Effect of several irreversibilities on the thermo-economic performance of a realistic Brayton heat engine cycle

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The thermo-economic study of a more realistic regenerative Brayton cycle heat engine with finite heat capacity of external reservoirs is presented in this paper. The external irreversibility is due to finite temperature differences between the external reservoirs and the heat engine while the internal irreversibilities are due to the non-isentropic processes in the compressor and turbine and the regenerative heat loss. The thermo-economic function is defined as the power output divided by the total cost plus the running and maintenance costs of the system. The objective function is optimized with respect to the cycle temperatures and the optimum performance parameters are calculated for a typical set of operating conditions. It is found that the effect of the turbine efficiency is more than that of the compressor efficiency on all the performance parameters. It is also found that the effects of the sink-side heat capacitance rate are more than those of the heat capacitance rates on the source-side and working fluid on all the performance parameters of the cycle for the same set of operating conditions.

Keywords: Thermo-economic function, Brayton heat engine cycle, Regenerator, Turbine, Compressor
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1 Introduction

Brayton cycles have been extensively used in power plants and airplanes. The concept of finite time thermodynamics was successfully applied to the Carnot cycle¹-⁵. Lefè⁶ found that the imposition of the maximum work add constraints on the Brayton cycle temperatures. Bejan⁷ showed that the thermal efficiency of the Brayton cycle is independent of how the total conductance is distributed between two heat exchangers. However, the power output is maximized when the cycle is positioned between the two heat reservoirs.

In recent years, the concept of finite time thermodynamics has been successfully used in the investigation of the Brayton heat engine by using power optimization⁸-⁳³, ecological optimization³⁴,³⁵, power density optimization³⁶-⁴² with a single heat source, while others⁴³-⁵⁰ have used two heat sources and found that there is a significant improvement in the efficiency (above 15%) of a Brayton cycle using two heat sources. Very recently, some researchers⁵¹-⁵⁷ have applied the concept of thermo-economic optimization on the various thermal cycles for different operating conditions.

In this paper, a more general analysis on thermo-economic optimization with a detailed parametric study of a more realistic regenerative Brayton heat engine cycle for the finite heat capacity of external reservoirs has been presented.

2 System Description

A regenerative closed Brayton heat engine cycle coupled with a heat source and a heat sink of finite heat capacities is shown on the T-S diagram of Fig. 1. The working fluid enters the compressor at state 4 and compressed up to state 1 and then enters the regenerator where it is partially heated up to state 1R by the turbine exhaust. In an ideal or real regenerator the working fluid leaves the regenerator at the temperature equal to or less than the turbine exhaust (T₃) i.e. T₁R ≤ T₃. The primary heat addition takes place between state 1 and 1R. The working fluid leaving the regenerator enters the hot side heat exchanger and heated up to state 2 by a heat source of finite heat capacity whose temperature varies from T_H₁ to T_H₂.

The working fluid enters the turbine and expands up to state 3 and the turbine exhaust enters the regenerator where it transfers heat partly to the compressor outlet. The working fluid then enters the cold side heat exchanger and cooled up to state 4 by rejecting the heat to the heat sink of finite heat capacity whose temperature increases from T_L₁ to T_L₂ thereby, completing the cycle. The processes 4-1S and 4-1 are respectively the ideal and real compression
processes in the compressor. Similarly, 2-3S and 2-3 are respectively the ideal and real expansion processes in the turbine. Thus, we consider a closed Brayton cycle 4-1-1R-2-3-3R-4 with real compression and expansion processes for finite heat capacity of external reservoirs.

