Influences of multiple holes on thermal stresses in a thermoplastic composite disc

Faruk Sen\textsuperscript{a,b,*} & Metin Sayer\textsuperscript{c}
\textsuperscript{a}Department of Mechanical Engineering, University of Sheffield, Sheffield, United Kingdom
\textsuperscript{b}Department of Mechanical Engineering, Aksaray University, Aksaray, Turkey
\textsuperscript{c}Department of Mechanical Engineering, Pamukkale University, Denizli, Turkey

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The aim of this investigation is to observe the influence of multiple holes on thermal stresses in a thermoplastic composite hollow disc. For this purpose, a thermoplastic composite disc is reinforced by steel fibers as curvilinear for radial direction, and thermal loads are carried out for uniform distribution at various temperatures. The solution is completed in two parts that are elastic and elastic-plastic analysis. Additionally, residual stresses are calculated using these analysis results. Finite element method (FEM) is utilized for calculation of thermal stresses. Therefore, the thermal stress analysis is performed using ANSYS\textsuperscript{®} finite element software. Due to the fact that composite disc have different thermal expansions in radial and tangential directions, thermal stresses are taken place in disc by the applied uniform thermal loadings. The magnitude of the tangential stress components both elastic and elastic-plastic solutions is higher than that of the radial stress components, except edges of multiple holes. The present study concludes that magnitudes and distributions of thermal and residual stresses on the composite disc are considerably affected by increasing the uniform temperature loadings and existence of multiple holes.

Keywords: Thermoplastic disc, Multiple holes, Residual stresses, Thermal stresses, FEM

The fiber reinforced composite materials are widely used in many structural applications such as marine, automotive and aviation industries, on account of their high strength, low weight, good fatigue life and corrosion resistance. Especially, the composite discs are used in automotive industry as a composite brake disc due to their specific advantages\textsuperscript{1-3}. Meanwhile, availability of the new, low cost, high performance fiber reinforced plastic (FRP) structural composites for infrastructure construction and rehabilitation has given an impetus to the development of a new technology in construction industry\textsuperscript{4}. The reinforced thermoplastics have been extensively used in many engineering applications due to the fact that they have high-resistance to corrosion, shrinking less in mould, less contraction in moist environments and less modification in dimension. Besides, reinforced thermoplastic plates softened through heating are widely used in not only avionics and space industry but also automotive sector for they are produced in unheated moulds and are appropriate for recyclable material utilization\textsuperscript{5}. Moreover, the load-carrying capacity of the reinforced discs is also much higher than that of the traditional isotropic steel discs of the same geometry and the weight of the former is also several times lower\textsuperscript{6,7}. Some thermal stress problems for beams, bars, cylinders and plates have been explained by Noda \textit{et al.}\textsuperscript{8}. A closed form solution for the stress analysis in curvilinear orthotropic disc and cylinders under pressure has been reported by Leknitski\textsuperscript{9}.

A lot of composite structures have multiple holes, i.e., pin, bolt etc to provide various purposes. These holes may also provide the contact from one side of the structure to the other. However, the calculation of the thermal stresses around multiple holes is rather difficult. In a previous study, Sen\textsuperscript{10} estimated the elasto-plastic thermal and residual stresses in a thermoplastic composite disc that was reinforced as rectilinear under uniform temperature effect. The designed hollow disc was not including any pin or bolt hole, so the mesh generation process was easier than present study, especially. Sen\textsuperscript{11} also investigated the effects on thermal and residual stresses of parabolic temperature loading in a thermoplastic composite disc having a central hole by FEM. It was reinforced by steel-fibers as unidirectional for radial direction. The obtained results pointed out that the
magnitude of thermal and residual stresses were considerably affected increasing of parabolic temperature loadings. Moreover, Sen et al.\textsuperscript{12} performed elastic-plastic stress analysis of steel fiber reinforced thermoplastic composite disc, curvilinearly. The thermal loading was applied as parabolic temperature distribution during the elastic and elastic-plastic solutions. It was completed both analytically and numerically using FORTRAN\textsuperscript{®} and ANSYS\textsuperscript{®} programs, respectively. According to obtained results these two solutions to be in good agreement. You et al.\textsuperscript{13} also improved a numerical method for the analysis of deformations and stresses in the elastic-plastic rotating discs with arbitrary cross-sections of continuously varying thicknesses and arbitrary variable density made of nonlinear strain-hardening materials. Sayman et al.\textsuperscript{14} carried out an elastic-plastic thermal stress analysis on thermoplastic composite disc reinforced curvilinearly with steel fibers under uniform temperature distribution. The solution was performed numerically via a program developed using FORTRAN\textsuperscript{®}. Sen and Aldas\textsuperscript{15} have also investigated the effects of increasing linear temperature loadings on the thermal stresses occurring in the disc which during the solution different linear thermal loads were selected. Besides, the thermal residual stresses were calculated by authors using elastic and elastic-plastic solution results via superposition method and they reported that the magnitudes and distributions of the thermal stresses. As a result they reported that the magnitudes and residual stresses were greatly influenced by the increase in linear temperature load. Furthermore, the calculation of loading and residual stresses, and related to strains and displacements were reported for shrink fitting rotating discs at elevated temperatures by Jahed and Shirazi\textsuperscript{16}. In that study, a range of material behaviour was considered including unloading described by an actual material curve, or modeled by isotropic or kinematic hardening with a variable Bauschinger effect. In addition, an analytical solution for the elastic-plastic stress distribution in rotating variable thickness solid disks was presented by Eraslan and Orcan\textsuperscript{17}. The analysis was based on Tresca’s yield criterion, its associated low rule and linear strain hardening material behaviour.

