

## Mathematical model for a multiple effect evaporator system with condensate-, feed- and product- flash and steam splitting

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In the present investigation a non-linear mathematical model is developed for the analysis of multiple effect evaporators (MEE) system. This model is capable of simulating process of evaporation and takes into account variations in -boiling point rise (BPR) due to variation in concentration and temperature of liquor, -overall heat transfer coefficient (OHTC) of effects and -physico-thermal property of the liquor. Using mass and energy balance around an effect, a cubic polynomial equation is developed to model an effect, which is solved using generalized cascade algorithm. For this purpose, a Septuple effect flat falling film evaporator (SEFFFE) system with backward feed flow sequence, being used for concentrating weak black liquor and operating in a near by paper mill, is selected as a typical MEE system. This system supports different operating strategies such as steam splitting and condensate-, feed- and product- flashing. The empirical correlations for BPR, OHTCs of flat falling film evaporators and heat loss have been developed using the plant data. Using these correlations, average errors of 3.4%,  $\pm 10\%$  and  $-33$  to  $+29\%$  have been observed for the predictions of BPR, OHTC and heat loss, respectively.

**Keywords:** Mathematical model, Flat falling film evaporator, Empirical correlations, Overall heat transfer coefficient, Heat loss

Evaporators are integral part of a number of process industries namely pulp and paper, chlor-alkali, sugar, pharmaceuticals, desalination, dairy and food processing, etc. During last seven decades, a number of models of multiple effect evaporator (MEE) systems for its analysis have been reported. A few of these were developed by Kern<sup>1</sup>, Itahara and Stiel<sup>2</sup>, Radovic *et al.*<sup>3</sup>, Nishitani and Kunugita<sup>4</sup>, Lambert *et al.*<sup>5</sup>, Mathur<sup>6</sup>, Bremford and Muller-Steinhagen<sup>7</sup>, El-Dessouky *et al.*<sup>8,9</sup>, Oliveira Souza da Costa and Lima<sup>10</sup>, Agarwal *et al.*<sup>11</sup> and Miranda and Simpson<sup>12</sup>.

These models are generally based on a set of non-linear equations and can accommodate effects of varying physical properties of vapour/steam and liquor with temperature. These models either use fixed overall heat transfer coefficient (OHTC) for different effects or variable OHTC based on empirical equations or mathematical models.

It has also been observed that model equations are developed for a given operating strategy and when this strategy changes, it forces to change the whole set of governing equations of the model to address the new operating strategy. This further adds difficulty in handling all the operating strategies through a single

model without changing the set of its governing equations. To eliminate this difficulty Stewart and Beveridge<sup>13</sup>, Ayangbile *et al.*<sup>14</sup> and Bhargava<sup>15</sup> developed generalized cascade algorithm in which the model of an evaporator body is solved repeatedly to address the different operating strategies of a MEE system. With the change in operating strategy the sequence of the solution of the model equation of the effect and the input data to it changes. The solution strategy, employed for solving the model, automatically selects the above sequence based on the input data file where the designer describes the operating strategy.

Further, the modeling technique proposed by Stewart and Beveridge<sup>13</sup> and Ayangbile *et al.*<sup>14</sup> has been extended and modified, in the present work, to accommodate different operating strategies such as steam splitting and to add provisions for condensate-, feed- and product- flash.

### Problem statement

The MEE system selected for above investigation is a Septuple Effect Flat Falling Film Evaporator (SEFFFE) system operating in a nearby Indian Kraft

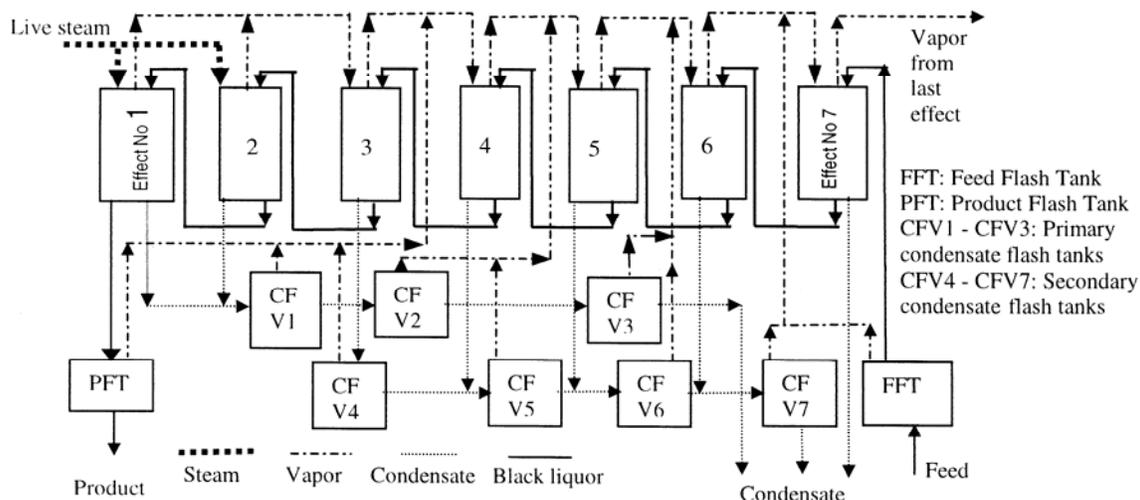


Fig. 1 — Schematic diagram of SEFFFE system

Paper Mill and is used for concentrating weak black liquor.

The schematic diagram of a SEFFFE system with backward feed flow sequence is shown in Fig. 1. The base case operating parameters and geometrical parameters for this system are given in Table 1. The first two effects of it are considered as finishing effects, which require live steam. This system employs feed- and product-flashing along with primary and secondary condensate flashing to generate auxiliary vapour, which are then used in vapour bodies of appropriate effects to improve overall steam economy of the system. The last effect, i.e. seventh effect in the present case, is attached to a vacuum unit.

### Mathematical modeling

For development of a mathematical model physico-thermal properties of black liquor involved in the process of evaporation and physical properties of steam, vapour and condensate are required. These are developed below:

#### Physico-thermal model for black liquor

The correlation for specific heat capacity of black liquor, proposed by Regested<sup>16</sup>, is given below:

$$C_{PL} = 4.187 * (1 - 0.0054 * x) \quad \dots(1)$$

Based on Eq. (1), specific enthalpy of the black liquor can be expressed as:

$$h_L = C_{PL} (T_L - C_5) \text{ J/kg} \quad \dots(2)$$

where,  $C_5 = 273$  and  $T_L$  is the black liquor temperature in Kelvin.

