

Intuitionistic fuzzy set approach for color region extraction

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An improvised technique is proposed in detecting and extracting color regions of an image using intuitionistic fuzzy set (IFS) theory. This will be very useful especially for remotely sensed images where different types of regions (agricultural, watery, or volcanic) are to be extracted. Comparing with non-fuzzy and fuzzy method, results of an extracted image were found better due to consideration of uncertainty in the form of a hesitation degree while defining membership function in IFS.

Keywords: Hesitation degree, Histogram back projection, Intuitionistic fuzzy set, Membership function, Region extraction

Introduction

Region extraction locates known objects, patterns or regions within an image. It is useful in unconstrained images where user doesn't have a prior knowledge of number of regions in an image. Extraction of regions is extremely tedious and time consuming for a large image database. Swain & Ballard¹ used histogram back projection algorithm to detect objects within an image. Smith² used histogram back projection algorithm in extracting color regions in an image. Chua *et al*³ developed a segmentation technique that uses color pairs. Hsu *et al*⁴ extended color pair techniques to perform object extraction. Chaira & Ray⁵ suggested a fuzzy based method for color region extraction. This study proposes a novel method using intuitionistic fuzzy set (IFS) theory for color region extraction.

Proposed Technique

Intuitionistic Fuzzy Set (IFS)

In fuzzy set theory, image is considered to be vague in the form of membership function, which is degree of belongingness of gray levels in an image. Another uncertainty term is hesitation degree (HD), which is 'personal error' or lack of knowledge in defining membership function. So, if HD is taken with membership degree (MD), results are expected to be better because error in defining membership function is also considered. CIELab color model was used for region

extraction. In fuzzy set theory, non-membership degree (NMD) is complement of MD. But manual representation of MD of a particular element cannot always express NMD as its complement. So, intuitionistic fuzzy set (IFS) theory⁶⁻⁸ was proposed to account for NMD, which is less than or equal to complement of MD due to hesitation or lack of knowledge in defining membership function. An IFS, A , in a finite set X may be represented as

$$A = \{(x, \mu_A(x), \nu_A(x)) \mid x \in X\}$$

where $\mu_A(x), \nu_A(x) : X \rightarrow [0,1]$ are membership and non-membership function of an element x respectively in finite set X with the condition,

Parameter $\pi_A(x)$, known as intuitionistic fuzzy index (IFI) or HD, satisfies following condition:

$$\dots(1)$$

where, $\nu_A(x) = 1 - \mu_A(x) - \pi_A(x)$ and $\pi_A(x)$ is IFI or HD with $0 \leq \pi_A(x) \leq 1$ for each

Construction of IFS

In order to construct IFS, intuitionistic fuzzy generators (IFGs)⁹ are used. A function $\varphi : [0, 1] \rightarrow [0, 1]$ is called IFG if $\varphi(x) \leq -x$ and $\varphi(1) < 0$ and $\varphi(0) = 1$, for all $x \in [0,1]$. IFG or fuzzy complement, created from Sugeno generating function¹⁰ as $g(\mu(x)) = 1/\lambda \log(1 + \lambda \cdot \mu(x))$, $g(\cdot)$, is an increasing function and $g : [0, 1] \rightarrow [0, 1]$.

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Sugeno's IFG (using inverse function) is written as

$$N(\mu(x)) = (1 - \mu(x)) / (1 + \lambda\mu(x)), \lambda > 0 \quad \dots(2)$$

where, $N(1) = 0$, $N(0) = 1$

Non-membership values are calculated from Sugeno type IFG $N(\mu(x))$. Thus, IFS is redefined as $A_{IFS} = \{x, \mu_A(x), (1 - \mu_A(x)) / (1 + \lambda\mu_A(x)) \mid x \in X\}$ with HD as

$$\pi_A(x) = 1 - \mu_A(x) - \frac{1 - \mu_A(x)}{1 + \lambda\mu_A(x)} \quad \dots(3)$$

where, denominator, $1 + \lambda\mu_A(x)$ in non-membership term

$\frac{1 - \mu_A(x)}{1 + \lambda\mu_A(x)}$ is > 1 . Thus non-membership term is $< 1 - \mu_A(x)$ for all $x \in X$.

For an image A, an intuitionistic fuzzy image A_{IFS} is defined as $A_{IFS} = (a_{ij}, \mu_A(a_{ij}), \nu_A(a_{ij}) \mid a_{ij} \in A)$, where $\mu_A, \nu_A : A \rightarrow [0, 1]$ are membership and non-membership functions of pixels of image A at $(i, j)^{th}$ point respectively. a_{ij} is pixel value at $(i, j)^{th}$ point of image.

