Chaotic analysis of pressure fluctuations in a gas-solid fluidized bed

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The aim of the present work is to determine the superficial gas velocity range for which chaos occurs in a gas-solid fluidized bed for various static-bed-height to bed diameter ratios and sand/plastic-bead mixtures and to observe the change in deterministic chaos parameters. Four piezo-resistive pressure transducer along the height of fluidized bed have been used to record pressure fluctuations at 600 Hz. Amplitude Adjusted Fourier Transform (AAFT) technique has been used for generating surrogate data. Data have been analyzed for the superficial gas velocity range of 0 to 16 times the minimum fluidization velocity. For the various static bed heights and sand plastic-bead mixtures it is seen that chaos is observed in the superficial gas velocity range of 8 to 12 times the minimum fluidization velocity. The Hurst exponent rises with superficial gas velocity and then falls after a maximum. The change in the fractal dimension between surrogate and original time series also follows the same trend. The superficial velocity range for which chaos occurs is confirmed when there is change in fractal dimension between original and surrogate series greater than 0.55, Hurst exponent is above 0.5 and Lyapunov exponent is positive.

Keywords: Chaotic analysis, pressure fluctuations, gas-solid fluidized bed, Lyapunov exponent, Hurst exponent.
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Fluidized beds are widely used in the chemical and fossil fuel processing industries to mix particulate solids and fluids. A typical fluidized bed consists of a vertically oriented chamber, a bed of particulate solids, and a fluid flow distributor at the bottom of the chamber. The fluid flows upward through the particles, creating a drag force which counteracts gravity. With sufficiently high flow, the solids are levitated and move in complex, turbulent patterns (hence the name “fluidized”). This turbulence promotes heat and mass transfer between the fluid and the solid particles.

In the present study, pressure fluctuations have been used to study the hydrodynamics of a gas fluidized bed. The advantage of the study of pressure signals is that they include the effects of many dynamical phenomena taking place in fluidized beds, such as gas turbulence, bubble formation, passage and eruption of bubbles, self-excited oscillations of fluidized particles, bubble coalescence and splitting1. The behavior of a fluidized bed exhibits sensitivity to initial conditions. The behavior associated with macroscopic motion such as slugging has been described as having low dimensional features in the sense that there is a clear transition from no bulk motion to periodic oscillations to apparently chaotic, intermittent bursts as the gas flow is increased2,3.

It is found that pressure and voltage measurements in fluidized beds exhibit the characteristic of low-dimensional deterministic chaos. Fractal dimension, Hurst exponent, and Lyapunov exponent are the methods used to predict chaos and change in the fractal dimension between original and surrogate time series is the method used to confirm chaos4. Deterministic chaos in bubbling fluidized beds can arise readily from non-linear bubble-to-bubble interactions5-8. The pressure waves can be very useful to characterize the gas bubble dynamics. Major source of pressure waves in a freely bubbling bed is due to bubble coalescence. Five different fluidization regimes have been identified in a circulating fluidized bed9,10.

Much work on chaotic study of fluidized bed has been carried out but there is no study that completely specifies the superficial gas velocity range for which chaos occurs for various static-bed-height to diameter ratios and sand/plastic-bead mixtures. The objective of the present study is to know the range of superficial gas velocity for which chaos occurs, and to know whether the range is affected by static bed height or type of particles.

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The work plan includes the following objectives: (i) to study the noise characteristics of the pressure transducer-amplifier-data acquisition card circuit assembly, (ii) to study the chaotic behavior of sand particles under fluidization conditions for various static bed height to bed diameter ratios, (iii) to study the chaotic behavior of mixture of plastic bead-sand under fluidization conditions for different vol% plastic beads. The chaotic behavior is to be studied using the various techniques of deterministic chaos: fractal dimension of state space plots using the box counting method, Lyapunov exponents, Hurst rescaled range analysis (R/S), periodogram of the data and generation of the surrogate data by AAFT method.

Experimental Procedure
A Perspex column with an inside diameter of 55 mm and height of 3000 mm was fabricated. It was provided with a perforated distributor plate that produces uniform air distribution in the bed. From an air compressor, the air was supplied to fluidize the bed. The superficial gas velocity was varied up to 16 times the minimum fluidization velocity. For a particular static bed height the minimum fluidization velocity was determined and the superficial gas velocity was supplied at various $V/V_{mf}$ ratios, where $V$ is the superficial gas velocity and $V_{mf}$ is the superficial gas velocity at minimum fluidization. All experiments were carried out with sand particles of mean size $d_p = 350 \, \mu m$ and density ($\rho$) = 2565 kg/m$^3$ and plastic beads of size $d_p = 1.5 \, mm$ and density ($\rho$) = 1081 kg/m$^3$. The static bed height was varied from 90 to 300 mm.