Table 1 — Base case operating parameters for the SEFFFE system

S. No	Parameter(s)	Value(s)	
1	Total number of effects	7	
2	Number of effects being supplied live steam	2	
3	Live steam temperature	Effect 1	140°C
		Effect 2	147°C
4	Black liquor inlet concentration	0.118	
5	Liquor inlet temperature	64.7°C	
6	Black liquor feed flow rate	56200 kg/h	
7	Last effect vapour temperature	52°C	
8	Feed flow sequence	Backward	
9	Feed flash tank position	As indicated	
10	Product flash tank position	in the Fig. 1	
12	Heat transfer area	Effect 1 and 2	540 m <sup>2</sup> each
		Effect 3 to 6	660 m <sup>2</sup> each
		Effect 7	690 m <sup>2</sup>

Boiling point rise (BPR) of the black liquor is given by a relation similar to TAPPI correlation, presented below

$$BPR_i = C_3 (C_2 + x_i)^2 \text{ K} \quad \dots(3)$$

Using the data of a paper mill, BPR correlation is obtained (Table 2).

Equation (3) is fitted against the data given in Table 2. The value of  $C_3$  is computed as 20, using the TAPPI recommended correlation value of  $C_2$  equal to

0.1. The fitted correlation as in Eq. (3) gave a maximum error of 6% with an average error of 3.4%.

#### Development of physical property correlations for steam, vapour and condensate

Values of saturated water vapour/steam - temperature, -specific enthalpy ( $H_v$ ) and condensate (water) specific enthalpy ( $h$ ) at different saturation pressures are taken from standard text of Smith *et al.*<sup>17</sup>. Properties for steam/water at a given saturation pressure, in the range of operation, are computed through a developed program using cubic spline interpolation<sup>18</sup> using the data of steam/water properties. Use of cubic spline was found to be more precise as compared to a polynomial for the whole range of data (0.02 to 25 bar).

Further, for computation of saturation pressure of steam/vapour at a given saturation temperature, three quadratic polynomials have been developed as shown in Table 3. Equations (4) to (6) are fitted using linear regression in the respective thermodynamic data taken from standard text of Smith *et al.*<sup>17</sup>.

#### Development of mathematical models of an effect, liquor flash tank and condensate flash tank

By taking mass and energy balances over  $i^{\text{th}}$  effect of a SEFFFE system (Fig. 2), following equations can be developed.

Overall mass balance

$$L_{i+1} + V_{i-1} = L_i + V_i + C_{i-1} \quad \dots(7)$$

As  $C_{i-1} = V_{i-1}$ , Eq. (7) reduces to

$$L_{i+1} = L_i + V_i \quad \dots(8)$$

Partial mass balance for solids provides

$$L_{i+1} x_{i+1} = L_i x_i = L_F x_F \quad \dots(9)$$

Overall energy balance gives

$$L_{i+1} h_{L_{i+1}} = L_i h_{L_i} + V_i H_{V_i} + \Delta H_i \quad \dots(10)$$

where

$$\Delta H_i = U_i A_i (T_{i-1} - T_{L_i}) \quad \dots(11)$$

and

$$T_{L_i} = T_i + BPR_i \quad \dots(12)$$

Energy balance on steam/vapour side gives rise to,

$$V_{i-1} = \Delta H_i / (H_{V_{i-1}} - h_{L_{i-1}}) \quad \dots(13)$$

Combining Eqs (1) to (3) and (8) to (12) and eliminating  $V_i$ ,  $x_i$ ,  $h_{L_i}$ ,  $\Delta H_i$  and  $T_{L_i}$  one gets the following cubic polynomial equation in terms of  $L_i$ :

$$a_1 L_i^3 + a_2 L_i^2 + a_3 L_i + a_4 = 0 \quad \dots(14)$$

where coefficients  $a_1$ ,  $a_2$ ,  $a_3$  and  $a_4$  of the cubic polynomial are functions of input liquor parameters and other known parameters like heat transfer area ( $A_i$ ) and overall heat transfer coefficient ( $U_i$ ) of different effects, calculated from correlation developed in the present investigation.

The expression for coefficients  $a_1$ ,  $a_2$ ,  $a_3$  and  $a_4$  are:

$$a_1 = H_{V_i} - C_1 T_i - C_1 C_2^2 C_3 + C_1 C_5 \quad \dots(14a)$$

$$a_2 = L_{i+1} h_{L_{i+1}} + U_i A_i (T_{i-1} - T_i - C_3 C_2^2) +$$

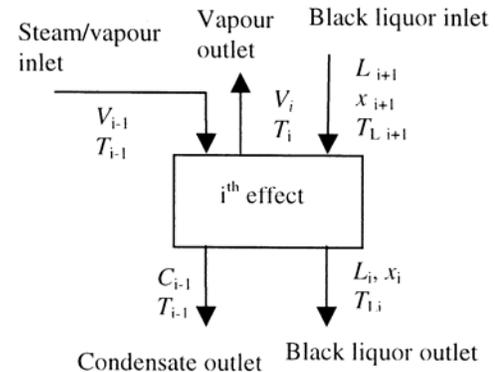


Fig. 2 — Block diagram of an evaporator

Table 2 — Data for the determination of BPR

$x_i$	0.0767	0.091	0.106	0.13	0.169	0.244	0.369	0.462	0.47
BPR, K	0.60	0.70	0.80	1.10	1.40	2.30	4.30	6.20	6.40

Table 3 — Correlations for computation of saturation pressure of steam/vapour at a given saturation temperature

$a_0$	$a_1$	$a_2$	$a_3$	$a_4$	Temperature range (°C)	Eq. No.
0.0105	$-1.999 \times 10^4$	$4.581 \times 10^{-5}$	$-4.268 \times 10^{-7}$	$9.874 \times 10^{-9}$	40 to 80	4
0.206	$-9.89 \times 10^{-3}$	$2.304 \times 10^{-4}$	$-2.03 \times 10^{-6}$	$1.521 \times 10^{-8}$	80 to 150	5
-0.439	$6.531 \times 10^{-3}$	$7.391 \times 10^{-5}$	$-1.37 \times 10^{-6}$	$1.42 \times 10^{-8}$	150 to 230	6

where,  $P = a_0 + a_1 T + a_2 T^2 + a_3 T^3 + a_4 T^4$

$$L_{i+1} x_{i+1} (C_1 C_4 T_i - 2C_1 C_2 C_3 + C_1 C_3 C_2^2 C_4 - C_1 C_4 C_5) - L_{i+1} H_{vi} \quad \dots(14b)$$

$$a_3 = (L_{i+1} x_{i+1})^2 (2C_1 C_2 C_3 C_4 - C_1 C_3) - 2C_2 C_3 U_i A_i L_{i+1} x_{i+1} \quad \dots(14c)$$

and

$$a_4 = (C_1 C_3 C_4 L_{i+1} x_{i+1} - C_3 U_i A_i) (L_{i+1} x_{i+1})^2 \quad \dots(14d)$$

