

Complex dielectric function and collective dynamics of one-dimensional weakly coupled quantum and classical hot plasmas

S P Tewari

Department of Physics and Astrophysics, University of Delhi, Delhi 110 007

and

Jyoti Sood

Department of Physics, Panjab University, Chandigarh 160 014

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The complete expressions for the wave vector and frequency dependent complex dielectric function for one-dimensional weakly coupled one component quantum and classical hot plasmas have been derived. These expressions, which are valid for the entire domain of frequency and wave vector, are used to obtain the knowledge about the collective dynamics of the plasma by computing the dynamical structure factor at different temperatures and wave vector. The results obtained are quite different from those in three- and two-dimensional cases.

[Keywords: Plasma, Dielectric function, Collective modes]

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1 Introduction

Theoretical study of a physical problem in one dimensional (1D), many a time yields¹ not only exact results, but also much needed insight into the physical problem, even though, such systems are strictly speaking not realizable. However, one can now make physical systems, which could be labeled as quite close to one dimension. For instance, quasi one-dimensional electron system can be realized in quantum wires, which are usually fabricated, on high mobility two dimension (2D) electron systems by adding an additional confinement along one of the remaining free directions using ultrafine lithographic techniques²⁻⁴. As 1D electron systems can achieve higher mobility than the 2D electron systems, since impurity content and distribution around these quantum wires can be selectively controlled, so they have become more important subject for research. This is also because in the low energy regime, the only relevant scattering mechanism is back scattering. Though there have been several theoretical studies⁵⁻¹⁰ in quasi 1D electron systems, these are for temperature equal to zero, which corresponds to highly degenerate quantum systems. However, there is, to our knowledge, no reported theoretical study on

classical and weakly quantum mechanical quasi-1D systems.

One dimensional weakly coupled, one component plasma behaves as classical or quantum mechanical depending upon the value of $n\lambda_{th}$, n being the linear number density and $\lambda_{th}(=h/(2mk_B T))^{1/2}$, T is the temperature, h and k_B are Planck's constant and Boltzmann's constant respectively, and m is the mass, th is the thermal de-Broglie wave length. If $n\lambda_{th} \ll 1$, plasma is classical otherwise it is quantum mechanical. The strength of coupling given by $\Gamma=(2ne^2/k_B T)$ should be equal to or less than 1 for weakly coupled plasma.

Complex dielectric function is an extremely important property, using which one can study various observable physical properties. In the present paper, the studies of generalised wave vector, \mathbf{q} , frequency, ω dependent complex dielectric function for the entire domain of \mathbf{q} and ω are reported. A similar form of dielectric function has recently been suggested for 2D^{11,12} and 3D^{13,14} plasmas. We have also reported the dynamical structure factor, $S(\mathbf{q}, \omega)$ of the quasi 1D system which yields its complete equilibrium dynamics.

2 Mathematical Formalism

The complex dielectric function $\varepsilon^Q(q, \omega)$ for a one-dimensional weakly coupled one component quantum plasma is related to the two particle quantum distribution function $F_{\pm}(q, \omega)$

$$\varepsilon^Q(q, \omega) = 1 + \frac{2\pi i}{h} [F_+(q, \omega) - F_-(q, \omega)] \quad \dots(1)$$

$F_{\pm}(q, \omega)$, after taking into consideration appropriate potential for one-dimensional plasma, is given^{11,13} by the equation:

$$F_{\pm}(q, \omega) = \frac{1}{2\pi i} \frac{e^2}{\pi} \int \frac{dp}{(\omega - \frac{qp}{m} + i\sigma)} f\left(p \pm \frac{hq}{2}\right) \quad \dots(2)$$

Here p is the one-dimensional linear momentum of the electron and σ is infinitesimal small quantity. The single particle momentum distribution function, $f(p \pm \frac{hq}{2})$ is a Maxwellian, given as;

$$f\left(p \pm \frac{hq}{2}\right) = A \exp\left(-\frac{\left(p \pm \frac{hq}{2}\right)^2}{2mk_B T}\right) \quad \dots(3)$$

