Mathematical model for a multiple effect evaporator system with condensate-, feed- and product- flash and steam splitting

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In the present investigation a non-linear mathematical model is developed for the analysis of multiple effect evaporators (MEE) system. This model is capable of simulating process of evaporation and takes into account variations in boiling point rise (BPR) due to variation in concentration and temperature of liquor, overall heat transfer coefficient (OHTC) of effects and physico-thermal property of the liquor. Using mass and energy balance around an effect, a cubic polynomial equation is developed to model an effect, which is solved using generalized cascade algorithm. For this purpose, a Septuple effect flat falling film evaporator (SEFFFE) system with backward feed flow sequence, being used for concentrating weak black liquor and operating in a nearby paper mill, is selected as a typical MEE system. This system supports different operating strategies such as steam splitting and condensate-, feed- and product- flashing. The empirical correlations for BPR, OHTCs of flat falling film evaporators and heat loss have been developed using the plant data. Using these correlations, average errors of 3.4%, ±10% and –33 to +29% have been observed for the predictions of BPR, OHTC and heat loss, respectively.

Keywords: Mathematical model, Flat falling film evaporator, Empirical correlations, Overall heat transfer coefficient, Heat loss

Evaporators are integral part of a number of process industries namely pulp and paper, chlor-alkali, sugar, pharmaceuticals, desalination, dairy and food processing, etc. During last seven decades, a number of models of multiple effect evaporator (MEE) systems for its analysis have been reported. A few of these were developed by Kern, Itahara and Stiel, Radovic et al., Nishitani and Kunugita, Mathur, Bremford and Muller-Steinhagen, El-Dessouky et al., Oliveira Souza da Costa and Lima, Agarwal et al. and Miranda and Simpson.

These models are generally based on a set of non-linear equations and can accommodate effects of varying physical properties of vapour/steam and liquor with temperature. These models either use fixed overall heat transfer coefficient (OHTC) for different effects or variable OHTC based on empirical equations or mathematical models.

It has also been observed that model equations are developed for a given operating strategy and when this strategy changes, it forces to change the whole set of governing equations of the model to address the new operating strategy. This further adds difficulty in handling all the operating strategies through a single model without changing the set of its governing equations. To eliminate this difficulty Stewart and Beveridge, Ayangbile et al. and Bhargava developed generalized cascade algorithm in which the model of an evaporator body is solved repeatedly to address the different operating strategies of a MEE system. With the change in operating strategy the sequence of the solution of the model equation of the effect and the input data to it changes. The solution strategy, employed for solving the model, automatically selects the above sequence based on the input data file where the designer describes the operating strategy.

Further, the modeling technique proposed by Stewart and Beveridge and Ayangbile et al. has been extended and modified, in the present work, to accommodate different operating strategies such as steam splitting and to add provisions for condensate-, feed- and product- flash.

Problem statement

The MEE system selected for above investigation is a Septuple Effect Flat Falling Film Evaporator (SEFFFE) system operating in a nearby Indian Kraft
Paper Mill and is used for concentrating weak black liquor.

The schematic diagram of a SEFFFE system with backward feed flow sequence is shown in Fig. 1. The base case operating parameters and geometrical parameters for this system are given in Table 1. The first two effects of it are considered as finishing effects, which require live steam. This system employs feed-and product-flashing along with primary and secondary condensate flashing to generate auxiliary vapour, which are then used in vapour bodies of appropriate effects to improve overall steam economy of the system. The last effect, i.e. seventh effect in the present case, is attached to a vacuum unit.

Mathematical modeling

For development of a mathematical model physico-thermal properties of black liquor involved in the process of evaporation and physical properties of steam, vapour and condensate are required. These are developed below:

Physico-thermal model for black liquor

The correlation for specific heat capacity of black liquor, proposed by Regested, is given below:

$$C_{PL} = 4.187 \times (1-0.0054*\times)$$  \hspace{1cm} (1)

Based on Eq. (1), specific enthalpy of the black liquor can be expressed as:

$$h_L = C_{PL}(T_L - C_5) \text{ J/kg}$$  \hspace{1cm} (2)

where, \(C_5 = 273\) and \(T_L\) is the black liquor temperature in Kelvin.

Boiling point rise (BPR) of the black liquor is given by a relation similar to TAPPI correlation, presented below

$$BPR_i = C_3 (C_2 + x_i)^2 \text{ K}$$  \hspace{1cm} (3)

Using the data of a paper mill, BPR correlation is obtained (Table 2).

Equation (3) is fitted against the data given in Table 2. The value of \(C_3\) is computed as 20, using the TAPPI recommended correlation value of \(C_2\) equal to
0.1. The fitted correlation as in Eq. (3) gave a maximum error of 6% with an average error of 3.4%.