Rossit and Laura\textsuperscript{18} investigated unsteady thermal stresses caused by the presence of a hot, central nucleus, in disc or cylinders. The temperature field was calculated in terms of a Fourier-Bessel expansion and then, radial and tangential stresses were evaluated analytically. The considered problem was of basic interest in mechanical and naval engineering systems. Trende et al.\textsuperscript{19} examined residual stresses and dimensional changes in the compression moulded glass-mat reinforced thermoplastic parts. A heat transfer and crystallization model with temperature dependent matrix properties was used to calculate input to the subsequent thermal stress analysis. Both an isotropic viscoelastic and a transversely isotropic elastic material model were examined through finite-element calculations. Sayer et al.\textsuperscript{20} investigated thermo-elastic stresses on a low-density thermoplastic composite disc reinforced curvilinearly with E-glass fibers. An analytical method was carried out to determine thermal stresses of the composite disc having a central hole for both uniform and linearly changing temperatures.

In the present study, an elastic-plastic thermal stress analysis was carried out on a thermoplastic composite hollow disc under various uniform thermal loadings. The composite disc reinforced with steel fibers as curvilinear and it was considered with multiple holes. Finite element method was used to calculate the thermal stresses. Additionally, residual stresses were computed using elastic and elastic-plastic solution results.

Materials and Methods

Production of thermoplastic composite disc

Mechanical properties of the steel fiber reinforced composite disc are presented in Table 1\textsuperscript{14}. According to Sayman et al.\textsuperscript{14} the composite disc consists of low-density polyethylene as a thermoplastic matrix and steel fibers as curvilinear. First of all the polyethylene layers were fabricated. The raw granules of the polyethylene were placed into the mold and heated by an electrical resistance and the temperature

<table>
<thead>
<tr>
<th>Table 1—Mechanical properties of the curvilinear composite disc</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modulus of elasticity for radial direction, ( E_r ) (MPa)</td>
</tr>
<tr>
<td>Modulus of elasticity for tangential direction, ( E_t ) (MPa)</td>
</tr>
<tr>
<td>Shear modulus, ( G_{12} ) (MPa)</td>
</tr>
<tr>
<td>Poisson ratio, ( \nu_{r\theta} )</td>
</tr>
<tr>
<td>Axial strength, ( X ) (MPa)</td>
</tr>
<tr>
<td>Transverse strength, ( Y ) (MPa)</td>
</tr>
<tr>
<td>Shear strength, ( S ) (MPa)</td>
</tr>
<tr>
<td>Plasticity constant, ( K ) (MPa)</td>
</tr>
<tr>
<td>Thermal expansion coefficient for radial direction, ( \alpha_r ) (1/°C)</td>
</tr>
<tr>
<td>Thermal expansion coefficient for tangential direction, ( \alpha_\theta ) (1/°C)</td>
</tr>
</tbody>
</table>
was increased up to 160°C. The raw material was held for 5 min under 2.5 MPa pressure at this temperature, and then the temperature was decreased to 30°C in 3 min under 15 MPa pressure at the mentioned temperature. Thus, a polyethylene layer with thickness of 1 mm was produced. Steel fibers were placed into two thermoplastic layers as curvilinearly, and then the above processes were applied for preparing the composite disc. Thus, a steel fiber reinforced curvilinear thermoplastic composite disc was obtained, and the fiber volume fraction was measured as 6%. According to mechanical test results, the yield point, the modulus of elasticity and the thermal expansion coefficient of the low density thermoplastic are 9.3 MPa, 170 MPa and $135 \times 10^{-6}$ (1/°C), respectively.