3 Thermodynamic Analysis

The heat transfer rates to and from the cycle ($Q_H$ and $Q_L$) are given by:

$$Q_H = U_H A_H (LMTD)_H = C_H (T_2 - T_1R) = C_H (T_{H1} - T_{H2})$$

\[ \text{\ldots (1)} \]

$$Q_L = U_L A_L (LMTD)_L = C_L (T_3R - T_4) = C_L (T_{L2} - T_{L1})$$

\[ \text{\ldots (2)} \]

and the regenerative heat transfer rate is given by:

$$Q_R = U_R A_R (LMTD)_R = C_w (T_{1R} - T_i) = C_w (T_3 - T_{3R})$$

\[ \text{\ldots (3)} \]

where

\begin{align*}
(LMTD)_H & = \frac{(T_{H1} - T_2) - (T_{H2} - T_{1R})}{\ln(T_{H1} - T_2) / (T_{H2} - T_{1R})} \text{\ldots (4)} \\
(LMTD)_L & = \frac{(T_{3R} - T_4) - (T_{3} - T_{L1})}{\ln(T_{3R} - T_4) / (T_3 - T_{L1})} \text{\ldots (5)} \\
(LMTD)_R & = \frac{(T_{3} - T_{1R}) - (T_{3R} - T_i)}{\ln(T_{3} - T_{1R}) / (T_{3R} - T_i)} \text{\ldots (6)}
\end{align*}

$U_H A_H$, $U_L A_L$ and $U_R A_R$ are respectively, the heat transfer coefficient-area products for the hot-, cold- and regenerative-side heat exchangers, and $C_H$, $C_L$ and $C_w$ are the heat capacitance rates of the fluids in the heat source, heat sink reservoirs and within the heat engine respectively. From Eqs (1-6) we have

$$Q_H = \varepsilon_H C_H \min (T_{H1} - T_{1R}) = C_w (T_2 - T_{1R})$$

\[ \text{\ldots (7)} \]

$$Q_L = \varepsilon_L C_L \min (T_{3R} - T_{L1}) = C_w (T_3 - T_4)$$

\[ \text{\ldots (8)} \]

$$Q_R = \varepsilon_R C_w (T_3 - T_i) = C_w (T_3 - T_{3R})$$

\[ \text{\ldots (9)} \]

where $\varepsilon_H$, $\varepsilon_L$ and $\varepsilon_R$ are the effectiveness of the hot-, cold- and regenerative-side heat exchangers respectively, and for counter flow heat exchangers, defined as:

$$\varepsilon_H = \frac{1 - e}{1 - \varepsilon_H \min} \left( \frac{C_H \max}{C_H \min} \right)$$

\[ \text{\ldots (10)} \]

$$\varepsilon_L = \frac{1 - e}{1 - \varepsilon_L \min} \left( \frac{C_L \max}{C_L \min} \right)$$

\[ \text{\ldots (11)} \]

$$\varepsilon_R = N_R / (1 + N_R)$$

\[ \text{\ldots (12)} \]

where $C_{J, \min} = \min(C_H, C_L, C_w)$, $C_{J, \max} = \max(C_H, C_L, C_w)$, $(J = H, L)$ and $N_H = U_H A_H C_{H, \min}$, $N_L = U_L A_L C_{L, \min}$ and $N_R = U_R A_R / C_w$, are respectively, the number of heat transfer units, based on the minimum thermal capacitance rates:

The compressor and turbine efficiencies are defined as:

$$\eta_C = \frac{(T_{1S} - T_4)}{(T_1 - T_4)}$$

\[ \text{\ldots (13)} \]

$$\eta_T = \frac{(T_2 - T_3)}{(T_2 - T_{3S})}$$

\[ \text{\ldots (14)} \]