**Development of physical property correlations for steam, vapour and condensate**

Values of saturated water vapour/steam temperature, specific enthalpy \((H_v)\) and condensate (water) specific enthalpy \((h)\) at different saturation pressures are taken from standard text of Smith et al.\(^{17}\). Properties for steam/water at a given saturation pressure, in the range of operation, are computed through a developed program using cubic spline interpolation\(^{18}\) using the data of steam/water properties. Use of cubic spline was found to be more precise as compared to a polynomial for the whole range of data (0.02 to 25 bar).

Further, for computation of saturation pressure of steam/vapour at a given saturation temperature, three quadratic polynomials have been developed as shown in Table 3. Equations (4) to (6) are fitted using linear regression in the respective thermodynamic data taken from standard text of Smith et al.\(^{17}\).

**Development of mathematical models of an effect, liquor flash tank and condensate flash tank**

By taking mass and energy balances over \(i^{th}\) effect of a SEFFFE system (Fig. 2), following equations can be developed.

**Overall mass balance**

\[L_{i+1} + V_{i-1} = L_i + V_i + C_{i-1} \quad \ldots (7)\]

As \(C_{i-1} = V_{i-1}\), Eq. (7) reduces to

\[L_{i+1} = L_i + V_i \quad \ldots (8)\]

**Partial mass balance for solids provides**

\[L_{i+1} x_{i+1} = L_i x_i = L_F x_F \quad \ldots (9)\]

**Overall energy balance gives**

\[L_{i+1} h_{L_{i+1}} = L_i h_{L_i} + V_i h_{V_i} + \Delta H_i \quad \ldots (10)\]

where

\[\Delta H_i = U_i A_i (T_{L_{i-1}} - T_{L_i}) \quad \ldots (11)\]

and

\[T_{L_i} = T_i + BPR_i \quad \ldots (12)\]

Energy balance on steam/vapour side gives rise to,

\[V_{i-1} = \frac{\Delta H_i}{(H_{V_{i-1}} - h_{L_{i-1}})} \quad \ldots (13)\]

Combining Eqs (1) to (3) and (8) to (12) and eliminating \(V_i\), \(x_i\), \(h_{L_i}\), \(\Delta H_i\) and \(T_{L_i}\) one gets the following cubic polynomial equation in terms of \(L_i\):

\[a_1 L_i^3 + a_2 L_i^2 + a_3 L_i + a_4 = 0 \quad \ldots (14)\]

where coefficients \(a_1\), \(a_2\), \(a_3\) and \(a_4\) of the cubic polynomial are functions of input liquor parameters and other known parameters like heat transfer area \((A_i)\) and overall heat transfer coefficient \((U_i)\) of different effects, calculated from correlation developed in the present investigation.

The expression for coefficients \(a_1\), \(a_2\), \(a_3\) and \(a_4\) are:

\[a_1 = H_{v_i} - C_1 T_i - C_1 C_2 C_3 + C_1 C_5 \quad \ldots (14a)\]

\[a_2 = L_i h_{V_i} + U_i A_i (T_{L_{i-1}} - T_i - C_3)^2 + \ldots (14b)\]

\[a_3 = L_i^2 x_i + a_1 L_i + a_2 \quad \ldots (14c)\]

\[a_4 = U_i A_i (T_{L_{i-1}} - T_i - C_3)^2 + U_i A_i (T_{L_{i-1}} - T_i - C_3)^2 \quad \ldots (14d)\]

**Table 3 — Correlations for computation of saturation pressure of steam/vapour at a given saturation temperature**

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y)</th>
<th>(z)</th>
<th>(u)</th>
<th>(v)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0105</td>
<td>-1.999*10^4</td>
<td>4.581*10^5</td>
<td>-4.268*10^7</td>
<td>9.874*10^9</td>
</tr>
<tr>
<td>0.206</td>
<td>-9.89*10^3</td>
<td>2.304*10^4</td>
<td>-2.03*10^6</td>
<td>1.521*10^8</td>
</tr>
<tr>
<td>-0.439</td>
<td>6.5310^{-3}</td>
<td>7.3910^{-5}</td>
<td>-1.37*10^{-6}</td>
<td>1.42*10^{-8}</td>
</tr>
</tbody>
</table>

where, \(P = a_0 + a_1 T + a_2 T^2 + a_3 T^3 + a_4 T^4\)
\[ L_{i+1} x_{i+1} (C_i C_4 T_i - 2 C_1 C_2 C_3 + C_1 C_3 C_2^2) C_4 - \\
C_1 C_4 C_3 = L_{i+1} H_v \quad \text{(14b)} \]

\[ a_3 = (L_{i+1} x_{i+1})^2 (2 C_1 C_2 C_3 + C_1 C_3) - \\
2 C_2 C_3 U A_i L_{i+1} x_{i+1} \quad \text{(14c)} \]

and

\[ a_4 = (C_1 C_3 C_4 L_{i+1} x_{i+1} - C_3 U A_i) (L_{i+1} x_{i+1})^2 \quad \text{(14d)} \]