**Modeling of composite disc and boundary conditions**

The steel fiber reinforced curvilinear thermoplastic hollow composite disc with multiple holes is illustrated schematically in Fig. 1. It is seen in this figure that the dimensions of it are modeled as inner radius $a=30$ mm, outer radius $b=60$ mm and diameter of each hole $d=10$ mm. In this study, finite element software, ANSYS® 10.0, was utilized in order to obtain the distributions of thermal elastic and elastic-plastic stress components. During the modeling process of disc, element type was chosen as PLANE 183 for mesh generation. This element type is a higher order 2-D, 8-node element. It has quadratic displacement behaviour and is well suited to modeling irregular meshes. Additionally, this element is defined by 8 nodes having two degrees of freedom at each node: translations in the nodal $x$ and $y$ directions. The element may be used as a plane element (plane stress, plane strain and generalized plane strain) or as an axisymmetric element. This element has plasticity, hyperelasticity, creep, stress stiffening, large deflection, and large strain capabilities. The geometry, node locations, and the coordinate system for this element are shown in Fig. 2. Orthotropic material directions match up to the element coordinate directions$^{21}$.

According to this information, the hollow composite disc with multiple holes was modeled in ANSYS® and boundary conditions and finite element model of the disc are shown in Fig. 3. The mesh details where surroundings of multiple holes are also shown in this figure as K and L. It is seen in this figure that mapped mesh structure is preferred for the mesh generation of the composite disc model because of some advantages comparing to the free mesh. It is also known, a mapped mesh is limited in terms of the element shape it contains and the pattern of the mesh$^{21,22}$. Furthermore, a good mesh generation is very important for FEM solutions. If the model has multiple holes, the generation of mapped mesh is very difficult. However, in this study, the mapped mesh was provided by the author. This advantage was supplied to decrease both element and node numbers.

Thermal stress problem were solved in two parts, which one of them elastic solution and other elastic-plastic solution. After these solutions, residual stresses were also calculated using superposition method. During the solution, the composite material was assumed to be linearly hardening, and also the mechanical properties of disc were assumed unchanged while temperature was increased. It was also desired to find out the effect on thermal and residual stresses of uniform temperature loading in a...
Fig. 3—Boundary conditions and FEM model with details of the composite disc
thermoplastic composite disc. Therefore, applied uniform temperature values were chosen from 50 to 100°C. The temperature increment was 5°C.

**Mathematical Formulation**

**Elastic solution**

In this study, thermal stress problem was solved analytically in two parts. Firstly, elastic solution was carried out on the composite disc under selected uniform temperature loadings. The composite disc was assumed that it is very thin therefore the temperature distribution didn’t change across the thickness of the composite disc and the solution was reduced to a plane stress case. The stress-strain relations in a composite disc reinforced curvilinear can be written as

\[
\varepsilon_r = \frac{du}{dr} = a_r\sigma_r + a_{r\theta}\sigma_\theta
\]

\[
\varepsilon_\theta = \frac{u}{r} = a_{r\theta}\sigma_r + a_{\theta\theta}\sigma_\theta
\]

where \( u \) is the displacement in the radial direction and \( a_r, a_{r\theta} \) and \( a_{\theta\theta} \) are the components of the compliance matrix, \( \alpha_r \) and \( \alpha_\theta \) are the thermal expansion coefficients in the radial and tangential directions, respectively. The equation of equilibrium in the plane stress case for a curvilinear orthotropic layer can be reduced to the next shape,

\[
r \frac{\partial \sigma_r}{\partial r} + \sigma_r - \sigma_\theta = 0
\]