For liquor flash tank (Fig. 3), a similar cubic model as developed for effects, presented in Eq. (14), is proposed. The modified expressions for constants  $a_1$  to  $a_4$  in Eq. (14) are described below:

$$a_1 = H_{ve} - C_1 T_e - C_1 C_2^2 C_3 + C_1 C_5 \quad \dots(14e)$$

$$a_2 = L_1 h_{L1} + L_1 x_1 (C_1 C_4 T_e - 2C_1 C_2 C_3 + C_1 C_2^2 C_3 C_4 - C_1 C_4 C_5) - L_1 H_{ve} \quad \dots(14f)$$

$$a_3 = (L_1 x_1)^2 (2C_1 C_2 C_3 C_4 - C_1 C_3) \quad \dots(14g)$$

$$a_4 = (L_1 x_1)^3 C_1 C_3 C_4 \quad \dots(14h)$$

The cubic equation, Eq. (14), is solved to get real roots. Out of the real roots only one root, which has a value equal or less than black liquor feed rate, provides the needed information and thus selected for further processing. Once, this root is known, other parameters like exit liquor –concentration, -temperature and vapour produced ( $V_i$ ) are calculated using Eqs (9), (12) and (8), respectively. Use of Eqs (11) and (13) provides the quantity of vapour required ( $V_{i-1}$ ) which is necessary to provide heat for the evaporation.

Material and energy balances over condensate flash tank yield following relation to determine exit condensate flow rate ( $C_j$ ), for a known condensate mass flow rate,  $C_i$ , entering at a temperature,  $T_i$ , with specific enthalpy,  $h_i$ , and being flashed at temperature,  $T_j$ . The overall mass and energy balance give:

$$C_j = C_i (H_{vj} - h_i) / (H_{vj} - h_j) \quad \dots(15)$$

and

$$V_j = C_i - C_j \quad \dots(16)$$

### Solution of the model using cascade algorithm

A MEE system consisting of 'n' effects has been modeled by Ayangbile *et al*<sup>14</sup>. This model is improved in the present work to account for feed sequencing, feed and steam splitting and primary and secondary

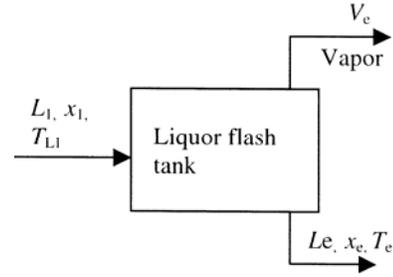


Fig. 3 — Block diagram of a liquor flash tank

condensate-, feed- and product- flashing. Detailed account of the model is given by Bhargava<sup>15</sup>.

The black liquor feed rate ( $L_{i+1}$ ) to  $i^{\text{th}}$  effect consists of two components: fresh feed and liquor coming from  $j^{\text{th}}$  effect. This fact accommodates any feed flow sequence and liquor splitting. The expression of  $L_{i+1}$  is given as:

$$y_{oi} L_F + \sum_{\substack{j=1 \\ j \neq i}}^n y_{ji} L_j \quad \dots(17)$$

where  $y_{oi}$  is the fraction of the feed (after feed flash), which enters into  $i^{\text{th}}$  effect and  $y_{ji}$  is the fraction of black liquor which is coming out from  $j^{\text{th}}$  effect and enters the  $i^{\text{th}}$  effect.

Total mass balance around  $i^{\text{th}}$  effect gives,

$$y_{o,i} L_F + \sum_{j=1}^n y_{j,i} L_j = L_i + V_i$$

or

$$\sum_{j=1}^n y_{j,i} L_j - L_i = V_i - y_{o,i} L_F \quad \dots(18)$$

This equation, developed for  $i^{\text{th}}$  effect and shown in Eq. (18), can be represented for all n effects by a matrix equation as given below:

$$\begin{bmatrix} y_{11}-1 & y_{21} & y_{31} & \dots & y_{n1} \\ y_{12} & y_{22}-1 & y_{32} & \dots & y_{n2} \\ y_{13} & y_{23} & y_{33}-1 & \dots & y_{n3} \\ \vdots & \vdots & \vdots & \dots & \vdots \\ y_{1n} & y_{2n} & y_{3n} & \dots & y_{nn}-1 \end{bmatrix} \begin{bmatrix} L_1 \\ L_2 \\ L_3 \\ \vdots \\ L_n \end{bmatrix} = \begin{bmatrix} V_1 - y_{o1} L_F \\ V_2 - y_{o2} L_F \\ V_3 - y_{o3} L_F \\ \vdots \\ V_n - y_{on} L_F \end{bmatrix}$$

$$\text{i.e., } [Y-I] L = V \quad \dots(19)$$

$$\text{or } L = [Y-I]^{-1} V = AV \quad \dots(20)$$

where,  $\mathbf{A}$  is the inverse of the matrix  $[\mathbf{Y}-\mathbf{I}]$ . As the value of  $y_{jj}$  is equal to zero, the diagonal elements of matrix  $[\mathbf{Y}-\mathbf{I}]$  become-1.

From Eq. (20), the exit liquor flow rate,  $L_i$ , from  $i^{\text{th}}$  effect can be written as:

$$L_i = \sum_{j=1}^n a_{ij} V_j - L_F \sum_{j=1}^n y_{oj} a_{ij} \quad \dots(21)$$

Similarly, component mass balance around  $i^{\text{th}}$  effect provides:

$$y_{oi} L_F X_F + \sum_{j=1}^n y_{ji} L_j X_j = L_i X_i$$

$$\text{or } \sum_{j=1}^n y_{ji} L_j X_j - L_i X_i = -y_{oi} L_F X_F$$

$$\text{or } [\mathbf{Y}-\mathbf{I}] \mathbf{LX} = -\mathbf{Y}_0 L_F X_F$$

$$\text{where, } \mathbf{LX} = [L_1 x_1 \quad L_2 x_2 \quad L_3 x_3 \quad \dots \quad L_n x_n]$$

$$\text{or } \mathbf{LX} = [\mathbf{Y}-\mathbf{I}]^{-1} (-\mathbf{Y}_0 L_F X_F) = \mathbf{A}(-\mathbf{Y}_0 L_F X_F) \quad \dots(22)$$

Using Eq. (22), the total solids coming out of  $i^{\text{th}}$  effect is:

$$L_i X_i = - \left( \sum_{j=1}^n y_{oj} a_{ij} \right) L_F X_F \quad \dots(23)$$

Combining Eqs (21) and (23)

$$X_i = \frac{\left( \sum_{j=1}^n y_{oj} a_{ij} \right) L_F X_F}{\left( \sum_{j=1}^n y_{oj} a_{ij} \right) L_F - \sum_{j=1}^n a_{ij} V_j} \quad \dots(24)$$