For liquor flash tank (Fig. 3), a similar cubic model as developed for effects, presented in Eq. (14), is proposed. The modified expressions for constants \( a_1 \) to \( a_4 \) in Eq. (14) are described below:

\[ a_1 = H_v e - C_i T_e - C_1 C_2 C_3 + C_1 C_5 \quad \text{(14e)} \]

\[ a_2 = L_1 h_1 + L_1 x_1 (C_i C_4 T_e - 2 C_1 C_2 C_3 + C_1 C_3 C_2^2) C_4 - \\
C_1 C_4 C_3 = L_1 H_v \quad \text{(14f)} \]

\[ a_3 = (L_1 x_1)^2 (2 C_1 C_2 C_3 - C_1 C_3) \quad \text{(14g)} \]

\[ a_4 = (L_1 x_1)^3 C_1 C_3 C_4 \quad \text{(14h)} \]

The cubic equation, Eq. (14), is solved to get real roots. Out of the real roots only one root, which has a value equal or less than black liquor feed rate, provides the needed information and thus selected for further processing. Once, this root is known, other parameters like exit liquor concentration, -temperature and vapour produced \( (V_i) \) are calculated using Eqs (9), (12) and (8), respectively. Use of Eqs (11) and (13) provides the quantity of vapour required \( (V_{i+1}) \) which is necessary to provide heat for the evaporation.

Material and energy balances over condensate flash tank yield following relation to determine exit condensate flow rate \( (C_i) \), for a known condensate mass flow rate, \( C_i \), entering at a temperature, \( T_i \), with specific enthalpy, \( h_i \), and being flashed at temperature, \( T_j \). The overall mass and energy balance give:

\[ C_j = C_i (H_j - h_i) / (H_j - h_j) \quad \text{(15)} \]

and

\[ V_j = C_i - C_j \quad \text{(16)} \]

**Solution of the model using cascade algorithm**

A MEE system consisting of ‘n’ effects has been modeled by Ayangbile et al.\(^\text{13}\). This model is improved in the present work to account for feed sequencing, feed and steam splitting and primary and secondary condensate-, feed- and product- flashing. Detailed account of the model is given by Bhargava\(^\text{15}\).

The black liquor feed rate \( (L_{i+1}) \) to \( i^{\text{th}} \) effect consists of two components: fresh feed and liquor coming from \((j+1)^{\text{th}}\) effect. This fact accommodates any feed flow sequence and liquor splitting. The expression of \( L_{i+1} \) is given as:

\[ y_{o_i} L_F + \sum_{j=i}^{n} y_{j_i} L_j \quad \text{(17)} \]

where \( y_{o_i} \) is the fraction of the feed (after feed flash), which enters into \( i^{\text{th}} \) effect and \( y_{j_i} \) is the fraction of black liquor which is coming out from \( j^{\text{th}} \) effect and enters the \( i^{\text{th}} \) effect.

Total mass balance around \( i^{\text{th}} \) effect gives,

\[ y_{o_i} L_F + \sum_{j=1}^{n} y_{j_i} L_j = L_i + V_i \]

or

\[ \sum_{j=1}^{n} y_{j_i} L_j - L_i = V_i - y_{o_i} L_F \quad \text{(18)} \]

This equation, developed for \( i^{\text{th}} \) effect and shown in Eq. (18), can be represented for all \( n \) effects by a matrix equation as given below:

\[
\begin{bmatrix}
 y_{i1} & y_{i2} & y_{i3} & \cdots & y_{i1} \\
 y_{i2} & y_{i2} & y_{i3} & \cdots & y_{i2} \\
 y_{i3} & y_{i3} & y_{i3} & \cdots & y_{i3} \\
 \vdots & \vdots & \vdots & \ddots & \vdots \\
 y_{in} & y_{2n} & y_{3n} & \cdots & y_{nn} - 1
\end{bmatrix}
\begin{bmatrix}
 L_1 \\
 L_2 \\
 L_3 \\
 \vdots \\
 L_n
\end{bmatrix}
= 
\begin{bmatrix}
 y_{i1} - y_{i1} L_F \\
 y_{i2} - y_{i2} L_F \\
 y_{i3} - y_{i3} L_F \\
 \vdots \\
 y_{in} - y_{in} L_F
\end{bmatrix}
\]

i.e., \([Y - I] L = V \quad \text{(19)}\)

or \( L = [Y - I]^{-1} V = AV \quad \text{(20)}\)
where, \( A \) is the inverse of the matrix \([Y-I]\). As the value of \( y_{jj} \) is equal to zero, the diagonal elements of matrix \([Y-I]\) become -1.