The constant A can be evaluated using the sum rule,

$$\int_0^{\infty} \omega \operatorname{Im} \varepsilon(q, \omega) d\omega = \frac{\pi}{2} \omega_p^2 \quad \dots(4)$$

where, ω_p is the angular plasma frequency of the one dimensional^{7,15} plasma given as:

$$\omega_p = \sqrt{\frac{2\pi n e^2 q^2 (\ln q)}{m}} \quad \dots(5)$$

To evaluate $F_{\pm}(q, \omega)$ Eq. (3) is substituted in Eq. (2). After solving one-dimensional integral and using the resultant in Eq. (1), we obtain complete frequency and wave vector dependent complex dielectric function as,

$$\varepsilon^Q(q, \omega) = \varepsilon_1^Q(q, \omega) + i\varepsilon_2^Q(q, \omega)$$

$$\varepsilon_1^Q(q, \omega) = 1 + \frac{2\pi i}{h} \frac{e^2}{\pi} \frac{1}{m} \left[\frac{1}{\omega - \frac{qp}{m} + i\sigma} - \frac{1}{\omega - \frac{qp}{m} - i\sigma} \right]$$

$$\left[\left(\frac{\omega}{\sqrt{2}qv} + \frac{hq}{2\sqrt{2}mv} \right) \left(1 + \frac{1}{3} \left(\frac{\omega}{\sqrt{2}qv} + \frac{hq}{2\sqrt{2}mv} \right)^2 + \dots \right) - \left(\frac{\omega}{\sqrt{2}qv} - \frac{hq}{2\sqrt{2}mv} \right) \left(1 + \frac{1}{3} \left(\frac{\omega}{\sqrt{2}qv} - \frac{hq}{2\sqrt{2}mv} \right)^2 + \dots \right) \right] \quad \dots(6)$$

$$\varepsilon_2^Q(q, \omega) = i \sqrt{\frac{\pi}{2}} \frac{\omega_p^2}{q^2 v^2} \frac{mv}{hq}$$

$$\left[e^{-\left(\frac{\omega}{\sqrt{2}qv} - \frac{hq}{2\sqrt{2}mv} \right)^2} - e^{-\left(\frac{\omega}{\sqrt{2}qv} + \frac{hq}{2\sqrt{2}mv} \right)^2} \right] \quad \dots(7)$$

where, $v = (k_B T/m)^{1/2}$ is the thermal velocity. In Eq. (7), when h is put equal to zero, it reduces to the corresponding expression for the classical plasma, which is similar to the expression obtained using Boltzmann transport equation approach,

$$\varepsilon^{Cl}(q, \omega) = \varepsilon_1^{Cl}(q, \omega) + i\varepsilon_2^{Cl}(q, \omega)$$

$$\varepsilon_1^{Cl}(q, \omega) = 1 + \frac{\omega_p^2}{q^2 v^2} - \frac{\omega_p^2}{q^2 v^2} \frac{\omega^2}{q^2 v^2} e^{-\frac{\omega^2}{2q^2 v^2}}$$

$$\left[1 + \frac{1}{3} \left(\frac{\omega^2}{q^2 v^2} \right) + \frac{1}{10} \left(\frac{\omega_p^2}{q^2 v^2} \right)^2 + \dots \right] \quad \dots(8)$$

$$\epsilon_2^{cl}(q, \omega) = i \sqrt{\frac{\pi}{2}} \frac{\omega_p^2}{q^2 v^2} \frac{\omega}{qv} e^{-\frac{\omega^2}{2q^2 v^2}} \quad \dots(9)$$

It may be emphasised that in the above Eqs 8 and 9, q is one-dimensional vector and ω_p^2 is given by Eq. (5), quite unlike the cases in 2D^{11,12} and 3D¹³ plasmas.