Now from Eqs. (7-14), we have:

$$T_{3R} = (1 - \varepsilon_R) T_3 + \varepsilon_R T_1$$

\[ \text{\ldots (15a)} \]

$$T_{1R} = (1 - \varepsilon_R) T_1 + \varepsilon_R T_3$$

\[ \text{\ldots (15b)} \]

$$T_4 = (1 - \varepsilon_R) T_3R + \varepsilon_R T_{3S}$$

\[ \text{\ldots (15c)} \]
The objective function of thermo-economic optimization as proposed by earlier researchers [2, 51-52, 57] is given by:

$$F = \frac{P}{C_i + C_e + C_m} \quad \ldots(22)$$

where $C_i$, $C_e$ and $C_m$ refer to annual investment, energy consumption and maintenance costs, respectively. The investment cost was considered the costs of the main system components that are the heat exchangers and the compression and expansion devices together. The investment cost of the heat exchangers is assumed to be proportional to the total heat transfer area [51,52]. On the other hand, the investment cost due to the compression and expansion devices is assumed to be proportional to their compression/expansion capacities. Thus, the total annual investment cost of the system can be given by:

$$C_i = a_i (A_H + A_L + A_R) + a_p (Q_h - Q_L)/t_{cycle} \quad \ldots(23a)$$

where the proportionality constant for the investment cost of the heat exchanger, $a_i$ is equal to the capital recovery factor times investment cost per unit heat exchanger area and the proportionality constant for the investment cost for the compression and expansion devices, $a_p$ is equal to the capital recovery factor times investment cost per unit power output. The initial investment cost is converted to equivalent yearly payment using capital recovery factor [51,52]. The annual energy consumption [51,52] and maintenance costs [2,57] are proportional to the energy input and power output respectively, as given below:

$$C_e = a_q Q_H \quad \ldots(23b)$$

$$C_m = b_p P = b_p (Q_H - Q_L) \quad \ldots(23c)$$

where the coefficient, $a_q$ is equal to the equivalent annual operation hours corresponding to energy input times price per unit energy [51,52] and the coefficient, $b_p$ is equal to the equivalent annual operation hours per unit power output. Substituting Eqs 23(a-c) into Eq. (22) we have:

$$F = \frac{P}{a_i (A_H + A_L + A_R) + a_p (Q_h - Q_L) + a_q Q_H} \quad \ldots(24)$$

where $a_i$, $a_q$ and $a_p$ refer to annual investment, energy consumption and maintenance costs, respectively. The investment cost was considered the costs of the main system components that are the heat exchangers and the compression and expansion devices together. The investment cost of the heat exchangers is assumed to be proportional to the total heat transfer area [51,52]. On the other hand, the investment cost due to the compression and expansion devices is assumed to be proportional to their compression/expansion capacities. Thus, the total annual investment cost of the system can be given by:

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$$C_e = a_q Q_H \quad \ldots(23b)$$

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where the coefficient, $a_q$ is equal to the equivalent annual operation hours corresponding to energy input times price per unit energy [51,52] and the coefficient, $b_p$ is equal to the equivalent annual operation hours per unit power output. Substituting Eqs 23(a-c) into Eq. (22) we have:

$$F = \frac{P}{a_i (A_H + A_L + A_R) + a_p (Q_h - Q_L) + a_q Q_H} \quad \ldots(24)$$
where \( b = a_p + b_p \). Thus, from Eqs (22-24), we have:

\[
bF = \frac{c_6 - a_6 T_1 - b_1 T_1}{k_1 k_1 + (c_5 - a_5 T_1 - b_5 T_1)} = \frac{c_6 - a_6 T_1 - b_6 T_1}{(c_5 - a_5 T_1 - b_5 T_1)} \quad \text{(25)}
\]

where

\[
k_1 = a_p/b, k_2 = a_q/b, a_q = (a_7 + k_3 a_9), b_q = (b_7 + k_3 b_9),
\]

\[
c_8 = (c_6 + k_2 c_7), c_9 = (k_2 k_3 + c_9), k_3 = (A_H + A_L + A_R),
\]

\[
A_H = (C_H / U_H) / (1 - C_{H,\text{max}} / C_{H,\text{min}})
\]

\[
\times \ln \left( \frac{1 - \varepsilon_H C_{H,\text{max}}}{C_{H,\text{min}}} \right) / (1 - \varepsilon_H)
\]

\[
A_L = (C_L / U_L) / (1 - C_{L,\text{min}} / C_{L,\text{max}})
\]

\[
\times \ln \left( \frac{1 - \varepsilon_L C_{L,\text{max}}}{C_{L,\text{min}}} \right) / (1 - \varepsilon_L)
\]

and

\[
A_R = \frac{C_w \varepsilon_R}{U_R (1 - \varepsilon_R)}
\]