From Eq. (20), the exit liquor flow rate, \( L_i \), from \( i \)th effect can be written as:

\[
L_i = \frac{a_{ij} v_j - L_F \sum_{j=1}^{n} y_{ij} a_{ij}}{\sum_{j=1}^{n} y_{ij} a_{ij}} \quad \text{...(21)}
\]

Similarly, component mass balance around \( i \)th effect provides:

\[
y_{ii} L_i x_i - \sum_{j=1}^{n} y_{ij} L_j x_j = -y_{ii} L_F x_F
\]

or

\[
[Y-I] L_i x_i = -\ Y_0 L_F x_F
\]

where,

\[
L_i = \begin{bmatrix} L_1, & L_2, & L_3, & \ldots & L_n \end{bmatrix}
\]

or \([Y-I] L_i = -\ Y_0 L_F x_F\) \quad \text{...(22)}

Using Eq. (22), the total solids coming out of \( i \)th effect is:

\[
L_i x_i = \frac{-\sum_{j=1}^{n} y_{ij} a_{ij} L_F x_F}{\sum_{j=1}^{n} y_{ij} a_{ij} L_F} \quad \text{...(23)}
\]

Combining Eqs (21) and (23)

\[
x_i = \frac{-\sum_{j=1}^{n} y_{ij} a_{ij} L_F x_F}{\sum_{j=1}^{n} y_{ij} a_{ij} L_F} \quad \text{...(24)}
\]

The model for an evaporator is solved to obtain \( L_i, x_i, V_i \) and \( V_{i-1} \). Further, \( V_{i-1} \), i.e. vapour required in the \( i \)th effect steam chest, calculated after solving the model of an effect, is re-designated as \( V_{bi} \) whereas, \( V_{i-1} \) (vapour available for supply to \( i \)th effect steam chest) can be modified to incorporate the flash vapour produced by feed-, product- and condensate- flashing with the vapour produced in the \((i-1)\)th effect. Further, if the concept of vapour bleeding is included in the model, vapour bled to re-heater is deducted from \( V_{i-1} \). Thus, the remaining vapour is obtained which can provide the required heat to the \( i \)th effect. This has been clearly shown in Fig. 4. The values of vapour denoted by \( V_{i-1} \) and \( V_{bi} \) should be equal for an exact solution. An index called “Performance Index (PI)” is defined as a measure of the difference in \( V_{i-1} \) and \( V_{bi} \).

\[
\text{PI} = \Sigma((V_{i-1} - V_{bi}) / V_{bi})^2 \quad \text{...(25)}
\]

The summation term shown in Eq. (25) is for \( n_s+1 \) to ‘n’ effects, where \( n_s \) effects are fed with live steam. The summation of \( V_{bi} \) for first \( n_s \) effects gives total steam consumption, and summation of \( V_i \) from first \( n_s \) effects is the vapour to \((n_s + 1)\)th effect.

Solution of the mathematical model starts with assumed values of operating pressures for effect number 1 to \((n-1)\). It gives the values of vapour required \((V_{bi})\) along with the vapour available \((V_i)\) for each effect and then Performance Index (PI) is calculated using Eq. (25). If it is greater than desired accuracy (say, \(5 \times 10^{-6}\)), next iteration is to be performed. This will require new and improved estimates of \( P_i \) for \( i = 1 \) to \((n-1)\).

Further, the effect of operating pressure \( P_i \) on other parameters such as \( T_i, T_{Li}, V_i \) and \( x_i \) of \( i \)th effect is studied. Though the relations between these parameters are not linear, but these have been approximated to be linear for variations of parameters within a small range.

On linearization, a set of \([2(n-n_s)+1]\) equations are obtained. These are solved to obtain the values of \( \delta P_i \), where \( i = 1, 2, \ldots, n-n_s \), which are used to modify the pressure of all the effects except last effect, as pressure of this effect is fixed. These improved values of pressure are used in next iteration. A matrix representation of these \([2(n-n_s)+1]\) equations, for a SEFFFE system shown in Fig. 1, is given in following equation form:

\[
D Z = E
\]

where the size of the coefficient matrix \( D \) is \([2(n-n_s)+1] \times [2(n-n_s)+1]\). Similarly, the size of \( Z \) and \( E \) matrices is \(1 \times [2(n-n_s)+1]\) each. The details of these matrices are given in Table 4.

where \( b_{ij} = \alpha_i a_{ij} \theta_i \)