The model for an evaporator is solved to obtain  $L_i$ ,  $x_i$ ,  $V_i$  and  $V_{i-1}$ . Further,  $V_{i-1}$ , i.e. vapour required in the  $i^{\text{th}}$  effect steam chest, calculated after solving the model of an effect, is re-designated as  $V_{bi}$  whereas,  $V_{i-1}$  (vapour available for supply to  $i^{\text{th}}$  effect steam chest) can be modified to incorporate the flash vapour produced by feed-, product- and condensate- flashing with the vapour produced in the  $(i-1)^{\text{th}}$  effect. Further, if the concept of vapour bleeding is included in the model, vapour bled to re-heater is deducted from  $V_{i-1}$ . Thus, the remaining vapour is obtained which can provide the required heat to the  $i^{\text{th}}$  effect. This has

been clearly shown in Fig. 4. The values of vapour denoted by  $V_{i-1}$  and  $V_{bi}$  should be equal for an exact solution. An index called ‘‘Performance Index (PI)’’ is defined as a measure of the difference in  $V_{i-1}$  and  $V_{bi}$ .

$$PI = \sum ((V_{i-1} - V_{bi}) / V_{bi})^2 \quad \dots(25)$$

The summation term shown in Eq. (25) is for ‘ $n_s+1$ ’ to ‘ $n$ ’ effects, where  $n_s$  effects are fed with live steam. The summation of  $V_{bi}$  for first  $n_s$  effects gives total steam consumption, and summation of  $V_i$  from first  $n_s$  effects is the vapour to  $(n_s + 1)^{\text{th}}$  effect.

Solution of the mathematical model starts with assumed values of operating pressures for effect number 1 to  $(n-1)$ . It gives the values of vapour required ( $V_{bi}$ ) along with the vapour available ( $V_{i-1}$ ) for each effect and then Performance Index (PI) is calculated using Eq. (25). If it is greater than desired accuracy (say,  $5 \times 10^{-6}$ ), next iteration is to be performed. This will require new and improved estimates of  $P_i$  for  $i = 1$  to  $(n-1)$ .

Further, the effect of operating pressure  $P_i$  on other parameters such as  $T_i$ ,  $T_{Li}$ ,  $V_i$  and  $x_i$  of  $i^{\text{th}}$  effect is studied. Though the relations between these parameters are not linear, but these have been approximated to be linear for variations of parameters within a small range.

On linearization, a set of  $[2(n-n_s)+1]$  equations are obtained. These are solved to obtain the values of  $\delta P_i$ , where,  $i = 1, 2, \dots, n-n_s$ , which are used to modify the pressure of all the effects except last effect, as pressure of this effect is fixed. These improved values of pressure are used in next iteration. A matrix representation of these  $[2(n-n_s)+1]$  equations, for a SEFFF system shown in Fig. 1, is given in following equation form:

$$\mathbf{DZ} = \mathbf{E}$$

where the size of the coefficient matrix  $\mathbf{D}$  is  $[2(n-n_s)+1] \times [2(n-n_s)+1]$ . Similarly, the size of  $\mathbf{Z}$  and  $\mathbf{E}$  matrices is  $1 \times [2(n-n_s)+1]$  each. The details of these matrices are given in Table 4.

where  $b_{ij} = \alpha_i a_{ij} \theta_i$

The terms used in linear equations, included in Table 4, are calculated as follows:

$$\alpha_i = V_i / \Delta T_i \quad k = 1$$

$$= (V_i^k - V_i^{k-1}) / (\Delta T_i^k - \Delta T_i^{k-1}) \quad k = 2, 3, \dots \dots(26)$$

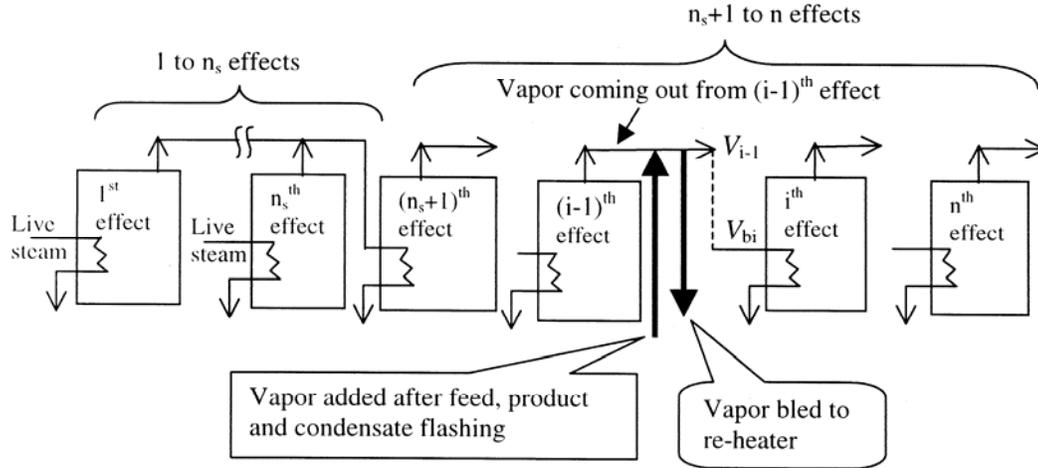


Fig. 4 — The schematic diagram of vapour flow in a MEE system consists of 'n' effects

Table 4 — Matrix representation of set of linear algebraic equations

1	$-\beta_2$	0	0	0	0	0	0	0	0	0	$\delta V_1$	$-\Delta V_2$
0	1	$-\beta_3$	0	0	0	0	0	0	0	0	$\delta V_2$	$-\Delta V_3$
0	0	1	$-\beta_4$	0	0	0	0	0	0	0	$\delta V_3$	$-\Delta V_4$
0	0	0	1	$-\beta_5$	0	0	0	0	0	0	$\delta V_4$	$-\Delta V_5$
0	0	0	0	1	$-\beta_6$	0	0	0	0	0	$\delta V_5$	$-\Delta V_6$
$1+b_{11}$	$b_{12}$	$b_{13}$	$b_{14}$	$b_{15}$	$b_{16}$	$\alpha_1\gamma'_1$	0	0	0	0	$\delta V_6$	= 0
$b_{21}$	$1+b_{22}$	$b_{23}$	$b_{24}$	$b_{25}$	$b_{26}$	$-\alpha_2\gamma_2$	$\alpha_2\gamma'_2$	0	0	0	$\delta P_1$	0
$b_{31}$	$b_{32}$	$1+b_{33}$	$b_{34}$	$b_{35}$	$b_{36}$	0	$-\alpha_3\gamma_3$	$\alpha_3\gamma'_3$	0	0	$\delta P_2$	0
$b_{41}$	$b_{42}$	$b_{43}$	$1+b_{44}$	$b_{45}$	$b_{46}$	0	0	$-\alpha_4\gamma_4$	$\alpha_4\gamma'_4$	0	$\delta P_3$	0
$b_{51}$	$b_{52}$	$b_{53}$	$b_{54}$	$1+b_{55}$	$b_{56}$	0	0	0	$-\alpha_5\gamma_5$	$\alpha_5\gamma'_5$	$\delta P_4$	0
$b_{61}$	$b_{62}$	$b_{63}$	$b_{64}$	$b_{65}$	$1+b_{66}$	0	0	0	0	$-\alpha_6\gamma_6$	$\delta P_5$	0