Pressure fluctuation measurements were performed using piezo-resistive Pressure Transducers of differential type (RS 286-686, 0-5 psi) imbedded in the wall of the fluidized bed. Pressure signals were sampled with a A/D data acquisition card (PCL-812, Dynalog make) and stored in a computer. Driver software built in BASIC language was used to control data acquisition. The signals were recorded at a frequency of 600 Hz. The location of the four transducers was set at different heights from the distributor plate (42, 79, 119 and 149 cm), which was far enough from the distributor plate to avoid end effects. The total acquisition length was 15000 to 45000 points for each experimental run in order to minimize statistical error.

Sand with $H_b/D$ values of 1.72, 2.74 and 4.23 and plastic-bead-sand mixtures of 30.1 vol% plastic beads in sand with $H_b/D$ of 2.74; and 43.75 vol% plastic beads in sand with $H_b/D$ of 3.09 have been taken in the present study. The details of experimental work are given elsewhere$^{11-13}$.

Data analysis
The pressure transducer converts the pressure signals into milli volts (mV). The signal was amplified by a two stage amplifier with an amplification factor of 1100. The amplified signals were converted to digital signals with the help of a data acquisition card. These digits were in the range between $-2047$ to $+2048$ corresponding to pressure transducer signals $-5$ to $+5$ mV, respectively. The data were recorded at the frequency of 600 Hz.

Construction of state space plots
For nonlinear analysis of time series data it is required to plot the signals in such a way so that time is not an independent variable. The time series of pressure signals were plotted in state space. A state space is constructed based on the method of delays. In these plots signals are plotted with its own time lagged signal. Time series of pressure signals are plotted in the state space to obtain attractors. In practical case, the selection of parameters such as time lag is important. It must be large enough so that it is not affected by noise in the data and small enough to discern local characteristics on the attractor rather than include the major folds that exist in the attractor since it is a bounded object. State space plots have been made by taking lag of 2000 points (of the total data consisting of about 15000 to 45000 data points) which is neither small nor large$^{14}$. For a particular system, state space plots were constructed for a wide range of superficial gas velocities. When a state space plot is observed to be a strange attractor, fluidized bed can be in chaotic state. An attractor is one whose data are bounded and a strange attractor is one which converges to a central point due to energy losses, may be due to friction. Attractors are geometrical structures or limit point sets in phase space on which the trajectories settle down eventually. The attractor underlying the chaotic motion (strange attractor) is a geometrical object whose dimension is fractional.

In the present study, mV signals were plotted in the form of $mV(t+\tau)$ versus $mV(t)$, where $mV(t)$ is the milli volt signal at time $t$ and $mV(t+\tau)$ is the milli volt signal after a time lag of $\tau$, that is at the time of $t+\tau$. The state-space plot was used for the prediction of chaos in the fluidized bed (once the strange attractor...
Calculation of fractal dimension

Fractal dimension is a measure used to characterize the state space plot quantitatively. The dimension of a derived unit may be defined as the power to which the fundamental units like mass, length and time must be raised to represent it. Box counting method was used to calculate the fractal dimension. Cubes were used and, therefore, the embedded dimension is 3. The time series data are shifted to the first quadrant by an addition of constant positive value according to the need of each of the data set in the time series. The new time series data are plotted in a three-dimensional state space, without any change in the pattern of its original state space characteristics. The cubes of size \( r^3 \) are defined from the origin (0,0) and then whole of the state space curve is covered with the small sized cubes. The size of the cubes is increased in each step of simulation and the corresponding minimum number of cubes required (\( n \)) for each value of \( r \) is calculated. The plot of log \( n \) versus log \( r \) gives a straight line with negative slope, where the absolute value of the slope is the fractal dimension of the state space plot. As \( r \) becomes very small, the fractal dimension is defined as

\[
d_f = -\lim_{r \to 0} \frac{\log n}{\log r}
\]

A computer program for counting the number of cubes for corresponding side \( r \) was made in C language. The listing of the C code is given elsewhere. Each time size of the cube was increased slightly and corresponding number of cubes \( n \) calculated. The slope of the best-fit straight line of log\( n \) versus log\( r \) plot was calculated. The negative slope of the log\( n \) versus log\( r \) plot gives the fractal dimension of that state space curve.