CIE Lab Color Model

RGB color space is a device dependant color model. CIE Lab color space is a perceptually human model and is device independent. It is a complete color space describing all colors visible to human eye. RGB color space is first converted to XYZ color model and then CIE Lab color model¹¹ as

$$L^* = 116 (Y / Y_N)^{(1/3)} - 16 \quad \text{for } (Y / Y_N) > 0.008856$$

$$= 903.3 (Y / Y_N) \quad \text{for } (Y / Y_N) \leq 0.008856$$

$$a^* = 500 [f(X / X_N) - f(Y / Y_N)]$$

$$b^* = 200 [f(Y / Y_N) - f(Z / Z_N)]$$

where, $f(t) = t^{(1/3)}$,

$$t > 0.008856 = 7.787t + 16 / 116, \quad t < 0.008856.$$

X_n, Y_n and Z_n are CIE tristimulus values with reference to white point (D_{65}).

Generalized Tversky's Measure (GTI) using IFS

Modified Tversky's measure using IFS is used in retrieving regions in scene image that find an exact match with query or model image provided. A Generalized Tversky's index (GTI) is defined as^{12,13}

$$GTI(A, B; \alpha, \beta) = \frac{f(A_n \cap B_n)}{f(A_n \cap B_n) + \alpha \cdot f(A_n - B_n) + \beta \cdot f(B_n - A_n)} \quad \dots(4)$$

where, f is non-negative and increasing function. A_n, B_n are histograms of images and $\alpha, \beta \geq 0$. Suffix 'n' is gray level of image.

GTI compares saliency of common features to saliency of distinctive features. GTI is very useful in similarity judgment. $A_n \cap B_n$ denotes common features while $A_n - B_n$ denotes distinctive features. Values α, β determine relative importance of distinctive features in similarity assessment. GTI incorporates fuzzy features only. Histogram A and B denotes counts of gray levels. Membership values of histograms are degree of belongingness in gray levels. In histogram, common feature is common membership value of histograms, $\min(\mu_A(n), \mu_B(n))$. Distinctive feature common membership values of histograms A and (1-B). Maximum membership value in fuzzy space is 1. In fuzzy space, distinctive feature is written as $\min(\mu_A(n), 1 - \mu_B(n))$. It is similar to that in ordinary set theory A-B, where elements present in A and not in B. Using min-t norm, Tversky's measure [Eq. (4)] is rewritten as

$$GTI(A, B; \alpha, \beta) =$$

$$\frac{\sum_{n=1}^L \min(\mu_A(n), \mu_B(n))}{\sum_{n=1}^L (\min(\mu_A(n), \mu_B(n)) + \alpha \cdot \min(\mu_A(n), 1 - \mu_B(n)) + \beta \cdot \min(1 - \mu_A(n), \mu_B(n)))} \quad \dots(5)$$

In modified GTI using IFS, HD is introduced. In Toliás's model, similarity is judged using MDs and change in membership values affects similarity. Likewise, in intuitionistic fuzzy GTI model, change in HDs also affects similarity. NMD also affects similarity. But with NMD in

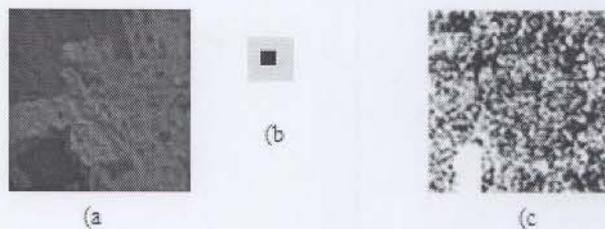


Fig. 1—Result using non- membership values: a) Original image; b) Query image; c) Region extracted image

GTI model, $v(1 - \mu - \pi)$ regions are not properly extracted.

In present work, IFS is used in Tversky's measure. Using Sugeno's IFG, NMD is computed and simultaneously HD is calculated from MD and NMD. Then modified Tversky's measure is defined as

$$GTI_{NT}(A, B, \alpha, \beta) = \frac{\sum_{n=0}^{L-1} \min(\pi_q(n), \pi_t(n))}{\sum_{n=0}^{L-1} [\min(\pi_q(n), \pi_t(n)) + \alpha \cdot \min(\pi_q(n), 1 - \pi_t(n)) + \beta \cdot \min(1 - \pi_q(n), \pi_t(n))]} \dots (6)$$

where $\pi_q(n)$ and $\pi_t(n)$ are HDs in membership of histograms of query image and target image respectively. Values of α and β are chosen as 0.5. If Eq.(6) is replaced with NMD, $v(1 - \mu - \pi)$ regions are not extracted properly (Fig. 1).