The compatibility equation is written from the relation between the displacement and strains as given in Eq. (3),

\[
\varepsilon_r = \varepsilon_\theta + r \frac{\partial \varepsilon_\theta}{\partial r}
\]

The stress function \( F \) satisfies the equation of equilibrium as,

\[
\sigma_r = \frac{F}{r}
\]

\[
\sigma_\theta = \frac{dF}{dr}
\]

The governing stress function is written by using the Eqs (1)-(4),

\[
a_{r\theta} r^2 \frac{d^2 F}{dr^2} + a_{0\theta} r \frac{dF}{dr} - a_{rr} F = -a_{0\theta} r^2 \frac{dT}{dr} + a_r Tr - a_\theta Tr
\]

If \( T \) is chosen as a constant value, \( T_0 \), the stress function \( F \) can be written as,

\[
F = C_1 r^k + C_2 r^{-k} + \frac{Ar}{1-k^2}
\]

where \( k^2 = \frac{a_r}{a_{\theta\theta}} \), \( C_1 \) and \( C_2 \) are the integration constants and last term is,

\[
A = \frac{T_0(\alpha_r - \alpha_\theta)}{a_{\theta\theta}}
\]

The stress components are found as,

\[
\sigma_r = \frac{F}{r} = C_1 r^{k-1} + C_2 r^{-k-1} + \frac{A}{1-k^2}
\]

\[
\sigma_\theta = \frac{dF}{dr} = C_1 kr^{k-1} - C_2 kr^{-k-1} + \frac{A}{1-k^2}
\]

Consequently, radial and tangential stress components (\( \sigma_r \) and \( \sigma_\theta \)) can be calculated from the boundary conditions.

**Elastic-plastic solution**

After the thermal elastic solution was completed, the thermal elastic-plastic solution was performed on the composite disc under same uniform thermal loadings as second part of the solution. The strain increments in the radial and tangential directions are written as

\[
d\varepsilon_r = a_{r\theta} d\sigma_r + a_{r\theta} d\sigma_\theta + d\varepsilon_\theta^p + \alpha_r dT
\]

\[
d\varepsilon_\theta = a_{r\theta} d\sigma_r + a_{\theta\theta} d\sigma_\theta + d\varepsilon_\theta^p + \alpha_\theta dT
\]

The strain increments for anisotropic materials are obtained from the plastic potential \( f \),

\[
d\varepsilon_\theta^p = \frac{\partial f}{\partial \sigma_\theta} d\lambda
\]
where \( f \) is the potential function which is equal to the equivalent stress \( (\bar{\sigma}) \), \( \frac{\partial\lambda}{\partial} \) is a scalar quantity and it is equal to the equivalent plastic strain increment \( d\varepsilon^p \). The equivalent stress for the Tsai-Hill criterion can be written as

\[
\bar{\sigma}^2 = \sigma_\theta \sigma_r + \frac{\sigma_r^2 X^2}{Y^2} \quad \ldots(11)
\]

where, \( X \) and \( Y \) are the yield strengths of the disc in the fiber and transverse directions, respectively. In this solution, it is assumed that the material hardens linearly.

\[
\sigma = \sigma_\theta + K\varepsilon_p \quad \text{where} \quad \sigma_\theta \quad \text{is equal to} \quad X \quad \text{and} \quad K \quad \text{is the plasticity constant.}
\]

The tangential stress can be written in terms of the radial stress component. Consequently, the Tsai-Hill criterion, \( d\sigma \) the radial stress increment is obtained as,

\[
d\sigma_r = \frac{-r\sigma_r \pm \sqrt{r^2\sigma_r^2 - 4r^2 \left( \frac{X^2}{Y^2} - \sigma_r - \sigma_\theta \right)}}{2r^2} dr \quad \ldots(12)
\]

where \( \bar{\sigma} \) is the equivalent stress, which is equal to \( X \). The plastic strain increments in the fiber and transverse directions are calculated from the plastic potential, according to the Prandtl-Reuss equations,

\[
d\varepsilon_r^p = \frac{\partial f}{\partial \sigma_r} d\lambda = -\frac{-\sigma_r + 2\sigma_r \frac{X^2}{2\sigma} \varepsilon_p}{2\sigma} 
\]

\[
d\varepsilon_\theta^p = \frac{\partial f}{\partial \sigma_\theta} d\lambda = \frac{2\sigma_\theta - \sigma_r}{2\sigma} d\varepsilon_p \quad \ldots(13)
\]

