Scattering of VLF signals from localized perturbations in the lower ionosphere

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Received 27 January 2009; revised received and accepted 19 April 2010

A wave-slab interaction model has been used to study the scattering of very low frequency (VLF) ground transmitter signals from localized enhancements of ionization in the lower ionosphere caused by transient luminous events (TLEs) such as sprites. The variation of reflection coefficients has been studied for a fixed frequency VLF transmitter signal of frequency 19.8 kHz (NWC, Australia) monitored at Agra for different slab thicknesses, incident angles, and enhancement factors. The results show that sufficient strength of the reflected signals is obtained for a slab thickness of 3 km, enhancement factor between 2.5 and 3, and incident angle around 75°. These results explain satisfactorily the VLF amplitude and phase anomalies in sub-ionospheric VLF transmitter signals recorded at Agra station which are interpreted as caused by transient luminous events.

Keywords: VLF signals scattering, Transient luminous events (TLEs), Sprites, Lower ionosphere perturbations, Reflection coefficient

PACS No.: 94.20.Vv

1 Introduction

The ionization at the lower boundary of the ionosphere is modified by many geophysical events which include precipitation of energetic particles$^1$, solar flares$^2$, lightening$^3$, and earthquakes$^4$. Some recently discovered new classes of events perturbing the lower ionosphere are transient luminous events (TLEs) like sprites and elves$^5$. The localized enhancement of ionization due to these events are found to cause sudden changes in the amplitude and phase of the fixed frequency transmitter signals propagated through earth-ionosphere waveguide space which, in turn, form a useful tool to study the ionospheric perturbations and their relation with naturally occurring events. A schematic diagram showing the interaction of sub-ionospheric VLF signals (shown by dotted line) with the perturbed region of the lower ionosphere caused by any of the above natural events is shown in Fig.1.

Many researchers have advanced modeling of the above phenomena by theoretical consideration. For example, Tolstoy et al.$^6$ have used a multi-slab model of the horizontal irregularities and 2-D mode theory$^7$ to calculate the change in the wave field produced by the perturbation; Dowden & Adams$^8$ have used a 3-D model based on “echoes” from lightening induced precipitation; Nunn & Stangeways$^9$ have studied Trimpi perturbations from large ionization enhancement patches; Rodger et al.$^{10}$ have studied modeling of sub-ionospheric VLF signal perturbations associated with earthquakes; and Soloviev et al.$^{11}$ have studied seismo-electromagnetic phenomena in terms of 3-D modeling of the perturbed region. In all these theoretical studies, the basic objective was to reproduce the VLF wave fields observed on the ground.

Monitoring of amplitude and phase of the fixed frequency VLF transmitter signals has also been carried out at Agra for a period of three years (1 August 2002-30 December 2005). For this purpose, AbsPAL receiver obtained from LF*EM Research Ltd, New Zealand has been used and the signals monitored are NWC, Australia (19.8 kHz), NPM, Hawaii (21.4 kHz), and NAA, Cutler (24 kHz). Seven cases of sudden change in amplitude and phase of these signals have been observed which are interpreted in terms of the ionospheric perturbations caused by distant sprites$^{12,13}$. An example of such changes is shown Fig. 2. In general, the amplitude changes in the range 3-7 dB and phase changes between 0.3 and 0.6 msec. In the present paper, the reflection of VLF signals from the localized perturbations has been studied by considering a simplified slab model on the lines of Tolstoy et al.$^6$. In
fact, the perturbation in the form of a slab located at the base of the ionosphere and VLF signals incident on it from below while propagating through the earth-ionosphere waveguide have been considered. The reflection coefficients for different slab geometry, ionization enhancements, and incident angles have been calculated and the results have been utilized to explain the amplitude and phase anomalies obtained in the VLF sub-ionospheric propagation studies.

2 Method of calculating the reflection coefficients

The equations for transmission and reflection of incident waves in such a wave-slab interaction model have been derived both for magnetic\(^4\) and electric field components\(^5\) of the waves. In the present case, the equations derived for electric field components have been considered. The equations for transmission and reflection coefficients are derived using boundary conditions\(^6\). The equation for transmission coefficient is given as:

\[
|\tau|^2 = \frac{4\gamma^2}{4\gamma^2 \cos^2 k_{zB} L + (1+\gamma^2)^2 \sin^2 k_{zB} L}
\]

\[
= \frac{4\gamma^2}{4\gamma^2 + (1-\gamma^2)^2 \sin^2 k_{zB} L}
\]

… (1)

where, \(\gamma\), is the ratio of the components of reflected and incident electric field; \(kzB = k \sin \theta\) [where, \(k\), is wave normal vector, and \(\theta\), angle between \(k\) and \(B_0\) (Earth’s magnetic field)]; \(L\), thickness of slab; \(\tau\), transmission coefficient; and \(\rho\), reflection coefficient.

Under this assumption the reflection coefficient \(|\rho|^2 = |E_{YR}/E_{YI}|^2\) satisfies the equation:

\[
|\tau|^2 + |\rho|^2 = 1
\]

… (2)

![Fig. 1 — Schematic diagram of ionosphere perturbation during some geophysical events](image1)

![Fig. 2 — Examples of sudden enhancement and decrement in phase and amplitude of VLF transmitter signals during lightening activities](image2)
Reflection coefficients are calculated with the help of a computer program for the above mentioned equations\textsuperscript{15}. The program requires wave frequency, gyro-frequency, thickness of the layer, enhancement factor and the incident wave normal angles as input parameters. The output yields the reflection coefficients for different variable parameters like enhancement factors, thickness of slab, and incident angles. Calculations are done for various thicknesses of the slab ranging 1-4 km, enhancement factors between 1 and 4, incident wave normal angles between 70\(^\circ\) and 80\(^\circ\) using the computer program which is in Fortran language.