The collective modes of the plasma can be obtained from the dynamical structure factor, $S(q, \omega)$, the quantum expression for which can be derived from fluctuation dissipation theorem^{11,13} and is given as:

$$S^Q(q, \omega) = \frac{1}{\pi \omega} \frac{q^2}{q_-^2} \frac{\epsilon_2^Q(q, \omega)}{[\epsilon_1^Q(q, \omega)^2 + (\epsilon_2^Q(q, \omega))^2]} \quad \dots(10)$$

Here,

$$q_-^2 = \frac{1}{\lambda_D^2} = \frac{\omega_p^2}{v^2}$$

λ_D is one-dimensional screening length. $\epsilon_1^Q(q, \omega)$ and $\epsilon_2^Q(q, \omega)$ are, respectively, the real and imaginary parts of quantum dielectric function given by Eqs 6(a) and 6(b). Although, collective modes of the plasma can be obtained from relation $\epsilon_l(q, \omega) = 0$, but the computations there are very involved as $\epsilon_l(q, \omega)$ is a polynomial of infinite order in q^2 . The singularity in $S(q, \omega)$ gives the value of collective modes corresponding exactly to the value given by $\epsilon_l(q, \omega) = 0$. The other parameters related to $S(q, \omega)$ such as zero frequency dynamical structure factor is given by the expression,

$$S^Q(q, 0) = \frac{1}{\sqrt{2\pi}} \frac{1}{(\epsilon_1^Q(q, 0))^2} \frac{1}{qv} e^{-\left(\frac{hq}{2\sqrt{2}mv}\right)^2} \quad \dots(11)$$

The static structure factor, $S(q)$ which is the zeroth sum rule,

$$S(q) = \int_{-\infty}^{\infty} S(q, \omega) d\omega \quad \dots(12)$$

is related to the dielectric function as,

$$S^Q(q) = -\frac{q^2}{q_-^2} \left(\frac{1}{\epsilon^Q(q, 0)} - 1 \right) \quad \dots(13)$$

The corresponding expressions for the classical plasma are,

$$S^{cl}(q, \omega) = \frac{1}{\pi \omega} \frac{q^2}{q_-^2} \frac{\epsilon_2^{cl}(q, \omega)}{[(\epsilon_2^{cl}(q, \omega))^2 + (\epsilon_2^{cl}(q, \omega))^2]} \quad \dots(14)$$

$$S^{cl}(q, 0) = \frac{1}{\sqrt{2\pi}} \frac{1}{(1 + q_-^2/q^2)^2} \frac{1}{qv} \quad \dots(15)$$

$$S^{cl}(q) = \frac{q^2}{q^2 + q_-^2} \quad \dots(16)$$

One must note that in computing $S(q, \omega)$, the value of $S(q)$ must be consistent from Eqs (12) and (13), for the quantum plasma and Eqs (12) and (16), for the classical plasma.

3 Results and Discussion

For the computation of dielectric function for one-dimensional plasma, given by Eqs 6, 7, 8 and 9, the linear number density, n , of electrons is taken to be $0.56 \times 10^6 \text{ cm}^{-1}$ and temperature $T = 1000 \text{ K}$ with the effective mass of electrons equal to the mass of an electron. The real and imaginary parts of the dielectric function, $\epsilon_1(q, \omega)$ and $\epsilon_2(q, \omega)$, respectively, have been computed for different values of quantum