$$\beta_i = V_{bi} / V_i \quad k = 1$$

$$= (V_{bi}^k - V_{bi}^{k-1}) / (V_i^k - V_i^{k-1}) \quad k=2,3,\dots \quad \dots(27)$$

$$\gamma'_i = \frac{\partial T_{Li}}{\partial P_i} = \frac{\partial (T_i + BPR_i)}{\partial P_i} = \frac{\partial T_i}{\partial P_i} \quad \dots(28)$$

$$\gamma_i = \frac{\partial T_{i-1}}{\partial P_{i-1}} \quad \dots(29)$$

$$\gamma''_i = \frac{\partial T_{Li}}{\partial x_i} = \frac{\partial (T_i + BPR_i)}{\partial x_i} = 2 * C_3 (C_2 + x_i) \quad \dots(30)$$

$$\theta_1 = \frac{\gamma_i \sum_{j=1}^n (y_{oj} a_{ij}) L_F X_F \sum_{j=1}^n a_{ij}}{\left[ \sum_{j=1}^n (y_{oj} a_{ij}) L_F - \sum_{j=1}^n a_{ij} V_j \right]^2} \quad \dots(31)$$

$$\Delta V_i = V_{i-1} - V_{bi} \quad \text{for } i = n, n-1, \dots, n-n_s \quad \dots(32)$$

$$\delta V_{i-1} = V_{i-1}^k - V_{i-1}^{k-1} \quad \dots(33)$$

$$\delta V_{bi} = V_{bi}^k - V_{bi}^{k-1} \quad \dots(34)$$

Here  $a_{ij}$ 's are the elements of the inverse matrix of  $[\mathbf{Y}-\mathbf{I}]$ , where  $y_{ji}$  is the element of flow fraction matrix  $\mathbf{Y}$ , given in Eq. (20), and  $\mathbf{I}$  is the identity matrix.

The above process is to be repeated iteratively till desired precision ( $\leq 5 \times 10^{-6}$ ) in the value of PI is obtained.

**Development of empirical correlations for OHTC and heat loss**

Data are collected from a nearby pulp and paper mill for SEFFFE system with backward feed flow sequence and live steam fed to first two effects. Its schematic diagram along with the positioning of flash tanks is given in Fig. 1.

Using material and energy balances, values of heat loss and OHTC for each evaporator are calculated from the plant data.

A simplified empirical model for heat losses from different effects, piping, heat exchangers, flash tanks, etc. of a SEFFFE system is developed using equation analogous to natural convection as given below:

$$Q_{\text{loss}} \propto (\Delta t)^{1.25}$$

where,  $\Delta t$ , is the difference between vapour body temperature of an effect and ambient temperature.

Regression, using calculated heat losses and corresponding values of  $(\Delta t)$  from plant data, yields following empirical correlation:

$$Q_{\text{loss}} = 1.9669 \times 10^3 (\Delta t)^{1.25} \quad \dots(35)$$

Predictions from the Eq. (35) show an error limit of -33 to +29%. The average total heat loss from SEFFFE system is 5.8% of total energy input through steam and liquor.

To develop the empirical correlation for the prediction of OHTC of an effect, parameters  $\Delta T$ ,  $x_{\text{avg}}$  and  $F_{\text{avg}}$  have been considered as a statistical analysis showed a high level of cross-correlation amongst these parameters. The results of this analysis are given in Table 5.

Further, a plot of calculated values of OHTCs of all seven effects for four different sets of plant data is shown in Fig. 5. The analysis of this figure clearly shows that the trend of OHTC for first two effects i.e. effect No. 1 and 2 is totally different than the trend of effect Nos. 3 to 7. The above observation may be due to the fact that in the vicinity of 45% solid concentration in black liquor, there is a rapid increase in its viscosity. It causes OHTC to fall drastically. Therefore, two different empirical correlations are

developed, one for effect Nos. 1 to 2 and the other for effect Nos. 3 to 7. Power law equation as shown below is used for both the correlations.

$$(U/2000) = a (\Delta T/40)^b (x_{\text{avg}}/0.6)^c (F_{\text{avg}}/25)^d \quad \dots(36)$$

The computed values of OHTCs from plant data, for all the seven effects, are used to estimate the unknown coefficients a, b, c and d of Eq. (36) using constrained minimization technique of Sigma Plot software. The estimated coefficients for both the correlations [Eqs 36(a) and 36(b)] are given in Table 6.

Figure 6 has been plotted to show the extent of fitting of the OHTC. From this figure it is clear that the correlations, Eqs 36(a) and 36(b), predict the OHTC data within an error limit of  $\pm 10\%$ .

**Simulation of SEFFFE system**

For the solution of model, developed in the present investigation, computer program is developed in Fortran 90 and run on Pentium IV machine, using

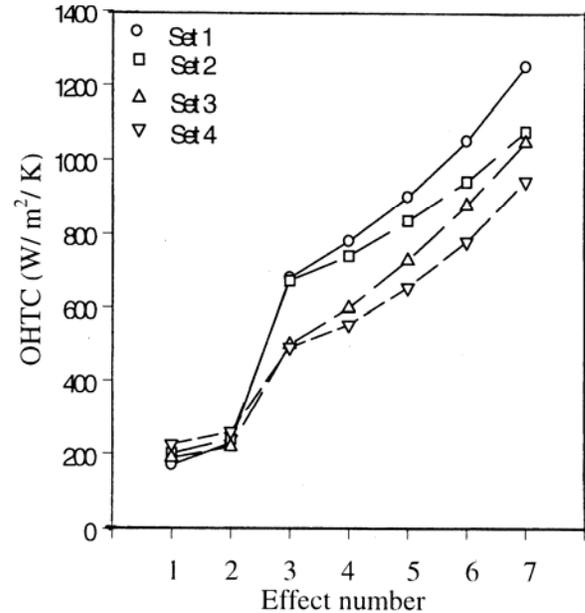


Fig. 5 — Profiles of OHTC from plant data

Table 5 — Cross-correlation coefficients between the parameters OHTC,  $\Delta T$ ,  $x_{\text{avg}}$  and  $F_{\text{avg}}$

	OHTC	Delta T	X avg	F avg
OHTC	1			
Delta T	-0.96999	1		
X avg	0.928086	-0.92326	1	
F avg	-0.81645	0.875356	-0.90815	1

Microsoft Fortran Power Station compiler. The specified input are  $n$ ,  $n_s$ ,  $\mathbf{B}$ ,  $A$ ,  $F$ ,  $x_F$ ,  $T_F$ ,  $T_S$  and  $T_{Le}$  whereas output parameters are pressures, temperatures of each effect, V1 to V7, heat transfer rate, heat loss, final product rate and concentration after product flash, total steam required and steam economy. The flow chart of the algorithm is shown in Fig. 7.