4 Results and Discussions

The various causes, which may perturb the lower region of the ionosphere, have been mentioned earlier. Out of these, the precipitation of energetic particle due to wave-particle interaction in the lower ionosphere of low L-shells is not possible. A global precipitation studies under this mechanism\textsuperscript{17} has shown that ionization enhancement in the low latitudes region is possible only for L>1.4. Wave-particle interactions and precipitation at low L-Shells (L<1.4) have been reported in the South–Atlantic anomaly region also known as Brazialian anomaly region\textsuperscript{18,19}. Hence, ionospheric perturbations at the latitudes (L=1.1) is not possible due to this mechanism. The effects of solar flares and earthquakes causing ionospheric perturbations in the present case are also not possible due to the reasons that no such events occurred on the days of the amplitude and phase anomalies observed in the study. Hence, the only possibility is that such anomalies were caused by transient luminous events like sprites.

The calculations for the reflection coefficients have been done for the frequency 19.8 kHz by varying ionization (enhancement factor) in the slab, thickness of the slab, and different incidents angles. The reflection coefficients corresponding to increasing thicknesses between 1 and 5 km for three incident angles of 70\(^\circ\), 75\(^\circ\), and 80\(^\circ\) have been plotted in Fig. 3. The calculations are done for three different enhancement factors of 1.5, 2, and 2.5 as shown in the

![Graph](image_url)

Fig. 3 — Variation of reflection coefficients with slab thickness and enhancement factors
three panels of the figure. Here, it can be seen that reflection coefficients show fluctuating characteristics corresponding to the thickness of the slab for different enhancement factors and incident angles. However, this figure has been examined critically by keeping in view the fact that maximum reflection coefficient means maximum scattering of the wave fields towards the receiver so that better amplitude and phase characteristics could be recorded. If this figure is examined in the light of this requirement, it is found that the enhancement factor of 2.5 is best suited as the range of reflection coefficient at y-axis is maximum for this enhancement factor (panel 3). Similarly, for thickness greater than 3 km, the reflection coefficient increases at least at incident angles of 70° and 75°. So, the most suitable values of the model parameters for best reception of the wave fields are $\delta = 2.5$, $L > 3$ km and $70° < \theta < 75°$. More stable values of the reflection coefficients may be obtained if the enhancement factor is increased larger than 3 at thickness between 1 and 3 km. The reflection coefficient as a function of incident angles, for different values of enhancement factor keeping the thickness between 1 and 3 km, has been plotted in Fig. 4. It is found that reflection coefficient
is between 0.6 and 0.9 at large range of incident angles between 73° and 79°. However, if $\delta$ is increased further, the range of angles is narrowed down though the reflection coefficient is increased to a little higher than 0.9 (panel 6). The behaviour of reflection coefficients noted above can be seen in Fig. 5 also in which reflection coefficients are plotted with enhancement factors. The reflection coefficients are maximum for almost all the incident angles at the enhancement factor 3.5.

Although the slab parameters considered above are good enough to explain perturbations caused by any of the geophysical events mentioned, a different model is needed to account for large solar flare event, because in such case the height range of perturbations may increase from 4 to 11 km (ref. 10) and the effect is spatially distributed. This requires a different model, since the model under study is suitable for low and moderate solar flare effect (thickness<5 km) and other events like earthquakes and heating by TLEs where perturbed region is localized and height range is low.

The change in amplitude and phase of the signal at two different angles of incidence may be calculated using the expressions $\Delta R = 20 \log_{10} \frac{R_1}{R_2}$; and $\Delta \phi = \frac{1}{\lambda} \frac{|R_1|^2}{|R_2|^2}$, respectively where, $R_1$ and $R_2$ are the reflection coefficients for two different parameters; and $R$, difference of $R_1$ and $R_2$. By considering the values of $R_1$ and $R_2$ from Fig. 5, where $R_1$ is 0.629 for the incident angles of 79°, enhancement factor 3, and thickness of the slab 1 km, and $R_2$ is 0.785 on increasing the thickness of slab upto 2 km and keeping other parameters same, the change in amplitude and phase are found to be 1.9 dB and 9.80 (0.07 msec), respectively using above mentioned formulae. These results differ slightly with those of observed anomalies of 3 dB and 0.3 msec recorded experimentally in the present study. The difference is because a crude model has been used for the present analysis. Inan & Carpenter3 have emphasized a realistic model of the perturbation to be considered. It is expected that an inverted bell type of irregularity model similar to that of Soloviev et al.11 may yield better results. This work will be undertaken soon and results reported.

5 Conclusions

The interaction of VLF transmitter signals ($f=19.8$ kHz) has been studied with perturbed ionosphere region due to sprite by considering the model in the form of a slab. In this slab model, all the possible parameters like gyrofrequency, electron density, ion density, dielectric constant, refractive index, etc. which are expected to influence the propagation, are taken into account. The results obtained from this model show that strength of reflected signals is better for slab thickness of 3 km, enhancement factor between 2.5 and 3, and incident angles around 75°. These results show the consistency with the anomalies observed in phase and amplitude of sub-ionospheric VLF signals recorded at the present station in the cases of both seismic and TLE events.

Acknowledgement

The authors are thankful to the Department of Science & Technology (DST), New Delhi for financial support for the research. One of the authors (VS) is also thankful to the Director General and Director of KP Engineering College for providing necessary facilities and moral support during the preparation of this manuscript.

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