Procedure

Initially, color scene image is converted into CIE Lab color space and a query image is selected from scene image, which is fuzzified by assigning MD to each pixel in an image. In this method, similarity between query image and window of scene image is calculated. Window of scene image is the region in scene image where query image was placed.

Membership values of gray levels are calculated by normalizing histograms of query image and non-overlapping windows in scene image. So membership values are normalized values. Normalizing factor (norm) is calculated by taking maximum amongst maximum counts of histogram of query image and windows of scene image as query image is made to slide over scene image. Mathematically, normalizing factor⁵ may be calculated as

$$norm = \max(h_q, z) \dots (7)$$

where, $z = \max_r (\max(h_r))$ $r = 0, 1, 2, \dots, r, \dots, R$, R is total number of windows in scene image. h_q, h_r are histograms of query image and r^{th} window of scene image respectively.

HDs (in membership calculation) are calculated using Eq. (3) as

$$\pi(n) = 1 - \mu(n) - \frac{1 - \mu(n)}{1 + \lambda \cdot \mu(n)} \dots (8)$$

where $\mu(n)$ is membership value of gray level of windows or model of image.

In this experiment, $\lambda=3$ was used and no remarkable change was observed in different values of λ . Then, GTI measure was used to find similarity between windows of scene image and query image. At each window position, hesitation ratio, $f_{hes}(n)$, was computed for each gray level of color histogram. $f_{hes}(n)$, calculated using GTI measure [Eq. (6)], is written as

$$f_{hes}(n) = \frac{\min(\pi_q(n), \pi_t(n))}{[\min(\pi_q(n), \pi_t(n)) + \alpha \cdot \min(\pi_q(n), 1 - \pi_t(n)) + \beta \cdot \min(1 - \pi_q(n), \pi_t(n))]} \dots (9)$$

where $\pi_q(n), \pi_t(n)$ are HDs of n^{th} gray level of histograms of query image and window of scene image respectively with ' n ' representing gray level; $n = \{0, 1, 2, \dots, L - 1\}$.

Each pixel gray level, n , in scene image window is replaced by hesitation ratio, $f_{hes}(n)$. This was done for all scene image windows as model image is moved over scene image. A new matrix of same size as that of scene image is formed. If each point in scene image is $I(i, j) = n$, then $I'(i, j) = f_{hes}(n)$. This is done for all three color channels. Thus, three matrices are formed for all three L^* , a , b color channels. Three matrices are then thresholded at highest peak and then added. Final image is again thresholded at highest peak and then filtered with a Gaussian filter to obtain a region extracted image.

Algorithm

Algorithm follows back projection algorithm (BPA)¹. Image is considered an intuitionistic fuzzy image instead of a non-fuzzy image as in BPA. Ratio or confidence score used by Swain & Ballard¹ to back project on to image is replaced by Eq. (7) for incorporating HDs. Algorithm is as follows: Step 1) Slide model or query image over scene image; Step 2) Compute histograms of model image and windows of scene image; Step 3) Compute membership values of gray level of windows and model image using a normalizing factor in Eq. (7); Step 4) Compute HDs of gray levels of model image and windows of scene image using Eq. (8); Step 5) Form a

likelihood ratio that window of scene image belongs to the model using hesitation ratio in Eq. (9); Step 6) Back project hesitation ratio on to image so that image values are replaced by values of $f_{hes}(n)$; Step 7) Threshold back projected image at its highest peak; and Step 8) Filter thresholded image with Gaussian filter to obtain a region extracted image.

Experimental Results

Experiment was performed on various types of color scene images and also on remotely sensed (RS) images. Results of proposed method on few scene images from VisTex database and RS images were displayed and compared with Smith's non-fuzzy/crisp method² and fuzzy method⁵. In all images, black portion in resultant image is the region in extracted image similar to query image.