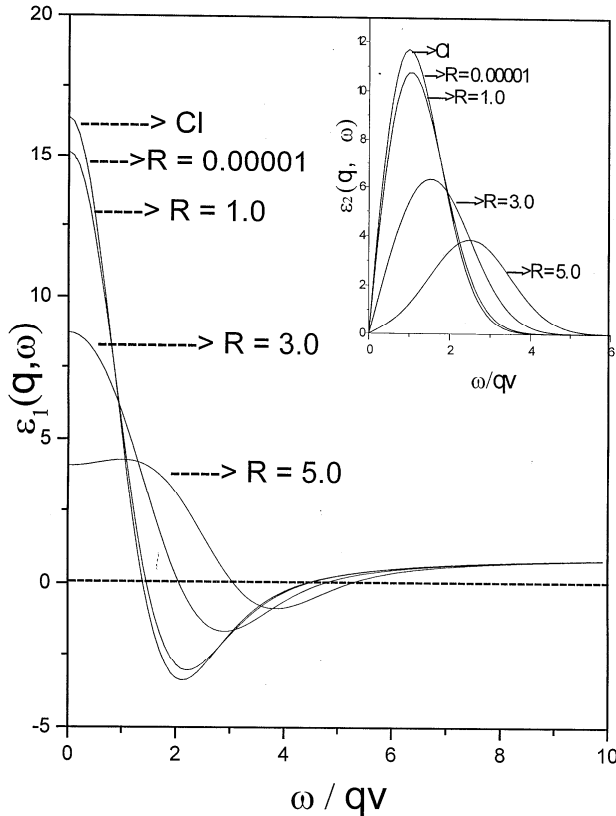


Fig. 1 — Variation of computed values of the real, $\varepsilon_1(q, \omega)$ and imaginary, $\varepsilon_2(q, \omega)$ parts of the dielectric function for one-dimensional plasma with ω/qv for different values of quantum parameter, $R = 0.00001, 1.0, 3.0, 5.0$ for quantum plasma when the linear number density n , of the electrons is considered to be $0.56 \times 10^6 \text{ cm}^{-1}$ and $T = 1000\text{K}$. Cl refers to classical variation

parameter $R(=h\bar{q}/mv)$, which indicates the deviation from classical description and are plotted in Fig.1. As can be seen from Fig. 1, if the value of R is taken to be very small, i.e., 0.00001, the quantum results approach the classical values. As R increases from 1 to 5, $\varepsilon_1^Q(q, \omega)$ becomes increasingly different from $\varepsilon_1^Cl(q, \omega)$. The result is more significant when ω/qv is small. In the inset of the figure are shown the results for imaginary part of the dielectric function. Here, also the difference between $\varepsilon_2^Q(q, \omega)$ and $\varepsilon_2^Cl(q, \omega)$ increases with the increase in R which results in broadening of the function and shift in the peak values to higher ω/qv values. For small ω/qv value, Landau damping is less while damping is more for larger values of ω/qv . $\varepsilon_1(q, \omega)$ consists of a series, which has to be properly summed (considering as many as 160 terms) to ensure the convergence.

In Fig. 2, the dynamical structure factor computed using Eqs (10) and (14) for quantum and

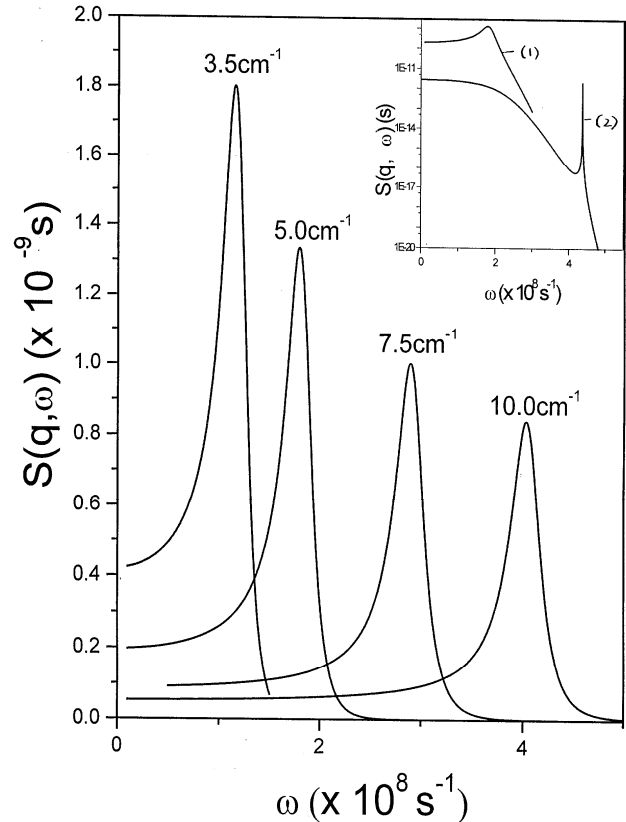


Fig. 2 — Variation of the dynamical structure factor, $S(q, \omega)$ with frequency, ω at $n = 0.56 \times 10^6 \text{ cm}^{-1}$ and $T = 1000\text{K}$ for different values of wave vector, $q = 3.5, 5.0, 7.5$ and 10 cm^{-1} . In the inset of the figure $S(q, \omega)$ with ω has been plotted for $n = 0.56 \times 10^6 \text{ cm}^{-1}$ and $n = 0.56 \times 10^7 \text{ cm}^{-1}$ marked as (1) and (2), respectively, for $q = 5.0 \text{ cm}^{-1}$ and $T = 1000 \text{ K}$