The total strain increments are calculated as,

\[
\varepsilon_r = \frac{du}{dr} = a_{rr}\sigma_r + a_{r\theta}\sigma_\theta + \sum d\varepsilon_r^p + \alpha_r dT + B_1 \]

\[
\varepsilon_\theta = \frac{u}{r} = a_{r\theta}\sigma_r + a_{\theta\theta}\sigma_\theta + \sum d\varepsilon_\theta^p + \alpha_\theta dT + B_2 \quad \ldots(14)
\]

Where \( B_1 \) and \( B_2 \) are the integration constants. \( B_1 \) and \( B_2 \) are found to be equal to zero by using the relation between the elastic and plastic strain increments which are equal at the beginning of plastic yielding.

**Calculation of residual stresses**

To find the thermal residual stresses, it is necessary to superpose on the thermal stress system. The superposition of the elastic and elastic-plastic stresses gives the residual stresses. Hence, the magnitudes of thermal residual stresses are calculated using the Eq. (15) as,

\[
(\sigma_r)_r = (\sigma_r)_p - (\sigma_r)_e \]

\[
(\sigma_\theta)_r = (\sigma_\theta)_p - (\sigma_\theta)_e \quad \ldots(15)
\]

**Results and Discussion**

The thermal stresses concentrations take place in the disc when the uniform temperature was increased from 50 to 100°C. Accordingly, the four points were selected on the modeling disc since they have critical importance in terms of thermal stresses. As seen in Fig. 3, two nodes of these are selected on inner and outer surfaces of the disc as named A and B, respectively. In addition, other two nodes labeled as C and D which are considered near the one of the multiple holes. The thermal elastic, plastic, residual stress and equivalent plastic strain components produced by uniform temperature effects at selected nodes on the disc are given in Table 2. It can be seen from this table that the magnitude of thermal stresses and other components increase by increasing uniform thermal loads. Therefore, the minimum values of it are calculated at 50°C, while the maximum values are obtained at 90°C. Thermal stresses for tangential direction are higher than that of radial direction on nodes A, B and C, although its values for radial direction are bigger than that of tangential direction on node D. In addition, the maximum values of thermal stresses are obtained on node A for tangential direction as compressive. The magnitudes of thermal stresses on node B for radial direction are estimated extremely small if it is compared with other stress components; therefore their values can be neglected. These all assumptions are also valid for both elastic and plastic stresses.

As seen clearly in Table 2 that a plastic strain is firstly began at 55°C applied uniform temperature on nodes A, B, and C. Namely, residual stress components are equal to zero for all nodes at 50°C.
The maximum value of residual stresses is obtained for tangential direction on node A at 90°C as 10.651 MPa for tangential direction. In other words, the highest value of residual stresses is created on the inner surface of the disc as tensile form for tangential direction. The magnitudes of residual stresses on node D for tangential direction are equal to zero under all thermal loadings. Meanwhile, the values of equivalent plastic strains are also calculated for only node B from 55 to 90°C. The maximum value of it is estimated as 4050 × 10^{-6} at 90°C, whereas it is equal to zero on all other selected nodes and applied uniform temperatures.

Thermal elastic and plastic stress distributions utilizing stress contours for radial and tangential directions for different applied thermal loadings are given in Figs 4 and 5, respectively. It is mentioned previously, thermal stresses at 50°C are calculated as only elastic form, since this thermal load does not produce any plastic strain on the composite disc. Therefore, stress contours are plotted as elastic form for 50°C in Fig. 4. Thermal stresses for tangential direction are higher than that of radial direction both elastic and plastic solutions as seen in Figures 4 and 5. These differences of thermal stress results between radial and tangential directions for elastic solution are bigger than that of plastic solution results. For instance, the maximum values of elastic stresses for radial and tangential directions are calculated as -5.598 and -25.250 MPa at 90°C, respectively.