Fig. 2a is a complex scene image (size 120 x 120). Light green color grass is query image (size 20 x 20), which is scattered all over the image (Fig. 2b). Resultant images are those using non fuzzy (Fig. 2c) and fuzzy (Fig. 2d) method. Black region in resultant image showed presence of query image. In crisp method, unwanted grassy regions (shown in black spots) were present more in non-grassy region (Fig. 2c)) in lower part of image than in fuzzy method (Fig. 2d)). But in proposed IFS method (Fig. 2e), grassy region i.e. query region in scene

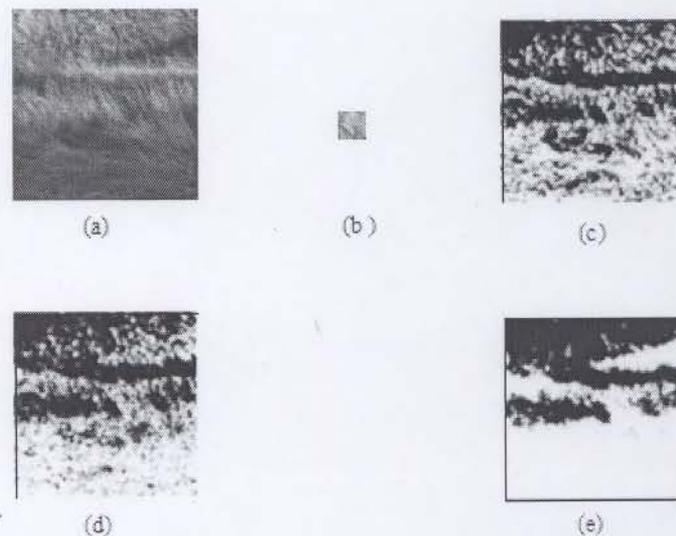


Fig. 2a)—Scene image, b) Query image, c) Region extracted using crisp method, d) Region extracted image using fuzzy method, e) Region extracted image using IFS method

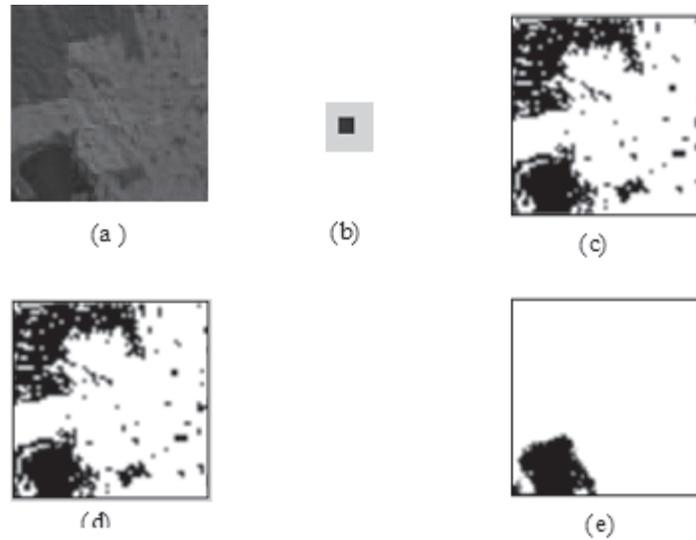


Fig. 3a)—Scene image, b) Query image, c) Region extracted using crisp method, d) Region extracted image using fuzzy method, e) Region extracted using IFS method

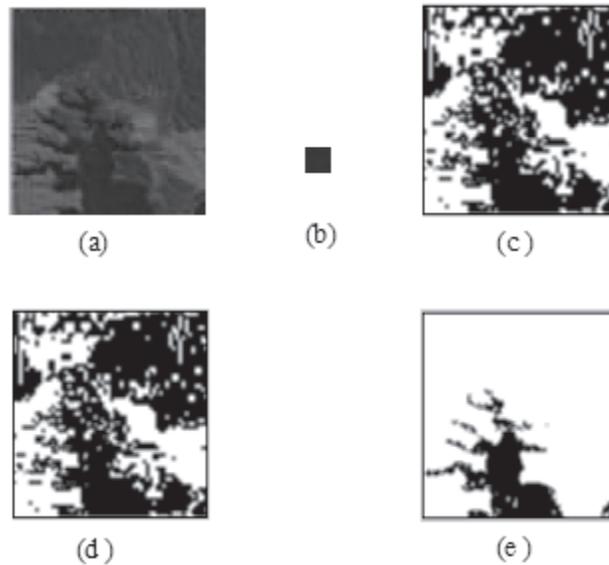


Fig. 4a)—Scene image, b) Query image, c) Region extracted using crisp method, d) Region extracted image using fuzzy method, e) Region extracted using IFS method

image was exactly extracted and query region was not present in non-grassy region as in case of other two methods.