Calculation of Lyapunov exponent

Chaos in deterministic systems implies a sensitive dependence on initial conditions. This means that two trajectories starting close to one another in phase space will move exponentially away from each other. Lyapunov exponent is a tool for diagnosing whether or not a system is chaotic. Lyapunov exponent is a measure of the divergence of nearby trajectories. If \( d_0 \) is a measure of the initial distance between two starting points, at a small but later time the distance is, \( d(t) = d_0 e^{2\lambda t} \), where \( \lambda \) is the Lyapunov exponent. It is well argued that a system’s behaviour is chaotic if its average Lyapunov exponent is a positive number. The criterion for chaos then becomes: \( \lambda > 0 \) (chaotic) and \( \lambda \leq 0 \) (regular motion). If Lyapunov exponent is negative, slightly separated trajectories converge and the evolution is not chaotic. If Lyapunov exponent is positive, nearby trajectories diverge, the evolution is sensitive to initial conditions and therefore chaotic. Coding for finding the largest Lyapunov exponent was carried out in Matlab 6.1. The details of the algorithm and code listing are recorded in Raj.

Calculation of Hurst exponent

To obtain information about the Hurst exponent \( H \), for a given time series, it can be resorted to the rescaled-range (R/S) analysis, which was originally proposed by Hurst. This method characterizes correlations in time-series data and is useful to distinguish three different cases. When a positive correlation in time-series data exists, the values of the Hurst exponent \( H \), are between 0.5 and 1. In addition, higher values of \( H \) indicate more persistent data exhibiting some trends. For uncorrelated data \( H \) equals to 0.5 and for negatively correlated data \( H \) takes the values 0-0.5. The Hurst exponent can be deduced from a time-series data as follows: first, from the time-series \( X(t) \), the cumulative departure \( B(t,u) \) to the average is computed in the range from \( t + \tau \):

\[
B(t,u) = \sum_{i=1}^{u} X(u) - \langle X \rangle_t
\]

Then the sample sequential range, \( R(t,\tau) \) is defined as

\[
R(t,\tau) = \max_{0 \leq t \leq \tau} B(t,u) - \min_{0 \leq t \leq \tau} B(t,u)
\]

Finally, the rescaled range \( R(t,\tau) / S(t,\tau) \), where \( S(t,\tau) \) is the mean square deviation of the time series, scales as a power function of \( \tau \) as,

\[
\frac{R(t,\tau)}{S(t,\tau)} \sim \tau^H
\]

Therefore, the value of \( H \) can be evaluated from the slope of the logarithmic plot of the rescaled range as a
function of $\tau$. Data of length 10000 data points was divided into elements of equal length ($\tau$). For each element mean and standard deviation were calculated. Cumulative departures from mean in each case were obtained. For $H > 0.5$, the system is chaotic. Coding for finding the Hurst exponent was carried out in Matlab 6.1. The details of the algorithm and code listing are recorded.

**Periodogram**

This is a plot of power versus frequency. Generally, it is plotted up to Nyquist frequency, since most of the data falls below this frequency. Most of the fluidized bed signals fall below 20 Hz, so the dominant frequency must be in this range. Sampling frequency more than 20 Hz is sufficient to extract all the information of the fluidized bed. Use of periodograms is made in the present study to check the surrogate data analysis. The periodogram of original time series and its corresponding surrogate data series must be of same nature. Coding for construction of periodogram was carried out in Matlab 6.1.

**Surrogate data generation**

For generating surrogate data consistent with the null hypothesis of linearly correlated Gaussian noise the approach used in the present study is to Amplitude Adjusted Fourier Transform (AAFT) the data set, randomize the amplitude, and then invert the transform. Surrogate data generation is used to confirm chaos. The phases were kept constant and amplitudes were adjusted according to Theiler et al. Inverse FFT gives surrogate data series whose fractal dimension (of the state space plot) was calculated using the box counting method. If the fractal dimension of original time series was found significantly different from fractal dimension of its corresponding surrogate series, chaos was confirmed as there is some information (other than noise) in the data which is changed due to amplitude adjusted Fourier transformation, else it is noise. The code for generating surrogate data series by AAFT method was written in Matlab 6.1.

**Results and Discussion**

The parameters that have been used to predict and confirm chaos are Lyapunov exponent, Hurst rescaled range analysis, and change in fractal dimension between original and surrogate series. First the characterization of noise was carried out and thereafter the effect of static bed height and the effect of mixing plastic beads in sand was studied. The whole data acquisition system from transducer to the data coming on the visual display unit of the computer was thoroughly calibrated and the amplifier was designed in such a way that the noise is minimized. The amplification factor was found to be constant for the input pressure signals from –5 to +5 mV.