The terms used in linear equations, included in Table 4, are calculated as follows:

\[
\alpha_i = V_i / \Delta T_i \quad k = 1
\]

\[
= (V_i^k - V_i^{k-1}) / (\Delta T_i^k - \Delta T_i^{k-1}) \quad k = 2, 3, \ldots \quad \text{...(26)}
\]
Fig. 4 — The schematic diagram of vapour flow in a MEE system consists of ‘n’ effects

Table 4 — Matrix representation of set of linear algebraic equations

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<thead>
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<th>δV1</th>
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<td>b14</td>
<td>b15</td>
<td>b16</td>
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<td>b24</td>
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<td>b26</td>
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<td>α2γ′2</td>
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<td>b32</td>
<td>1+b33</td>
<td>b34</td>
<td>b35</td>
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<td>0</td>
<td>-α3γ3</td>
<td>α3γ′3</td>
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<td>b41</td>
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<td>α4γ′4</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>-α6γ6</td>
<td>0</td>
</tr>
</tbody>
</table>

\[ \beta_i = \frac{V_{bi}}{V_i} \quad k = 1 \]

\[ \beta_i = (V_{bi}^k - V_{bi}^{k-1}) / (V_i^k - V_i^{k-1}) \quad k = 2, 3, \ldots \quad (27) \]

\[ \gamma'_i = \frac{\partial T_{Li}}{\partial P_i} = \frac{\partial (T_i + \text{BPR},i)}{\partial P_i} = \frac{\partial T_i}{\partial P_i} \quad (28) \]

\[ \gamma_i = \frac{\partial T_{i-1}}{\partial P_{i-1}} \quad (29) \]

\[ \gamma''_i = \frac{\partial T_{Li}}{\partial x_i} = \frac{\partial (T_i + \text{BPR},i)}{\partial x_i} = 2C_a(C_2 + x_i) \quad (30) \]

\[ \theta_i = \frac{\gamma_i \sum_{j=1}^{n} (y_{oj}a_{ij})L_Fx_F \sum_{j=1}^{n} a_{ij}}{\sum_{j=1}^{n} (y_{oj}a_{ij})L_F - \sum_{j=1}^{n} a_{ij}V_j} \quad (31) \]

\[ \Delta V_i = V_{i-1} - V_{bi} \quad \text{for} \quad i = n, n-1, \ldots, n-n_s \quad (32) \]

\[ \delta V_{i-1} = V_{i-1}^k - V_{i-1}^{k-1} \quad (33) \]

\[ \delta V_{bi} = V_{bi}^k - V_{bi}^{k-1} \quad (34) \]
Here \( a_{ij} \)'s are the elements of the inverse matrix of 
\([Y-I]\), where \( y_{ji} \) is the element of flow fraction matrix 
\( Y \), given in Eq. (20), and \( I \) is the identity matrix. 
The above process is to be repeated iteratively till 
desired precision (\( \leq 5 \times 10^{-6} \)) in the value of \( P_I \) is 
obtained.

Development of empirical correlations for OHTC and heat 
loss

Data are collected from a nearby pulp and paper 
mill for SEFFFE system with backward feed flow 
sequence and live steam fed to first two effects. Its 
schematic diagram along with the positioning of flash 
tanks is given in Fig. 1.

Using material and energy balances, values of heat 
loss and OHTC for each evaporator are calculated 
from the plant data.

A simplified empirical model for heat losses from 
different effects, piping, heat exchangers, flash tanks, 
etc. of a SEFFFE system is developed using equation 
alogous to natural convection as given below:

\[
Q_{\text{loss}} \propto (\Delta t)^{1.25}
\]

where, \( \Delta t \) is the difference between vapour body 
temperature of an effect and ambient temperature.

Regression, using calculated heat losses and 
corresponding values of \( \Delta t \) from plant data, yields 
following empirical correlation:

\[
Q_{\text{loss}} = 1.9669 \times 10^3 (\Delta t)^{1.25} \quad \ldots(35)
\]

Predictions from the Eq. (35) show an error limit of 
\(-33\) to \(+29\%\). The average total heat loss from 
SEFFFE system is \(5.8\%\) of total energy input through 
steam and liquor.

To develop the empirical correlation for the 
prediction of OHTC of an effect, parameters \( \Delta T \), \( x_{\text{avg}} \) 
and \( F_{\text{avg}} \) have been considered as a statistical analysis 
showed a high level of cross-correlation amongst 
these parameters. The results of this analysis are given 
in Table 5.