### Table 2—Thermal elastic, plastic, residual stress and equivalent plastic strain components at selected nodes on the composite disc

<table>
<thead>
<tr>
<th>$T_o$ (°C)</th>
<th>Node</th>
<th>Elastic stress (σ_r)</th>
<th>Plastic stress (σ_θ)</th>
<th>Residual stress (σ_r)</th>
<th>Eqv. Plastic Strain (×10^{-6})</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(MPa)</td>
<td>(MPa)</td>
<td>(MPa)</td>
<td>ε_eqv</td>
</tr>
<tr>
<td>50</td>
<td>A</td>
<td>-0.012</td>
<td>-14.032</td>
<td>-0.012</td>
<td>-14.032</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>-0.55×10^{-3}</td>
<td>5.570</td>
<td>-0.55×10^{-3}</td>
<td>5.570</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>0.029</td>
<td>7.536</td>
<td>0.029</td>
<td>7.536</td>
</tr>
<tr>
<td></td>
<td>D</td>
<td>-2.985</td>
<td>-0.059</td>
<td>-2.985</td>
<td>-0.059</td>
</tr>
<tr>
<td>55</td>
<td>A</td>
<td>-0.013</td>
<td>-15.436</td>
<td>-0.012</td>
<td>-14.059</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>-0.61×10^{-3}</td>
<td>6.127</td>
<td>-0.61×10^{-3}</td>
<td>6.127</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>0.032</td>
<td>8.289</td>
<td>0.032</td>
<td>8.287</td>
</tr>
<tr>
<td></td>
<td>D</td>
<td>-3.284</td>
<td>-0.065</td>
<td>-3.284</td>
<td>-0.065</td>
</tr>
<tr>
<td>60</td>
<td>A</td>
<td>-0.015</td>
<td>-16.839</td>
<td>-0.064</td>
<td>-14.100</td>
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<tr>
<td></td>
<td>B</td>
<td>-0.66×10^{-3}</td>
<td>6.684</td>
<td>-0.66×10^{-3}</td>
<td>6.687</td>
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<tr>
<td></td>
<td>C</td>
<td>0.035</td>
<td>9.043</td>
<td>0.035</td>
<td>9.032</td>
</tr>
<tr>
<td></td>
<td>D</td>
<td>-3.582</td>
<td>-0.071</td>
<td>-3.583</td>
<td>-0.071</td>
</tr>
<tr>
<td>65</td>
<td>A</td>
<td>-0.016</td>
<td>-18.242</td>
<td>-0.061</td>
<td>-14.171</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>-0.72×10^{-3}</td>
<td>7.241</td>
<td>-0.72×10^{-3}</td>
<td>7.249</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>0.038</td>
<td>9.796</td>
<td>0.038</td>
<td>9.767</td>
</tr>
<tr>
<td></td>
<td>D</td>
<td>-3.881</td>
<td>-0.077</td>
<td>-3.883</td>
<td>-0.077</td>
</tr>
<tr>
<td>70</td>
<td>A</td>
<td>-0.017</td>
<td>-19.645</td>
<td>-0.061</td>
<td>-14.260</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>-0.77×10^{-3}</td>
<td>7.798</td>
<td>-0.78×10^{-3}</td>
<td>7.814</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>0.041</td>
<td>10.550</td>
<td>0.041</td>
<td>10.482</td>
</tr>
<tr>
<td></td>
<td>D</td>
<td>-4.179</td>
<td>-0.083</td>
<td>-4.184</td>
<td>-0.083</td>
</tr>
<tr>
<td>75</td>
<td>A</td>
<td>-0.018</td>
<td>-21.048</td>
<td>-0.062</td>
<td>-14.341</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>-0.83×10^{-3}</td>
<td>8.355</td>
<td>-0.84×10^{-3}</td>
<td>8.382</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>0.044</td>
<td>11.304</td>
<td>0.043</td>
<td>11.180</td>
</tr>
<tr>
<td></td>
<td>D</td>
<td>-4.478</td>
<td>-0.089</td>
<td>-4.485</td>
<td>-0.089</td>
</tr>
<tr>
<td>80</td>
<td>A</td>
<td>-0.020</td>
<td>-22.452</td>
<td>-0.062</td>
<td>-14.432</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>-0.89×10^{-3}</td>
<td>8.912</td>
<td>-0.90×10^{-3}</td>
<td>8.951</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>0.047</td>
<td>12.057</td>
<td>0.046</td>
<td>11.849</td>
</tr>
<tr>
<td></td>
<td>D</td>
<td>-4.776</td>
<td>-0.095</td>
<td>-4.784</td>
<td>-0.095</td>
</tr>
<tr>
<td>85</td>
<td>A</td>
<td>-0.021</td>
<td>-23.855</td>
<td>-0.063</td>
<td>-14.518</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>-0.94×10^{-3}</td>
<td>9.469</td>
<td>-0.81×10^{-3}</td>
<td>9.336</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>0.050</td>
<td>12.811</td>
<td>0.049</td>
<td>12.543</td>
</tr>
<tr>
<td></td>
<td>D</td>
<td>-5.075</td>
<td>-0.101</td>
<td>-5.083</td>
<td>-0.101</td>
</tr>
<tr>
<td>90</td>
<td>A</td>
<td>-0.022</td>
<td>-25.258</td>
<td>-0.063</td>
<td>-14.670</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>-0.001</td>
<td>10.026</td>
<td>-0.62×10^{-3}</td>
<td>9.502</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>0.053</td>
<td>13.564</td>
<td>0.052</td>
<td>13.306</td>
</tr>
<tr>
<td></td>
<td>D</td>
<td>-5.373</td>
<td>-0.107</td>
<td>-5.379</td>
<td>-0.107</td>
</tr>
</tbody>
</table>
Nevertheless, the maximum values of plastic compressive stresses for radial and tangential directions are obtained as -5.612 MPa and -14.607 MPa at 90°C, respectively. For tangential direction, thermal stresses on the inner and outer surfaces are obtained as compressive and tensile forms, respectively. Besides, the magnitudes of compressive stresses are higher than that of tensile stresses both radial and tangential directions.

