^* \; \ldots \; P_i^* \; \ldots \; P_{20}^*\right]$ for Model-2, where P_i^* is steady state probability that system is in state i and not going to next state. Probability that system is in service (availability) is given as

$$A_v(\lambda_m) = P_1^* + P_2^* + P_3^* \qquad \ldots(7)$$

for Model-1, where P_1^*, P_2^*, P_3^* are probabilities that system is in operational state D_i ($i = 1 \div 3$) over a stated period of time, and

$$A_v(\lambda_{m1}, \lambda_{m2}) = P_1^* + P_2^* + P_3^* + P_4^* + P_5^* + P_6^* + P_7^* + P_8^* + P_9^* \qquad \ldots(8)$$

for Model-2, where $P_1^*, P_2^*, P_3^*, P_4^*, P_5^*, P_6^*, P_7^*, P_8^*, P_9^*$ are probabilities that system is in operational state $D_{i,j}$ ($i = 1 \div 3$ and $j = 1 \div 3$) over a stated period of time.

Optimal value of mean time to PM l_m (l_{m1} & l_{m2}) is calculated by maximizing system availability.

Results and Discussion

A Real Life Case Study

System Description

Function of CTD of waste on city of Niš (Serbia) is performed by Public utility company, Mediana Niš, whose transportation system comprises 22 special purpose vehicles – GCT. The company covers 75,000 locations comprising 250,000 service users. No other vehicle can be used for the performance of defined function aside from those that are found within observed system. From the aspect of system function, city is divided into two groups of areas: i) where waste is collected in bins and ii) where waste is collected in containers. In a similar way, according to the type of superstructure of basic vehicle (working device), division of GCT into two construction-technical-exploitation (CTE) groups is conducted: i) CTE1 – trucks with device for lifting and emptying containers; and ii) CTE2 – trucks without device for lifting, i.e. with manual unloading of bins. In this study, defined CTE groups are observed as separate subsystems, for which optimal values of mean time to PM are calculated by maximizing availability of whole system by applying methodology already defined.

Numerical Results

For both subsystems of system for CTD of waste on city of Niš (Serbia), by using data from database of real information system (Public utility company Mediana – Niš, Serbia), continuous time dependent failure rate functions (bath tub curves) and their approximations can be determined in the form of discrete step-by-step functions (Fig. 4). Degradation and random failures were recorded from January 1, 2005 to December 31, 2009. In total, 1442 failures were observed. Continuous time dependent failure rate function was defined by Weibull distribution W1(2.156; 185.843) for Subsystem 1 (KTE1) and W2(1.871; 105.178) for Subsystem 2 (KTE2). If system was to be observed in its entirety (all GCT), failure intensity function would be W(1.719; 152.641). In the case of a model with 4 degradation levels $n_D = 4$, which corresponds to GCT maintenance organization in Public utility company, Mediana Niš, approximations of continuous time Weibull distributions for both subsystems are shown (Fig. 4). By dividing value of step-by-step failure rate functions l^{i*} by coefficient k_{n_D}, values used in model (l^i) are determined. For different values of number of degradation levels ($n_D = 4, 7, 11$), transition rates (l^i) are given (Table 1).

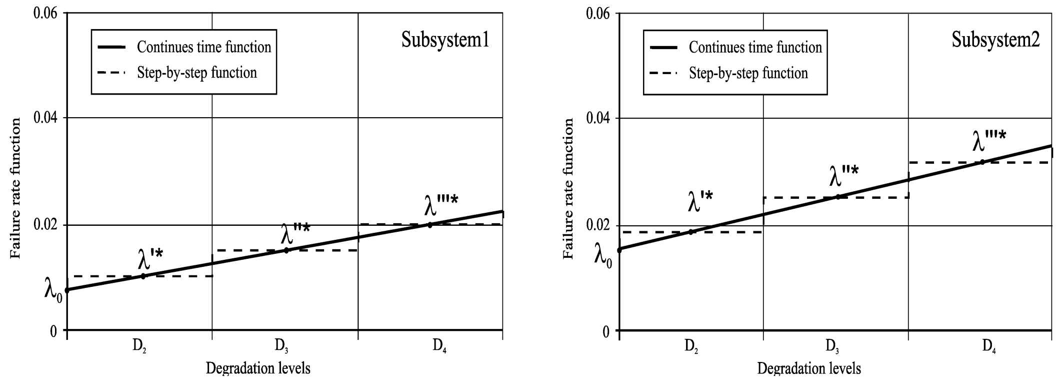

Fig. 4—Failure rate functions for Subsystem 1 (CTE1) and Subsystem 2 (CTE2)