Fig. 3a and Fig. 4a showed remotely sensed images (size 120 x120) containing 500 MB of raw pixel information in 6 bands (3 in visible region and 3 in near

IR region). It covered an area of 204, 105 km² having an urban, agriculture and forestry areas. Query images (size 15 x15) were dense black portion of scene image (Fig. 3b & Fig. 4b). It was observed that non-fuzzy (Fig. 3 c-d) and fuzzy methods (Fig. 4 c-d) did not extract regions properly. Query image was dense black portion selected from lower portion of image, but regions that

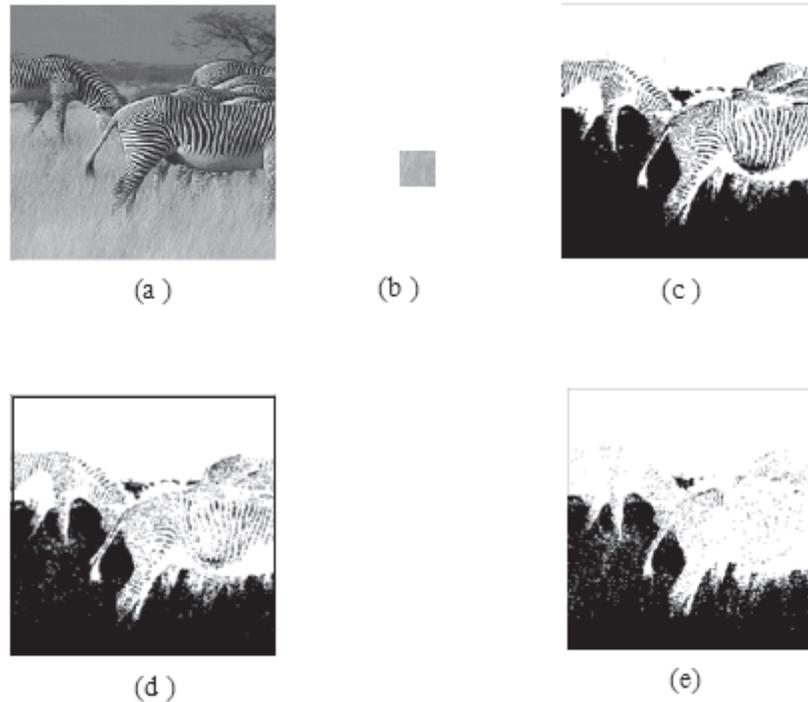


Fig. 5a)—Scene image, b) Query image, c) Region extracted using crisp method, d) Region extracted image using fuzzy method, e) Region extracted image using IFS method

contained a mixture of red and black were also extracted, which should not be the case, as shown in black color in the region extracted images. But in intuitionistic fuzzy method, regions were clearly extracted as similar to query image (Fig. 3e & Fig. 4e).

Fig. 5a that showed zebras in a forest (size 340 x 340) was obtained from Smith². A small query image (size 40 x 40) is a part of scene image that consists of yellow grass (Fig. 5b). Under region extracted images using non-fuzzy (Fig. 5c), fuzzy (Fig. 5d), and intuitionistic fuzzy (Fig. 5e) methods, black region corresponded to query image. It was observed that in crisp method, there were few black stripes in the body of zebras and those were mismatch errors. In fuzzy method, mismatch errors or black stripes in the body of zebras were less. But in intuitionistic fuzzy method, there was hardly any mismatch error and query image was exactly extracted from scene image.

Discussion

In fuzzy set theory, degree of belongingness is defined in terms of membership function. But membership function may vary from person to person and it may or

may not be well defined. Intuitionistic fuzzy sets consider another uncertainty parameter, HDs or personal error in defining membership function. So when membership function is not properly defined, then intuitionistic fuzzy method gives better result than fuzzy method. In non-fuzzy method, image is considered precise and regions/boundaries are assumed to be properly defined; this is not so in a remotely sensed or natural or real time images. So results using non-fuzzy method on real time images are not better.

Conclusions

This paper presents a novel technique for extracting regions of an image using IFS theory. Results were found better or almost similar to the result using fuzzy sets. This improvement was due to consideration of uncertainty in form of HD while defining membership function in Attanassov's IFS. This novel region extraction method will be useful in remote sensing application.

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