It is also seen in Figs 4 and 5, multiple holes caused stress concentrations around itself. These concentrations are clearly seen for radial direction. Additionally, the maximum values of compressive stresses occur near the multiple holes for radial direction. This means that stress concentrations are created by presence of multiple holes in the disc. Meanwhile, it is known that holes are necessary in many composite structures including discs for some
purposes such as pin, bolt and rivet applications. Consequently, in practice the number of multiple holes and their sizes must be considered to avoid from a weakest construction by the constructor.

The effects of uniform temperature loads on residual stresses on selected nodes for radial and tangential directions are shown in Fig. 6. This figure points out that the magnitude of residual stress components for tangential direction is higher than that of radial direction. The value of residual stresses on node A are extremely bigger than that of other nodes both radial and tangential directions, especially. Its forms are compressive and tensile on node A for radial and tangential directions, respectively. On the other hand the maximum values of residual stresses are come into being on the inner surface of the disc. It is mentioned previously, the magnitudes of residual stresses are equal to zero both radial and tangential directions at 50°C. Other important node is seemed as node D since higher values of residual stresses are obtained on this node for radial direction. Briefly, the
Fig. 6—The effect of uniform temperature loads on residual stresses on selected nodes
presence of any multiple holes produces residual stresses on the composite disc.

The distributions of equivalent plastic strains with color contours for 70 and 90°C are shown in Fig. 7. It is seen clearly that the distributions and magnitudes of equivalent plastic strains are enlarged related to increasing of uniform thermal loads. The values of it are obtained as zero at outer surface of the disc and around the multiple holes both 70 and 90°C, while the maximum value is calculated as $4050 \times 10^{-6}$ on the inner surface of the disc at 90°C.

**Conclusions**

According to the elastic and elastic-plastic thermal stress analyses results of the curvilinear steel fiber reinforced thermoplastic composite disc with multiple holes utilizing FEM, the following conclusions can be drawn:

(i) Thermal and residual stresses occurred in the disc under uniform temperature loads, due to the fact that different thermal expansion coefficients in principal material directions.

(ii) In the disc, plastic flow starts both edges of multiple holes and inner surface of the disc at 55°C, initially.

(iii) Plastic yielding expands, if the uniform temperature load is bigger than ever.

(iv) The magnitude of tangential stress components are higher than that of radial stress components both inner and outer surfaces of the disc.

(v) The maximum values of thermal stresses for radial direction are obtained edges of the multiple holes.

(vi) The multiple holes cause stress concentrations on the disc.

(vii) Because of the causing stress concentrations, the designer must be selected number of multiple holes and its size to avoid from weakest structure in practice.

(viii) Thermal residual stresses are obtained on the disc due to uniform temperature loadings. The maximum values of residual stresses are produced on the inner surface of the disc.

(ix) Residual stresses are also produced from the existence of the multiple holes on the disc.

**References**