For the generalized cascade algorithm the equation developed for an effect is solved repeatedly for different effects depending upon the sequence of computation determined by the selected flow sequence which in turn is decided by the Boolean matrix  $\mathbf{B}$ .

For the SEFFFE system, order of the matrix is  $8 \times 8$ , where the first column denotes the feed stream and subsequent columns are source effects 1 to 7 and first 7 rows are sink effects and last row is product stream. A unit value of element  $b_{ij}$  indicates that liquor exiting from  $(j-1)^{\text{th}}$  effect enters  $i^{\text{th}}$  effect. The backward flow sequence is represented by the Boolean matrix, given below, in which element  $b_{13} = 1$  shows that liquor exits from  $2^{\text{nd}}$  effect and enters first effect.

Similarly, Boolean matrices for other flow sequences, feed, product and condensate flash tanks etc. can be represented.

(Feed) F	1	2	3	4	5	6	7	← Source effect
								↓ Sink effect

$$\mathbf{B} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ P \text{ (Product)} \end{matrix}$$

### Validation of model

To establish the reliability of the present model it is required to compare the results obtained from simulation with the plant data obtained from the paper mill.

A scaling factor for Eqs 36(a), 36(b) and (35) for OHTC and heat loss, respectively, has been used as a tuning constant during simulation. It was used to

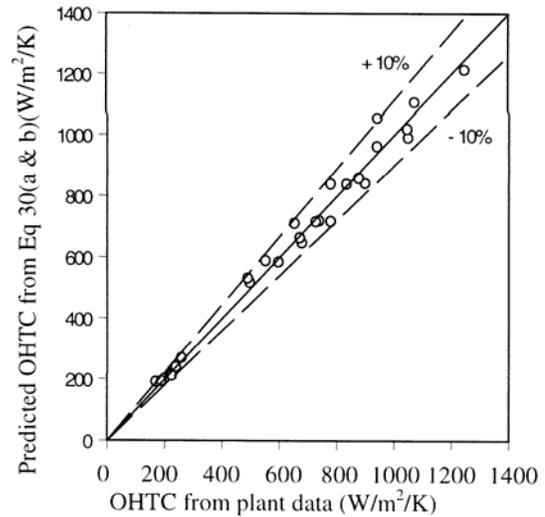


Fig. 6 — Comparison of OHTC from plant data for all effects and those predicted from Eq. (36) (a & b)

Table 6 — Value of coefficients of Eq. (36)

Effect No.	a	b	c	d	% Error band	Eq. No.
1 and 2	0.0604	-0.3717	-1.227	0.0748	-11.32 to 7.25	36(a)
3 to 7	0.1396	-0.7949	0.0	0.1673	-11.75 to 8.20	36(b)

match output parameters such as live steam consumption and exit concentration of simulation run with plant data. In general tuning constant varied from 0.935 to 1.045 for four sets of plant data. This is within the error limit of the Eqs (35) and 36 (a & b). For base case parameters, shown in Table 1, tuning constant is taken as 1.01.

Figures 8 and 9 have been plotted to show the comparison between experimental data obtained from the mill for concentration of black liquor and vapour body temperature of different effects with that obtained from model, respectively. Predicted results show that the liquor concentration match within an error band of  $-0.2$  to  $+0.4\%$ , and the vapour temperature of different effects match within an error limit of  $-0.26$  to  $+1.76\%$ . The present model computes the temperature difference ( $\Delta T$ ) for each effect with a maximum relative error of 23% between plant data and simulation result. However, for the similar MEE system the published model<sup>7</sup> showed a maximum error of 43.43% for the prediction of  $\Delta T$  in each effect. Thus, it appears that the present model predicts the plant data fairly well in comparison to the previously published model.