classical plasma respectively has been plotted for different values of $q = 3.5, 5.0, 7.5$ and 10 cm^{-1} for $n = 0.56 \times 10^6 \text{ cm}^{-1}$ and $T = 1000 \text{ K}$. In these parameters, value of R considered is very small so one hardly finds any difference in quantum and classical results. The collective modes become less well defined although the damping is not very evident in Fig. 2. The details of this graph have been shown in Fig. 3. Here one sees a striking difference from 2D and 3D plasmas. The value of $S(q, 0)$ decreases with the increase in the value of the wave vector as shown in Fig. 3(a), while in 2D and 3D plasmas it increases with the increase in q . The variation in the value of the peak in the collective modes, $S(q, \omega_c)$ decreases with q as seen from Fig. 3(b). ω_c , at which $S(q, \omega)$ peaks, is shifting to higher frequency as seen in Fig. 3(c). The static structure factor, $S(q)$ as calculated from Eq. (16) and found to be same as that given by Eq. (12) has been plotted in Fig. 3(d). So from the

Table 1 — Values of $S(q,0)$, $S(q,\omega_c)$ and ω_c for classical plasma for $n = 0.56 \times 10^6 \text{ cm}^{-1}$ and $T = 0.9 \text{ K}$

$q(\text{cm}^{-1})$	$S(q,0)(s)$	$S(q,\omega_c)(s)$	$(\omega_c)(s^{-1})$
1.0×10^1	1.907×10^{-15}	3.16×10^{-68}	6.15×10^7
1.0×10^2	4.77×10^{-17}	1.2×10^{-70}	6.14×10^8
1.0×10^3	2.12×10^{-18}	5.217×10^{-71}	6.16×10^9
1.0×10^4	1.186×10^{-19}	6.708×10^{-70}	6.15×10^{10}
1.0×10^5	8.913×10^{-21}	2.486×10^{-73}	6.16×10^{11}

Table 2 — Values of $S(q,0)$, $S(q,\omega_c)$ and ω_c for quantum plasma for $n = 0.56 \times 10^6 \text{ cm}^{-1}$ and $T = 0.9 \text{ K}$

$q(\text{cm}^{-1})$	$S(q,0)(s)$	$S(q,\omega_c)(s)$	$(\omega_c)(s^{-1})$
1.0×10^1	1.907×10^{-15}	1.294×10^{-97}	7.712×10^7
1.0×10^2	4.77×10^{-17}	1.609×10^{-102}	7.76×10^8
1.0×10^3	2.12×10^{-18}	6.26×10^{-103}	7.8×10^9
1.0×10^4	1.186×10^{-19}	6.789×10^{-105}	7.82×10^{10}
1.0×10^5	8.913×10^{-21}	3.398×10^{-104}	8.03×10^{11}

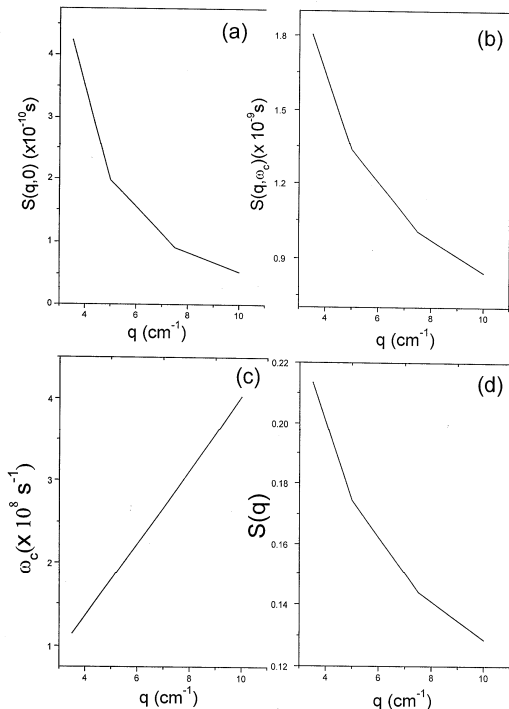


Fig. 3 — Variation of the parameters: (a) zero frequency dynamical structure factor, $S(q,0)$; (b) peak value of dynamical structure factor, $S(q,\omega_c)$; (c) collective mode frequency, ω_c and (d) static structure factor, $S(q)$ has been shown with wave vector, q