Further, a plot of calculated values of OHTCs of all 
seven effects for four different sets of plant data is 
shown in Fig. 5. The analysis of this figure clearly 
shows that the trend of OHTC for first two effects i.e. 
effect No. 1 and 2 is totally different than the trend of 
effect Nos. 3 to 7. The above observation may be due 
to the fact that in the vicinity of 45\% solid 
concentration in black liquor, there is a rapid increase 
in its viscosity. It causes OHTC to fall drastically. 
Therefore, two different empirical correlations are 
developed, one for effect Nos. 1 to 2 and the other for 
effect Nos. 3 to 7. Power law equation as shown 
below is used for both the correlations.

\[
(U/2000) = a (\Delta T/40)^b (x_{\text{avg}}/0.6)^c (F_{\text{avg}}/25)^d \quad \ldots(36)
\]

The computed values of OHTCs from plant data, 
for all the seven effects, are used to estimate the 
unknown coefficients \( a, b, c \) and \( d \) of Eq. (36) using 
constrained minimization technique of Sigma Plot 
software. The estimated coefficients for both the 
correlations [Eqs 36(a) and 36(b)] are given in 
Table 6.

Figure 6 has been plotted to show the extent of 
fitting of the OHTC. From this figure it is clear that 
the correlations, Eqs 36(a) and 36(b), predict the 
OHTC data within an error limit of \(\pm10\%\).

Simulation of SEFFFE system

For the solution of model, developed in the present 
investigation, computer program is developed in 
Fortran 90 and run on Pentium IV machine, using
Microsoft Fortran Power Station compiler. The specified input are \( n, n_s, B, A, F, x_F, T_F, T_S \) and \( T_L \) whereas output parameters are pressures, temperatures of each effect, \( V_1 \) to \( V_7 \), heat transfer rate, heat loss, final product rate and concentration after product flash, total steam required and steam economy. The flow chart of the algorithm is shown in Fig. 7.

For the generalized cascade algorithm the equation developed for an effect is solved repeatedly for different effects depending upon the sequence of computation determined by the selected flow sequence which in turn is decided by the Boolean matrix \( B \).

For the SEFFFE system, order of the matrix is \( 8 \times 8 \), where the first column denotes the feed stream and subsequent columns are source effects 1 to 7 and first 7 rows are sink effects and last row is product stream. A unit value of element \( b_{ij} \) indicates that liquor exiting from \((j-1)\)th effect enters \(i\)th effect. The backward flow sequence is represented by the Boolean matrix, given below, in which element \( b_{13} = 1 \) shows that liquor exits from \(2\)nd effect and enters first effect.

Similarly, Boolean matrices for other flow sequences, feed, product and condensate flash tanks etc. can be represented.

\[
B = \begin{bmatrix}
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

(Feed) \[ F \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \]

Source effect \[ \rightarrow \quad \text{Sink effect} \]

Validation of model

To establish the reliability of the present model it is required to compare the results obtained from simulation with the plant data obtained from the paper mill.

A scaling factor for Eqs 36(a), 36(b) and (35) for OHTC and heat loss, respectively, has been used as a tuning constant during simulation. It was used to match output parameters such as live steam consumption and exit concentration of simulation run with plant data. In general tuning constant varied from 0.935 to 1.045 for four sets of plant data. This is within the error limit of the Eqs (35) and 36 (a & b). For base case parameters, shown in Table 1, tuning constant is taken as 1.01.

Figures 8 and 9 have been plotted to show the comparison between experimental data obtained from the mill for concentration of black liquor and vapour body temperature of different effects with that obtained from model, respectively. Predicted results show that the liquor concentration match within an error band of -0.2 to +0.4%, and the vapour temperature of different effects match within an error limit of –0.26 to +1.76%. The present model computes the temperature difference \( \Delta T \) for each effect with a maximum relative error of 23% between plant data and simulation result. However, for the similar MEE system the published model\(^7\) showed a maximum error of 43.43% for the prediction of \( \Delta T \) in each effect. Thus, it appears that the present model predicts the plant data fairly well in comparison to the previously published model.

![Graph](image-url)
Results and Discussion

The results of the simulation run for the base case operating data are given in Appendix A. Further, it is interesting and useful to find out the contributions of different flash streams towards the total vapour produced including the live steam. For the base case data reported in Table 1, this analysis is carried out. Fig. 10 shows the share of different vapour/steam streams towards the total vapour in each effect. It is interesting to note that out of the total vapour produced and live steam used in the network (for all effects), live steam comprises 18.6%, vapours produced in effects No. 1 to 6, V1 to V6, comprise 74% and condensate-, feed- and product- flash comprise 6.1%, 0.7% and 0.6%, respectively.

Effect of variation in input parameters on output parameters

After establishing the reliability of the present model, it was thought logical to study the variation of output parameters such as live steam consumption (SC), steam economy (SE) and product concentration (xP) with change in input parameters so that better operating conditions can be identified which will give maximum steam economy.