**Characterization of noise**

When pressure transducer was inserted in the fluidized bed, but no air was flowing, the signals recorded were totally coming from noise. Analysis of noise is done to check the reliability of readings of the experiment. It was found that state space plot of noise is not a strange attractor. The digital signals on the visual display unit generated from the noise data are only between –20 to +20, which is small, as compared to pressure signals when the bed was fluidized, which were in the range –2000 to +2000.

For the noise data the pressure fluctuation signals were recorded and the state space plot plotted and the fractal dimension determined using the box counting method. Figure 1 shows the log $n$ versus log $r$ plot for noise data. The slope of the line is –1.3128 and, therefore, the fractal dimension is 1.3128, as fractal dimension is the negative of the slope of the log $n$ versus log $r$ graph. Figure 2 shows the log $n$ versus log $r$ plot for surrogate data of noise and the fractal dimension is 1.3135. Since the change in the fractal dimension between the original time series and surrogate series is small, therefore chaos is not present in the noise data. Figure 3 shows the log R/S versus log $r$ plot and the Hurst exponent is 0.3421. Since the Hurst exponent is less than 0.5, so chaos does not exist in noise data. The Lyapunov exponent is also calculated to be negative. It shows that no information comes from transducer signals and circuit assembly and, therefore, the data acquisition system is fit to be used in the fluidized state.

![Fig. 1—log n versus log r plot of original time series of noise data](image-url)
Effect of static bed height

For a particular static bed height the gas was fed at a particular superficial gas velocity and the pressure fluctuations were recorded with time. The superficial gas velocity was changed and again the fluctuations were recorded. For superficial gas velocities of 1, 5, 8, 10, 12, 14 and 16 $V_{mf}$ the procedure was repeated. Thereafter, the static bed height was changed and again the whole procedure was repeated. Figure 4 shows the log$n$ versus log$r$ plot of the state space curve made from the time series of pressure fluctuations by taking the time lag to be 2000 data points recorded at a frequency of 600 Hz ($\tau = 3.33$ s). Thus, the number of data points plotted are 13000-43000 out of the total data points for various experiments taken, that is, 15000-45000. The fractal dimension at the superficial gas velocity of 10 $V_{mf}$ for sand bed of static bed height to bed diameter ratio, $H_b/D=2.74$ and at height of 42 cm above the distributor plate is 2.1927 as shown in Fig. 4.

Figure 5 shows the log$n$ versus log$r$ plot of the surrogate data series of the conditions of Fig. 4 and the fractal dimension is 2.8762. Since the fractal dimension has appreciably changed it implies that there is some information (and not simply noise) present in the pressure fluctuations. Figure 6 shows the change in fractal dimension between the surrogate data and original time series data ($\Delta d_f$) versus $V/V_{mf}$ for static bed height to diameter ratio $H_b/D=1.72$. The curve rises and falls. It has been reported in literature that there should be appreciable change in the fractal dimension only then chaos is confirmed but the quantitative value of the change varies with the method used for generating the surrogate data. When the state space plot of time series is a strange attractor chaos was predicted and the confirmation of chaos was carried out by generating surrogate data using the AAFT method. Periodograms plots of original time series and surrogate series are of the same nature. Similar results were also obtained by Hay et al. $^3$. Figure 7 which is the $\Delta d_f$ versus $V/V_{mf}$ plot for static bed height to bed diameter ratio of 4.23 also shows the similar trend as in Fig. 6 and in both the figures the maximum change in the fractal dimension occurs at superficial gas velocity of 10 $V_{mf}$. So chaos should also occur around that range.
Figures 8 and 9 show the logR/S versus log τ plot for $H_b/D=2.74$ at superficial gas velocity of 5 and 8 $V_{mf}$, respectively. At 5 $V_{mf}$ the Hurst exponent is 0.4004 and at 8 $V_{mf}$ it is 0.5259. It has been reported in literature that chaos occurs when Hurst exponent is greater than 0.5, so at 8 $V_{mf}$ or somewhat before that chaos begins.