derived expressions for the dielectric function it is not only possible to obtain the collective modes but also

the static structure factor. Results have also been checked by satisfying the relation, $\epsilon_l(q,\omega_c=0)$. In the inset of Fig. 2, two linear densities have been compared. The densities considered are $0.56 \times 10^6 \text{ cm}^{-1}$ and $0.56 \times 10^7 \text{ cm}^{-1}$ for $q=5.0 \text{ cm}^{-1}$ marked as (1) and (2), respectively, in Fig. 2. The collective modes for higher density are much more well defined as can be seen from the peak value, which occurs at higher frequency as compared to lower density. One may note that the parameters considered here apply mainly to classical plasmas, i.e., $R \ll 1$. We have performed calculations for appropriate values of q and temperature, which give the value of R nearly equal to 1, therefore making the plasma quantum mechanical. The values obtained, which are quite different for classical and quantum plasma have been given in Table 1 and Table 2, respectively, for $n = 0.56 \times 10^6 \text{ cm}^{-1}$ and $T = 0.9 \text{ K}$ for different values of q . The value of $S(q,0)$ is same for both the plasmas, classical and quantum mechanical. With the increase in frequency, values of $S(q,\omega)$ show a sharp decrease. Although, there is a peak at some higher value of frequency, but it is not significant enough to be seen in the graph. The quantum plasma peaks at higher frequency than the classical plasma.

Lastly in Fig. 4, $S(q,\omega)$ has been calculated at different temperatures: 400, 600, 800, 1000, 1200K

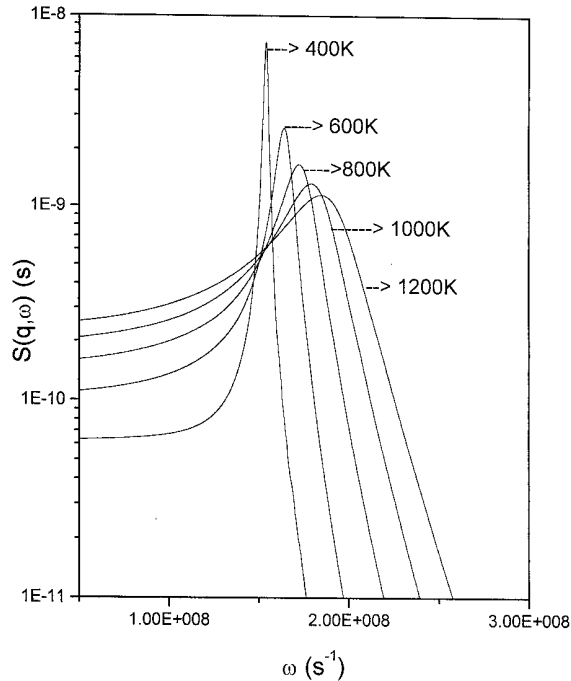


Fig. 4 — Variation of dynamical structure factor, $S(q, \omega)$ with ω for $n = 0.56 \times 10^6 \text{ cm}^{-3}$, $q = 5.0 \text{ cm}^{-1}$ at different temperatures: 400, 600, 800, 1000 and 1200 K

for one-dimensional classical plasma. As expected, the collective modes become less well defined with the increase in temperature. The peak at which collective modes occur, shifts to higher frequency and becomes broadened with the increase in temperature.

4 Conclusion

We may conclude that derived expressions for the complex dielectric function for both quantum and

classical one-dimensional plasmas can be computed for the entire range of frequency. The complete dynamics of the collective modes can be studied by evaluating dynamical structure factor using real and imaginary parts of the dielectric function.

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