Generally, changes made in the input parameters such as, TS, TLE, TF, xF and F, do affect output parameters SC, SE and xP. In the present investigation, the input parameters are varied within a range as given in Table 7 to study its effect on output parameter. The ranges of input parameters are considered after analyzing the prevailing practices in
Indian paper mills. To study the effect of different input parameters, it is necessary to figure out a standard set of input parameters, given in Table 1, around which these parameters are varied.

The input-output interaction is a complex phenomena in a SEFFFE system, as a change in input parameters trigger a intricate chain reaction affecting almost all parameters, including, vapour body and liquor temperatures of all the effects, almost all flash vapour fractions, heat transfer coefficients, rate of evaporation in each effect, etc. Further, it appears that the steam economy, SE, is the single most prominent parameter to evaluate the efficiency of the SEFFFE system as it varies with variation in operating parameters and geometrical parameters as well. Moreover, the contributions of SC and $x_p$ are also included in it as the value of SE is the ratio of total water evaporated to total SC. In addition to it, the amount of evaporated water is also related to the product concentration, $x_p$, directly. Though by monitoring SE one can keep a watch on the economics of evaporation, the study of variations in parameters such as SC and $x_p$ with input parameter offers better understanding of the process. The trends of behaviours of SC, SE and $x_p$ with change in $T_s$, $T_{Le}$, $T_f$, $x_F$ and $F$ are shown in Table 8.

![Graph 1](image1)

**Fig. 8 — Comparison between solid concentration in liquor from plant data and that predicted by model**

![Graph 2](image2)

**Fig. 9 — Comparison between vapour temperature of different effects from plant data and that from model**

![Graph 3](image3)

**Fig. 10 — Effectwise steam/vapour consumption obtained from simulation**

<table>
<thead>
<tr>
<th>S. No.</th>
<th>Parameters</th>
<th>Variation in value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Steam temperature ($T_s$)</td>
<td>120°C - 160°C</td>
</tr>
<tr>
<td>2.</td>
<td>Inlet liquor concentration ($x_F$)</td>
<td>8% - 16%</td>
</tr>
<tr>
<td>3.</td>
<td>Last effect (7th) temperature ($T_{Le}$)</td>
<td>42°C - 62°C</td>
</tr>
<tr>
<td>4.</td>
<td>Liquor temperature ($T_f$)</td>
<td>44.7°C - 84.7°C</td>
</tr>
<tr>
<td>5.</td>
<td>Liquor flow rate ($F$)</td>
<td>56200 - 78680 kg/h</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>SC</th>
<th>SE</th>
<th>$x_p$</th>
<th>Specified parameter</th>
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<tr>
<td>$T_s$</td>
<td>$x_F$, $F$, $T_f$ and $T_{Le}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$x_F$</td>
<td>$T_s$, $F$, $T_f$ and $T_{Le}$</td>
<td></td>
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<tr>
<td>$T_f$</td>
<td>$T_s$, $x_F$, $F$ and $T_{Le}$</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>$F$</td>
<td>$T_s$, $x_F$, $T_f$ and $T_{Le}$</td>
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<tr>
<td>$T_{Le}$</td>
<td>$x_F$, $F$, $T_f$ and $T_s$</td>
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</tr>
</tbody>
</table>

Table 7 — Ranges of operating parameters of a SEFFFE system

Table 8 — Trends of SC, SE and $x_p$ with change in $T_s$, $T_{Le}$, $T_f$, $x_F$ and $F$
Based on above investigation, it is seen that for values of parameters $T_S$, $T_L$, $T_F$, $x_F$ and $F$ equal to 120°C, 84.7°C, 52°C, 0.118 and 56200 kg/h respectively, the SEFFFE system exhibits maximum SE of 5.57 with $x_P$ and SC equal to 0.49 and 7700 kg/h, respectively. This value of SE is 11.6% more than the SE at which SEFFFE system is being operated currently. Thus, from above analysis it can be suggested that only by changing the operating conditions, SE of the system can be improved without any prior modification in layout of the paper mill.

Conclusions
The salient conclusions drawn from the present investigations are given as:
(i) The correlation developed for BPR predicts the plant data within an average error of 3.4%.
(ii) The empirical correlation of OHTC for a SEFFFE system predicts the plant data within ±10%.
(iii) The present model predicts the liquor concentration profile and vapour body temperature profile of different effect within maximum error limit of ±2%. Also it simulates the plant data with considerably smaller amount of error in comparison to published model.
(iv) The steam economy of the SEFFFE system can be improved by proper selection of values of operating parameters without any prior modification in the plant layout.