Figures 10 and 11 show the Hurst exponent versus $V/V_{mf}$ for $H_b/D=1.72$ and 4.23, respectively. The Hurst exponent is greater than 0.5 (for which chaos occurs) for superficial gas velocity range from 7.1 to 12.2 $V_{mf}$ in Fig. 10 and from 7.7 to 12.0 $V_{mf}$ in Fig. 11. For these superficial gas velocity range it can be seen from Figs 6 and 7 that to quantify the $Δd_f$ for which chaos occurs it is seen that $Δd_f$ should be greater than 0.55 if chaos is to be confirmed. In Fig. 6 for $Δd_f >0.55$, $V/V_{mf}$ lies in the range 7.3 to 12.5 and in Fig. 7 for $Δd_f >0.55$, $V/V_{mf}$ lies in the range 7.6 to 12.2. Thus, it is seen that for various static bed height to bed diameter ratios though the minimum fluidization velocity changes but the ratio of $V/V_{mf}$ for which chaos is observed remains the same and for the various experiments done it was seen, that, for all the $H_b/D$ ratios chaos is confirmed for the superficial gas velocity range from 8 to 12 $V_{mf}$. Beyond the superficial gas velocity of 12 $V_{mf}$ turbulence comes in the fluidized bed and therefore chaos is not observed.

As chaotic motion is observed between periodic and turbulent motion.

For chaos to occur it is not essential that the fractal dimension change be quite appreciable after AAFT but also that the Hurst exponent be more than 0.5 and Lyapunov exponent is positive. There is appreciable change in the fractal dimension for superficial gas velocities of $V_{mf}$ and 5 $V_{mf}$ but the Hurst exponent is below 0.5 so chaos is not confirmed. Therefore, the change in the fractal dimension between the surrogate and original series should be greater than 0.55, for chaos to occur.
Effect of plastic-bead-sand ratio

The vol% plastic beads in sand of 43.75 and 30.10 have been used in the present study. The $H_b/D$ ratio is not the same for both the cases but it is seen from the previous section that $H_b/D$ ratio has no effect on the ratio $V/V_{mf}$ for which chaos is observed. In this section the ratio of $V/V_{mf}$ for which chaos occurs if plastic beads are added to sand is determined.

Figure 12 shows the $\Delta d_f$ versus $V/V_{mf}$ plot for 43.75 vol% plastic beads in sand and it is seen, that, $\Delta d_f > 0.55$ for $V/V_{mf}$ lying in the range 7.7 to 12.6 which is almost similar to that for pure sand. It is seen that the curve rises and falls more steeply when plastic beads are added to sand. Figure 13 shows the logR/S versus log $\tau$ plot for superficial gas velocity of 8 $V_{mf}$ for 30.10 vol% plastic beads in sand and the slope of the line is 0.5486 which shows that chaos occurs for this superficial gas velocity as Hurst exponent is greater than 0.5. Figure 14 shows the Hurst exponent versus $V/V_{mf}$ for 43.75 vol% plastic beads in sand and it is seen that for $H > 0.5$ the $V/V_{mf}$ lies in the range 7.8 to 12.5. Thus, it is seen that plastic beads also have no effect on the ratio $V/V_{mf}$ for which chaos occurs.

For the case of noise, the Lyapunov exponent is negative and positive values are obtained even for velocities as low as $V_{mf}$, so Lyapunov exponent alone cannot predict the presence of chaos. Thus, it is the change in fractal dimension of the state space plots of surrogate and original series (to be greater than 0.55), Lyapunov exponent (to be positive) and the Hurst exponent (to be greater than 0.5) together, that characterizes the presence of chaotic behavior in a gas-solid fluidized bed.

Conclusions

For various static bed height to fluidized bed diameter ratios of sand and for sand/plastic beads ratios, chaos occurs for superficial gas velocities from 8 to 12 times the minimum fluidization velocity. For chaos to occur, the Hurst exponent should be greater than 0.5 and the change in the fractal dimension (of the state space plots) between original and surrogate data series (generated using AAFM method) should be greater than 0.55. The noise data report negative Lyapunov exponent, Hurst exponent below 0.5 and no appreciable change in fractal dimension. This study can be of immense use in studying the biomass or plastic pyrolysis/gasification in fluidized bed reactors where fluidization of sand along with biomass/plastics is carried out.

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Nomenclature

- $d_f$: Fractal dimension of the state space curve
- $\Delta d_f$: Change in fractal dimension between surrogate and original time series data
- $D$: Diameter of the column
- $H$: Hurst exponent
- $H_b$: Static bed height
- $n$: Number of self-similar cubes required to fill the state space curve in Box counting algorithm
- $r$: Length of side of a cube in box counting method for fractal dimension calculation
- $R/S$: Rescaled range analysis
- $t$: Time
- $V$: Superficial gas velocity
- $V_{mf}$: Superficial gas velocity at minimum fluidization
- $\lambda$: Lyapunov exponent
- $\tau$: Time lag

References