Table 1—Parameters of step-by-step failure rate functions

Parameter	All GCT, day^{-1}			Subsystem 1, day^{-1}			Subsystem 2, day^{-1}		
	Model with 4 degradation levels	Model with 7 degradation levels	Model with 11 degradation levels	Model with 4 degradation levels	Model with 7 degradation levels	Model with 11 degradation levels	Model with 4 degradation levels	Model with 7 degradation levels	Model with 11 degradation levels
MTBF day		136.0930			164.5835			93.3802	
l_0	0.01003	0.01003	0.01003	0.00767	0.00767	0.00767	0.01553	0.01553	0.01553
l'	0.01382	0.01912	0.02238	0.01092	0.01492	0.01742	0.01979	0.02758	0.03194
l''	0.02655	0.03613	0.04394	0.02261	0.02945	0.03520	0.03912	0.05227	0.06333
l'''	0.03838	0.05235	0.06480	0.03478	0.04437	0.05331	0.05782	0.07643	0.09424
l^{iv}		0.06793	0.08504		0.05962	0.07172		0.10015	0.12473
l^{v}		0.08298	0.10476		0.07517	0.09041		0.12349	0.15485
l^{vi}		0.09755	0.12399		0.09099	0.10935		0.14650	0.18462
l^{vii}			0.14280			0.12853			0.21409
l^{viii}			0.16121			0.14793			0.24328
l^{ix}			0.17928			0.16754			0.27220
l^{x}			0.19702			0.18735			0.30089

Table 2—Parameters of minimal maintenance and repairs

Parameter, day^{-1}	All GCT	Subsystem 1	Subsystem 2
m_o	0.26389	0.24138	0.30086
m_1	0.09143	0.08462	-
m_2	-	-	0.10409
m_m	1	1	1

Transition rates (m_0, m_1, m_2 and m_m), which represent reciprocal values of mean repair times (after random failure and after degradation failure) and minimal maintenance, for both subsystems and for all GCT (entire system without division into subsystems) are given (Table 2). Optimization parameters are mean times to minimal maintenance (l_{m1} and l_{m2}), which can take values in interval $l_{m1}^{-1} = 1 \div 1000$ days and $l_{m2}^{-1} = 1 \div 1000$ days with the step of 1 day. By solving system of Eq. (2), for Model-1 ($n_D = 4$), changes of availability (Fig. 5) of both subsystems as individual components (SUS) are obtained. Availability increases quickly, until mean time to minimal PM exceeds its optimal value and it decreases slowly if mean time to minimal PM is greater than optimal value. Also, as value of m_1^{-1} or m_2^{-1} decreases (for 10 and 20%), mean time to minimal PM and availability increases. On the other hand, as value of m_0^{-1} decreases (for 10 and 20%), mean time to minimal