4 Results and Discussion

In order to have the numerical appreciation of the results on thermo-economic optimization of an irreversible regenerative Brayton heat engine, we continue to investigate the effects of the turbine and compressor efficiencies (\( \eta_T \) and \( \eta_C \)), the effectiveness of the heat exchangers (\( \varepsilon_H \) and \( \varepsilon_L \)), the economic parameters (\( k_1 \) and \( k_2 \)) and the heat capacitance rates (\( C_{H}, C_{L} \) and \( C_w \)). While the effect of each one of these parameters is examined and the rest of the parameters are kept constant as \( \varepsilon_H = \varepsilon_L = \varepsilon_R = 0.75 \), \( T_{H1} = 1250 \text{K} \), \( T_{L1} = 300 \text{K} \), \( k_1 = 0.50 \), \( k_2 = 0.10 \), \( C_H = C_L = 1.0 \text{ kW/K} \), \( C_w = 1.05 \text{ kW/K} \), \( \eta_T = \eta_C = 0.80 \) and \( U_H = U_L = U_R = 2.0 \text{ kW/K-m}^2 \) and discussion of the result is as follows:

4.1 Effect of effectiveness

Figures 2(a-c) show the effects of the hot-, cold- and regenerative side effectiveness on the maximum objective function, the corresponding power output and thermal efficiency of an irreversible regenerative Brayton heat engine. It is seen from Figs 2(a-c) that the maximum objective function first increases, attains its maximum and then slightly decreases, while the corresponding power output and thermal efficiency increase as the effectiveness on either side heat exchanger increases. It is also seen that the effect of the cold side effectiveness is more pronounced for all the performance parameters viz. the maximum objective function and the corresponding power output and thermal efficiency of an irreversible regenerative Brayton heat engine cycle.

The result obtained in this section can be explained on the basis of the heat transfer area. For the typical parameters given above, when the effectiveness increases, it is required to increase the heat transfer area, so that the cost of the system and the power output increase. However, in general, the objective function is not a linear function of the power output. As a result, there is a maximum for the maximum objective function as the effectiveness increases.

4.2 Effect of heat capacitance rates

The effects of different side heat capacitance rates on the maximum objective function, the
corresponding power output and thermal efficiency are shown in Figs 3(a-c). It is clear from the point of view of thermodynamics that the larger the heat capacitance rates of the external reservoirs are, the smaller the temperature difference of heat-transfer between the working fluid and the external reservoirs is required, the smaller the irreversibility of finite-rate heat transfer, and the better the performance of the cycle. On the other hand, the larger the heat capacitance rate of the working fluid is, the larger the temperature difference of heat-transfer between the
working fluid and the external reservoirs is required, the larger the irreversibility of finite-rate heat transfer, and the worse the performance of the cycle. Thus, the maximum objective function, the corresponding power output and thermal efficiency increases as the heat capacitance rates on hot- and cold-side reservoir increase, while all the parameters i.e. the maximum objective function, the corresponding power output and thermal efficiency decrease as the heat capacitance rate within the heat engine increases. Also, it is seen that the effect of cold-side heat capacitance rates is more pronounced than the other side heat capacitance rates on all the performance parameters.

4.3 Effect of economic parameters
The effect of economic parameters \( (k_1 \text{ and } k_2) \) on the maximum objective function is shown in Fig 4. The maximum objective function decreases as the economic parameters increase but the effect of \( k_2 \) is more pronounced than \( k_1 \) (Fig. 4). As the cost of the system is dependent on the economic parameters, as a result the objective function will decrease as the cost of the system increases. Since the economic parameter \( k_1 \) belongs only to the investment cost while the economic parameter \( k_2 \) belongs not only to the investment cost but also to the running cost of the system, so the running cost is more effective than that of the investment cost.