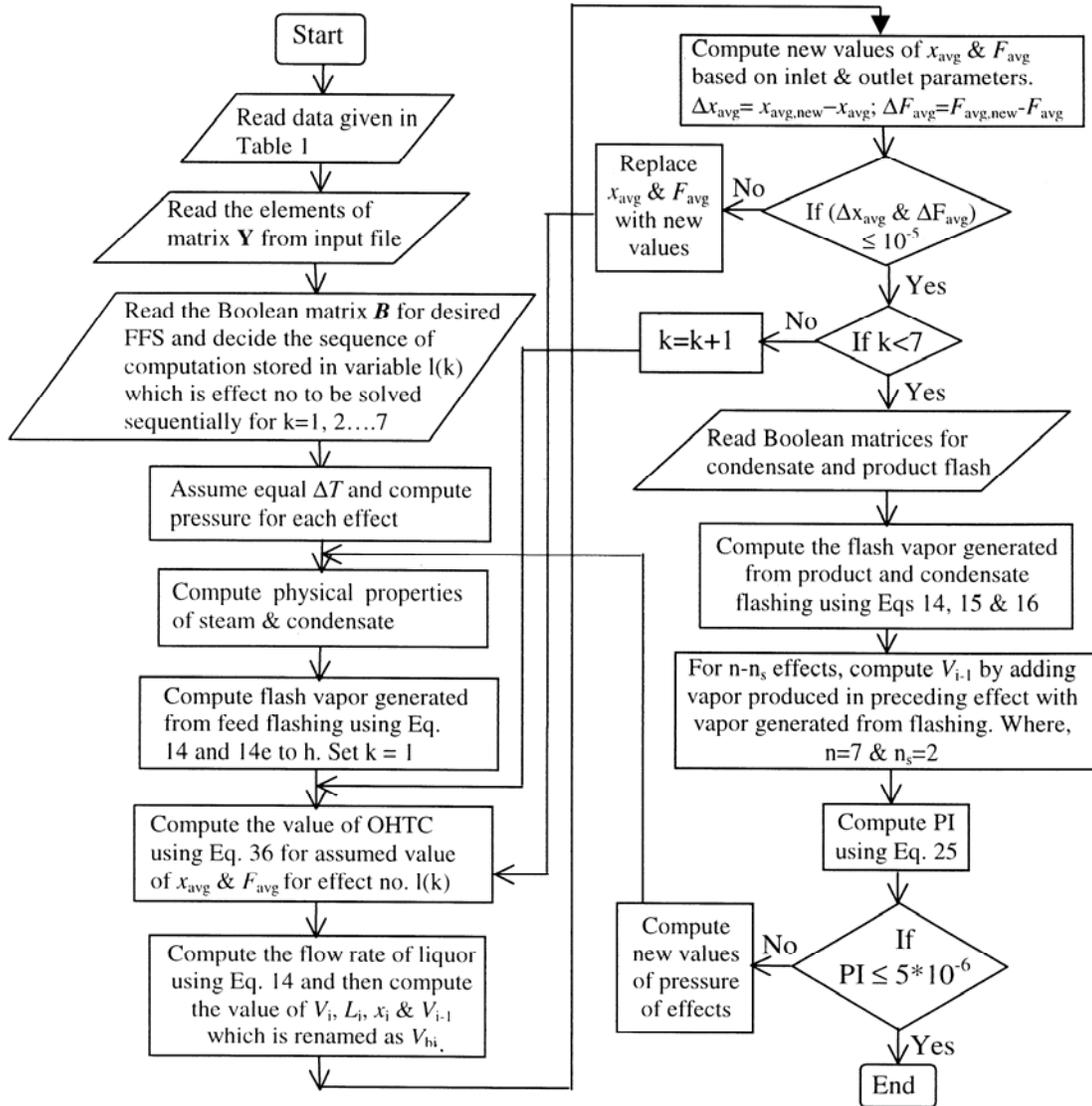


Fig. 7 — Flow chart of the algorithm for solution of the model

## Results and Discussion

The results of the simulation run for the base case operating data are given in Appendix A. Further, it is interesting and useful to find out the contributions of different flash streams towards the total vapour produced including the live steam. For the base case data reported in Table 1, this analysis is carried out. Fig. 10 shows the share of different vapour/steam streams towards the total vapour in each effect. It is interesting to note that out of the total vapour produced and live steam used in the network (for all effects), live steam comprises 18.6%, vapours produced in effects No. 1 to 6, V1 to V6, comprise 74% and condensate -, feed- and product- flash comprise 6.1%, 0.7% and 0.6%, respectively.

## Effect of variation in input parameters on output parameters

After establishing the reliability of the present model, it was thought logical to study the variation of output parameters such as live steam consumption (SC), steam economy (SE) and product concentration ( $x_p$ ) with change in input parameters so that better operating conditions can be identified which will give maximum steam economy.

Generally, changes made in the input parameters such as,  $T_s$ ,  $T_{Le}$ ,  $T_F$ ,  $x_F$  and  $F$ , do affect output parameters SC, SE and  $x_p$ . In the present investigation, the input parameters are varied within a range as given in Table 7 to study its effect on output parameter. The ranges of input parameters are considered after analyzing the prevailing practices in

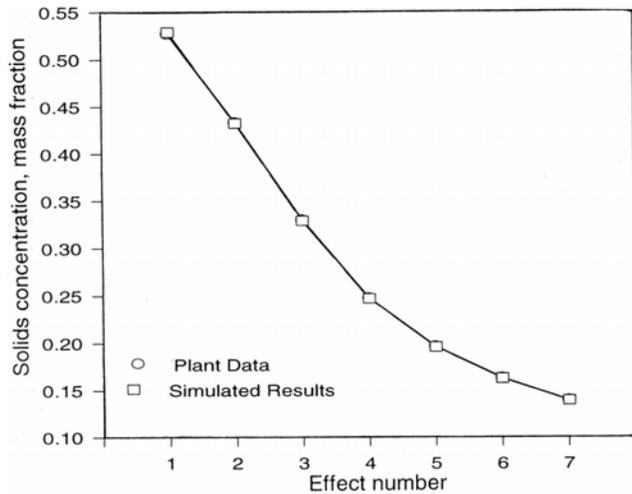


Fig. 8 — Comparison between solid concentration in liquor from plant data and that predicted by model

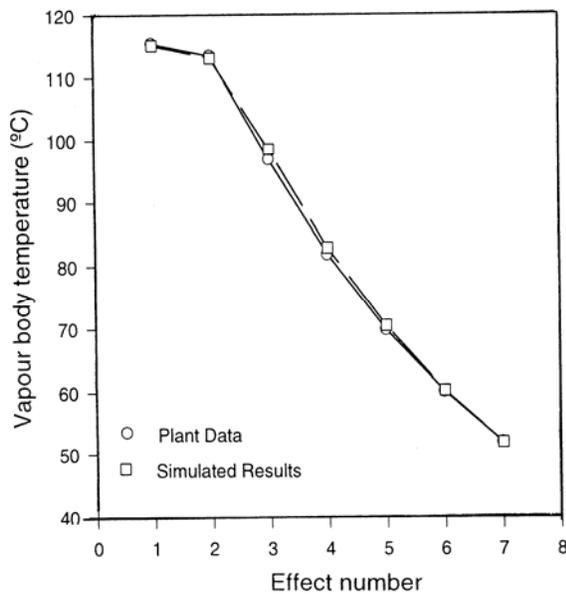


Fig. 9 — Comparison between vapour temperature of different effects from plant data and that from model

Indian paper mills. To study the effect of different input parameters, it is necessary to figure out a standard set of input parameters, given in Table 1, around which these parameters are varied.

The input-output interaction is a complex phenomena in a SEFFFE system, as a change in input parameters trigger a intricate chain reaction affecting almost all parameters, including, vapour body and liquor temperatures of all the effects, almost all flash vapour fractions, heat transfer coefficients, rate of evaporation in each effect, etc. Further, it appears that the steam economy, SE, is the single most prominent

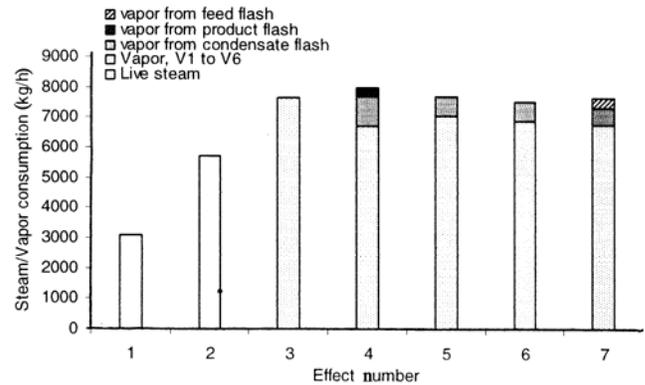


Fig. 10 — Effectwise steam/vapour consumption obtained from simulation

Table 7 — Ranges of operating parameters of a SEFFFE system

S. No.	Parameters	Variation in value
1.	Steam temperature ( $T_S$ )	120°C - 160°C
2.	Inlet liquor concentration ( $x_F$ )	8% - 16%
3.	Last effect (7th) temperature ( $T_{Le}$ )	42°C - 62°C
4.	Liquor temperature ( $T_F$ )	44.7°C - 84.7°C
5.	Liquor flow rate ( $F$ )	56200 - 78680 kg/h

Table 8 — Trends of SC, SE and  $x_p$  with change in  $T_s$ ,  $T_{Le}$ ,  $T_F$ ,  $x_F$  and  $F$

Parameter	SC	SE	$x_p$	Specified parameter
$T_s$				$x_F$ , $F$ , $T_F$ and $T_{Le}$
$x_F$				$T_s$ , $F$ , $T_F$ and $T_{Le}$
$T_F$				$T_s$ , $x_F$ , $F$ and $T_{Le}$
$F$				$T_s$ , $x_F$ , $T_F$ and $T_{Le}$
$T_{Le}$				$x_F$ , $F$ , $T_F$ and $T_s$

parameter to evaluate the efficiency of the SEFFFE system as it varies with variation in operating parameters and geometrical parameters as well. Moreover, the contributions of SC and  $x_p$  are also included in it as the value of SE is the ratio of total water evaporated to total SC. In addition to it, the amount of evaporated water is also related to the product concentration,  $x_p$ , directly. Though by monitoring SE one can keep a watch on the economics of evaporation, the study of variations in parameters such as SC and  $x_p$  with input parameter offers better understanding of the process. The trends of behaviours of SC, SE and  $x_p$  with change in input parameters are also shown in Table 8.