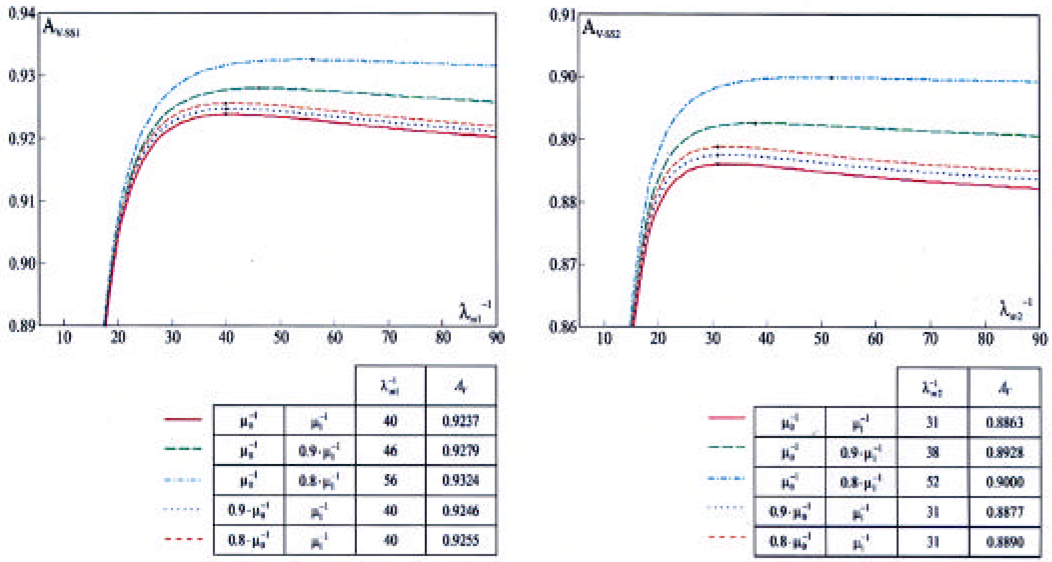

Fig. 5—Availability of both subsystems as single-unit components

Table 3—Optimal values of mean time to minimal maintenance and adequate availability – single-unit system

Number of degradation levels	All GCT		Subsystem 1		Subsystem 2	
n_D	l_m^{-1} day	A_V	l_{m1}^{-1} day	A_V	l_{m2}^{-1} day	A_V
4	37	0.9138	40	0.9237	31	0.8863
5	33	0.9126	36	0.9226	29	0.8845
6	34	0.9096	35	0.9198	34	0.8811
7	40	0.9073	39	0.9175	60	0.8796
9	281	0.9033	75	0.9129	>1000	-
11	>1000	-	>1000	-	>1000	-

PM and availability remain approximately constant. In this way, activities of PM that cannot influence occurrence of random failures is confirmed. Increase in intensity of PM is of importance only in the case of system (component) degradation in the sense of prolonging life cycle (MTBF). In the case of subsystem1, reduction of time spent in state of CM after a degradation failure for 10%, i.e. 20% (10% represents approximately one day) leads to increase of PM interval for 15%, i.e. 40%. In the case of subsystem2, reduction of 10%, i.e. 20% leads to increase of PM interval for 22.58%, i.e. 67.71%. This analysis indicates need to reduce the time, which subsystems are spending in state of CM after a degradation failure, because reduction of only a single day leads to increase in availability and significant decrease of need for PM.

This study analyses changes in availability A_V and mean times to minimal maintenance l_m depending on the number of degradation levels n_D. Changes A_V and l_m for three categories of GCT (all GCT, subsystem 1 and subsystem 2) and for the number of degradation levels ($n_D = 4, 5, 6, 7, 9, 11$) are given (Table 3). Public utility company, Mediana Niš organized maintenance of GCT into 4 age categories. This study investigates not only whether or not this number of degradation levels is chosen correctly, but to point out need to deal more seriously with this problem when number of degradation levels is not known in advance. Main point of interest is to present and analyze a model for predicting availability of multi-state degradation systems with two dependent subsystems. By maximizing Eq. (8), optimal values of mean time to minimal maintenance (l_{m1} and l_{m2}) are determined (Fig. 6a). Furthermore, by maximizing Eq. (7), optimal values of mean time to minimal maintenance l_m of all GCT as a single-unit system are determined (Fig. 6b).

Model-2 represents a different manner of maintenance in relation to Model-1. In model-2, frequency of entering state of minimal maintenance from state $D_{1,1}$ is greater in relation to frequency of minimal maintenance in Model-1. In model-1, system returns to the state identical to the state of new system after minimal maintenance. On the other hand, in order for Model-2 to be closer to real system, intensity of entering maintenance state M_1 from state $D_{1,1}$ is equal to half of