4.4 Effect of component efficiencies
The effects of the component efficiency \( \eta_T \text{ and } \eta_C \) on the maximum objective function, the corresponding power output and thermal efficiency of an irreversible regenerative Brayton heat engine are shown in Figs 5(a-c). It is obvious from the point of view of thermodynamics that the larger the component efficiency is, the better the performance of the cycle. Thus, the maximum objective function, the corresponding power output and thermal efficiency increase as the efficiency of either component i.e. the

![Fig. 4—Objective function versus economic parameters](image1)

![Fig. 5(a)—Objective function versus component efficiency](image2)

![Fig. 5(b)—Power output versus component efficiency](image3)

![Fig. 5(c)—Thermal efficiency versus component efficiency](image4)
compressor or the turbine increases. Again, the effects of the turbine efficiency are found to be more than that of the compressor efficiency not only on the thermodynamic performance but also on the economic performance of the cycle.

5 Conclusions

A more realistic regenerative Brayton heat engine cycle model including external and internal irreversibilities for the finite heat capacities of external reservoirs has been studied in detail. The thermo-economic function which is the power output per unit cost of the system, adopted as an objective function for maximization. The objective function is maximized with respect to the cycle temperatures and the corresponding power output and thermal efficiency are calculated on the different operating conditions. The thermo-economic function is found to be the increasing function of the hot- and cold-side heat capacitance rates, component efficiencies while it is found to be the decreasing function of the heat capacitance rate of the working fluid and the economic parameters \(k_1\) and \(k_2\). On the other hand, there are optimal values of the various effectiveness at which the objective function attains its maximum for a typical set of operating condition. It is seen that the effects of the turbine efficiency are more pronounced than that of the compressor efficiency on the maximum objective function as well as on the corresponding power output and thermal efficiency. It is also found that effects of the cold-side heat capacitance rate are more than those of the other parameters on all the performance parameters of the cycle. The heat capacitance rates \(C_H, C_I\) and \(C_S\) also play an important role in the optimal performance of this cycle and it is found that these heat capacitance rates must be in the order of \(C_L > C_H > C_W\) for the better performance of the cycle. The results obtained here are useful to understand and design an irreversible regenerative Brayton cycle from the point of view of thermodynamics as well as from the point of view of economics.

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Nomenclature

\[ A = \text{Area (m}^2\) \]
\[ C = \text{Heat capacitance rates (kW/K)} \]
\[ k = \text{Specific heat ratio} \]
\[ N = \text{Number of heat transfer units} \]
\[ P = \text{Power output (kW)} \]
\[ Q = \text{Heat transfer rates (kW)} \]
\[ S = \text{Isentropic} \]
\[ T = \text{Temperature (K)} \]
\[ U = \text{Overall heat transfer coefficient (kW/m}^2\text{/K)} \]
\[ 1, 2, 3, 4 = \text{State points} \]

Greeks

\[ \eta = \text{Thermal efficiency} \]
\[ \epsilon = \text{Effectiveness} \]

Subscripts

\[ a = \text{Area cost} \]
\[ C = \text{Compressor} \]
\[ H = \text{Hot side/heat source} \]
\[ L = \text{Sink/Cold side,} \]
\[ R = \text{Regenerator} \]
\[ S = \text{Ideal/isentropic} \]
\[ T = \text{Turbine} \]
\[ W = \text{Working fluid} \]
\[ \text{max} = \text{Maximum/optimum} \]
\[ q = \text{Heat input cost} \]
\[ p = \text{Power cost} \]
\[ m = \text{Maintenance cost} \]
\[ i = \text{Investment cost} \]
\[ e = \text{Energy input cost} \]

References