Based on above investigation, it is seen that for values of parameters  $T_S$ ,  $T_L$ ,  $T_F$ ,  $x_F$  and  $F$  equal to 120°C, 84.7°C, 52°C, 0.118 and 56200 kg/h respectively, the SEFFFE system exhibits maximum SE of 5.57 with  $x_p$  and SC equal to 0.49 and 7700 kg/h, respectively. This value of SE is 11.6% more than the SE at which SEFFFE system is being operated currently. Thus, from above analysis it can be suggested that only by changing the operating conditions, SE of the system can be improved without any prior modification in layout of the paper mill.

### Conclusions

The salient conclusions drawn from the present investigations are given as:

- (i) The correlation developed for BPR predicts the plant data within an average error of 3.4%.
- (ii) The empirical correlation of OHTC for a SEFFFE system predicts the plant data within  $\pm 10\%$ .
- (iii) The present model predicts the liquor concentration profile and vapour body temperature profile of different effect within maximum error limit of  $\pm 2\%$ . Also it simulates the plant data with considerably smaller amount of error in comparison to published model.
- (iv) The steam economy of the SEFFFE system can be improved by proper selection of values of operating parameters without any prior modification in the plant layout.

### Nomenclature

$A$	= Heat transfer area of an effect, $m^2$
$a_1 - a_4$	= Coefficients of cubic polynomial (Eq. 14)
$a_{ij}$	= Element of the inverse matrix of $[Y - I]$
$b_{ij}$	= A parameter used in Table 4
$B$	= Boolean matrix
$BPR$	= Boiling point rise, K
$C$	= Condensate flow rate, kg/s
$C_1 - C_5$	= Constant in mathematical model
$C_p$	= Specific heat capacity, J/kg/K
$F$	= Flow rate of liquor, kg/h
$h$	= Specific enthalpy of liquid phase, J/kg
$H$	= Specific enthalpy of vapour phase, J/kg
$I$	= Identity matrix
$k$	= Iteration number
$L$	= Liquor flow rate, kg/s
$n$	= Number of total effects
$n_s$	= Number of effects supplied with live steam
$P$	= Pressure of steam and vapour bodies of evaporator, $N/m^2$
$PI$	= Performance index as defined in Eq. (25)

$Q_{loss}$	= Heat loss from an evaporator, W
$T$	= Vapour body temperature of an effect, K
$U$	= Overall heat transfer coefficient, $W/m^2/K$
$V$	= Rate of vapour produced in an evaporator, kg/s
$x$	= Mass fraction of solids in liquor
$Y$	= Flow fraction matrix

### Greek letters

$\alpha$	= A parameter as defined by Eq. (26)
$\beta$	= A parameter as defined in Eq. (27)
$\Delta$	= Difference between two parameter
$\delta$	= Difference between same variable over two successive iterations
$\gamma$	= A parameter as defined in Eq. (28)
$\gamma'$	= A parameter as defined in Eq. (29)
$\gamma''$	= A parameter as defined in Eq. (30)
$\theta$	= A parameter as defined in Eq. (31)

### Subscripts

$avg$	= Average of inlet and outlet conditions
$e$	= Exit condition
$F$	= Feed
$i$	= Effect number
$L$	= Black liquor
$Le$	= Last effect
$S$	= Steam
$V$	= Vapour
$V1$ to $V7$	= Vapours produced in one to seventh effects

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### Appendix A

The simulation results for base case operating parameter are shown below:

Final Results Iterations used are 19

Steam temps. Are 140.02 °C and 147.00 °C

Steam Pressures are 3.616 bar and 4.389 bar

Feed rate= 56200.0 kg/h, Initial Feed concn = .1180, Feed temp. = 64.7 °C

Feed to 7<sup>th</sup> effect, after initial feed flash rate = 55873.4 kg/h

Flashed Feed Concentration = .1187

Amount of vapour generated due to feed flash = 326.64 kg/h

Feed Sequence

Product ← 1 ← 2 ← 3 ← 4 ← 5 ← 6 ← 7 ← Feed

Live steam in Effect Nos: 1 and 2

Last effect temp, °C: 52.00 Last effect pressure, bar: .136

Effect No.	1	2	3	4	5	6	7
Pressure, bar	1.70	1.59	.96	.53	.32	.20	.14
Vapour temp, °C	115.1	113.1	98.5	82.9	70.5	60.3	52.0
Liquor temp, °C	123.0	118.8	102.2	85.3	72.3	61.6	53.1
Feed flash vap, kg/h	.00	.00	.00	.00	.00	326.64	.00
Prod flash vap, kg/h	.00	.00	275.48	.00	.00	.00	.00
Cond flash vap, kg/h	.00	.00	986.06	668.97	674.04	543.95	.00
Vap to Preheat, kg/h	.00	.00	.00	.00	.00	.00	.00
Vapour added, kg/h	.00	.00	1261.54	668.97	674.04	870.59	.00
Exit vapour, kg/h	2801.7	4843.4	6707.5	7039.5	6868.0	6773.2	8294.6
Total vapour, kg/h	2801.7	4843.4	7969.0	7708.4	7542.1	7643.8	8294.6
Steam/Vap reqd, kg/h	3096.8	5705.5	7644.8	7960.2	7712.2	7548.9	7636.5
Exit liquor, kg/h	12545.5	15347.2	20190.6	26898.1	33937.6	40805.6	47578.8
Exit concn.	.5286	.4321	.3285	.2465	.1954	.1625	.1394
delta T, °C	16.97	28.20	11.93	13.19	10.61	8.92	7.11
OHTC, W/sq m/K	191.75	215.38	589.29	568.08	698.97	825.56	1014.80
Heat Transfer, MW	1.758	3.279	4.642	4.945	4.895	4.858	4.978
Energy loss, MW	.0867	.0846	.0697	.0539	.0413	.0308	.0224

Total Product Rate = 12270.0 kg/h, at liquid concn = .5405

Total Steam Consumption = 8802.3 kg/h

Primary and secondary condensate after flashing = 7637.11 and 36800.6 kg/h respectively

Overall Energy Balance

Energy in = 10.656 MW, Energy out = 10.263 MW, Energy loss = .3926 MW (in percent 3.6845%)

Delta P = -.114E+03 -.382E+02 -.127E+03 -.753E+02 .351E+02 N/sq m

Performance Index = .3185E-05, Steam Economy = 4.9907

Facu = 1.01000 and floss = .76000