Nomenclature
A = Heat transfer area of an effect, m²
$a_1$ - $a_4$ = Coefficients of cubic polynomial (Eq. 14)
$a_{ij}$ = Element of the inverse matrix of $[Y – I]$
$b_{ij}$ = A parameter used in Table 4
B = Boolean matrix
BPR = Boiling point rise, K
C = Condensate flow rate, kg/s
$C_1$ - $C_5$ = Constant in mathematical model
$C_P$ = Specific heat capacity, J/kg/K
$F$ = Flow rate of liquor, kg/h
$h$ = Specific enthalpy of liquid phase, J/kg
$H$ = Specific enthalpy of vapour phase, J/kg
$I$ = Identity matrix
$k$ = Iteration number
$L$ = Liquor flow rate, kg/s
$n$ = Number of total effects
$n_s$ = Number of effects supplied with live steam
$P$ = Pressure of steam and vapour bodies of evaporator, N/m²
$PI$ = Performance index as defined in Eq. (25)
$Q_{loss}$ = Heat loss from an evaporator, W
$T$ = Vapour body temperature of an effect, K
$U$ = Overall heat transfer coefficient, W/m²K
$V$ = Rate of vapour produced in an evaporator, kg/s
$x$ = Mass fraction of solids in liquor
$Y$ = Flow fraction matrix

Greek letters
$\alpha$ = A parameter as defined by Eq. (26)
$\beta$ = A parameter as defined in Eq. (27)
$\Delta$ = Difference between two parameter
$\delta$ = Difference between same variable over two successive iterations
$\gamma$ = A parameter as defined in Eq. (28)
$\gamma'$ = A parameter as defined in Eq. (29)
$\gamma''$ = A parameter as defined in Eq. (30)
$\theta$ = A parameter as defined in Eq. (31)

Subscripts
avg = Average of inlet and outlet conditions
e = Exit condition
$F$ = Feed
$i$ = Effect number
$L$ = Black liquor
$Le$ = Last effect
$S$ = Steam
$V$ = Vapour
$V1$ to $V7$ = Vapours produced in one to seventh effects

References
4 Nishitani H & Kunugita E, Comp Chem Eng, 3 (1979) 261.
Appendix A

The simulation results for base case operating parameter are shown below:

Final Results  Iterations used are 19
Steam temps. Are 140.02 °C and 147.00 °C
Steam Pressures are 3.616 bar and 4.389 bar
Feed rate= 56200.0 kg/h, Initial Feed concn = .1180, Feed temp. = 64.7 °C
Feed to 7th effect, after initial feed flash rate = 55873.4 kg/h
Flashed Feed Concentration = .1187
Amount of vapour generated due to feed flash = 326.64 kg/h

Feed Sequence
Product  Feed
Live steam in Effect Nos: 1 and 2
Effect No.  1  2  3  4  5  6  7  Last effect temp, °C: 52.00  Last effect pressure, bar: .136
Pressure, bar  1.70  1.59  .96  .53  .32  .20  .14
Vapour temp, °C  115.1  113.1  98.5  82.9  70.5  60.3  52.0
Liquor temp, °C  123.0  118.8  102.2  85.3  72.3  61.6  53.1
Feed flash vap, kg/h 0.00 0.00 0.00 0.00 0.00 326.64 .00
Prod flash vap, kg/h 0.00 0.00 275.48 .00 .00 .00 .00
Cond flash vap, kg/h 0.00 0.00 986.06 668.97 674.04 543.95 .00
Vap to Preheat, kg/h 0.00 0.00 .00 .00 .00 .00 .00
Vapour added, kg/h 0.00 0.00 1261.54 668.97 674.04 870.59 .00
Exit vapour, kg/h 2801.7 4843.4 6707.5 7039.5 6868.0 6773.2 8294.6
Total vapour, kg/h 2801.7 4843.4 7969.0 7708.4 7542.1 7643.8 8294.6
Steam/Vap reqd, kg/h 3096.8 5705.5 7644.8 7960.2 7541.2 7548.9 7635.6
Exit liquor, kg/h 12545.5 15347.2 20190.6 26898.1 33937.6 40805.6 47578.8
Exit concn. .5286 .4321 .3285 .2465 .1954 .1625 .1394
Delta T, °C 16.97 28.20 11.93 13.19 10.61 8.92 7.11
OHTC, W/sq m/K 191.75 215.38 589.29 568.08 698.97 825.56 1014.80
Energy loss, MW .0867 .0846 .0697 .0539 .0413 .0308 .0224

Total Product Rate =12270.0 kg/h, at liquid concn = .5405
Total Steam Consumption = 8802.3 kg/h
Primary and secondary condensate after flashing = 7637.11 and 36800.6 kg/h respectively

Overall Energy Balance
Energy in = 10.656 MW, Energy out = 10.263 MW, Energy loss = .3926 MW (in percent 3.6845%)

Delta P = -.114E+03 -.382E+02 -.127E+03 -.753E+02 .351E+02 N/sq m
Performance Index = .3185E-05, Steam Economy = 4.9907
Facu = 1.01000 and floss = .76000