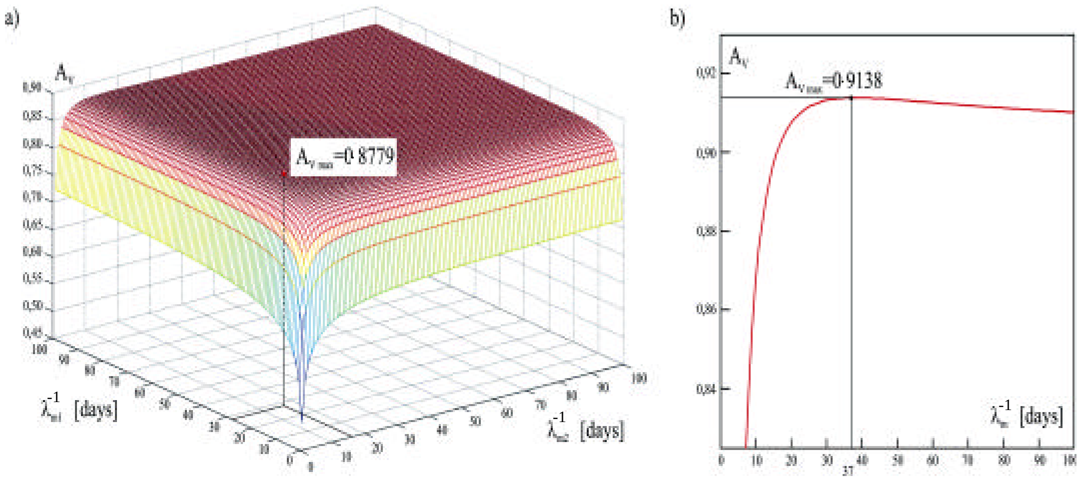

Fig. 6—Availability of: a) Two-component system with different $l_{m1} \neq l_{m2}$; b) Single-unit system

Table 4—Comparison of model-1 and model-2 of GCT maintenance

Mark	Description	Model	l_{m1}^{-1} day	l_{m2}^{-1} day	A_V
SS1	Subsystem 1 as a single-unit system	model-1	40	-	$A_{V1}(40) = 0.9237$ $A_{V2}(40) = 0.8887$
SS2	Subsystem 2 as a single-unit system	model-1	-	31	$A_{V2}(31) = 0.8863$ $A_{V1}(31) = 0.9231$
All GCT	All GCT as a single-unit system	model-1	37		$A_V(37) = 0.9138$
SS1+SS2	All GCT as a multi-component system $l_{m1} \neq l_{m2}$	model-2	27	16	$A_V(27,16) = 0.8779$
SS1+SS2	All GCT as a multi-component system $l_{m1} = l_{m2}$	model-2	21		$A_V(21,21) = 0.8771$

the sum of intensities l_{m1} and l_{m2}. Considering two subsystems have similar functions of failure intensity, as well as maintenance parameters (m_0, $m_1 \approx m_2$, and m_m), intervals of minimal maintenance l_{m1} and l_{m2} have similar values. Availability of both subsystems (Table 4) remains approximately same regardless of whether entire system has minimal maintenance interval of 40 (optimal value for subsystem 1) or 31 days (optimal value for subsystem 2). In case the entire system is observed (all GCT), minimal maintenance interval would be 37 days with availability of 0.914. In the case of Model-2, intervals of minimal maintenance are significantly shorter (27 and 16). Regardless of whether or not subsystems enter state of minimal maintenance simultaneously, availability remains approximately the same. This is also a consequence of observing two subsystems of similar characteristics.

Conclusions

This paper discussed use of model to find optimum maintenance policy of multi-component degraded systems. This model can be applied to many practical multi-component systems, since it takes care of situations where one cannot restore systems back to as good as new condition with minimal maintenance actions and where total failure degradation state occurs when performance of the system exceeds acceptable limit. One of the best ways to reduce mean time to minimal maintenance is to reduce mean time of repairing a degradation failure. In other words, random failures cannot be prevented through PM. Analysis indicates the need to reduce time that GCT spend in the state of corrective maintenance after a degradation failure, because a single day reduction leads to increase in availability and significant decrease in the need for PM. Both models are easy to use and represent a practical and systematic procedure to perform maintenance activities. Model-1 offers better results in the case of single- and multi-component systems with elements that have similar failure and maintenance parameters. In the case when there are significant differences in failure and maintenance parameters, it is necessary to use Model-2.

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