Comparison of hexagon and octagon cylinder models with conventional models for effective thermal conductivity estimation of suspension systems

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This study presents hexagon and octagon cylinder models for effective thermal conductivity estimation of suspension systems. Algebraic equations were derived based on isotherm approach for two dimensional spatially periodic medium. Model prediction was achieved with minimum (±10.83%) and maximum (±15.58%) deviation from experimental data for suspension systems. Model prediction was comparable with conventional models.

Keywords: Conventional models, Effective thermal conductivity, Isotherm, Resistance based unit cell model, Suspension systems

Introduction

Estimation of effective thermal conductivity (TC) of two-phase materials (TPMs) (ceramics, soils, foams, emulsion systems, porous and suspension systems, solid-solid mixtures, fiber reinforced materials and composites) is becoming increasingly important in microelectronic chip cooling, space craft structures, catalytic reactors, heat recovery process, heat exchangers, heat storage systems, petroleum refineries, solar collectors, and nuclear reactors. A resistance based unit cell approach has been applied for various TPMs to predict effective TC. Wiener1 gave upper and lower limits to TC of TPMs based on parallel and series resistance. Most restrictive bounds2-3 (lower and upper) have been proposed for predicting effective TC of TPMs. Bruggeman4 extended Maxwell’s result for lower concentration of dispersed phases to full range of concentration by assuming quasi-homogeneous mixture. Zehner & Schlunder5 proposed a model considering effect of primary and secondary parameters for cylindrical unit cell containing spherical inclusions. TC modeling6 was carried out based on unit cell approach with constant heat flux conditions for random distribution of spheres in a continuum of different materials. Hsu et al7 obtained algebraic expressions for effective TCs of a number of porous media by applying lumped parameter method. Deisser & Boregli8 calculated TC of a saturated porous medium for a two-layer model representing as electrical resistance in an electrical circuit. A unit cell model9 was proposed consisting of spherical particles contacting each other with point to point. Hadley10 used empirically weighted averages of Hashin & Shtrikman3 bounds to model effective TC of several TPMs. Brailsford & Major11 formulated solid or fluid component as a continuous phase, based on Maxwell’s model. Lichteneker12 proposed empirical models for effective TC of binary metallic mixture. Pandé13 developed geometries for effective TC estimation of TPMs. Krupiczka14 carried out numerical study for effective TC based on a model made up of spheres in cubic lattice. Samantray et al15 proposed a comprehensive conductivity model by considering primary parameters based on unit cell and field solution approaches. Validity of model was extended to predict effective TC of various binary metallic mixtures16. Collocated parameter model17 was developed based on unit cell approach for predicting effective TC of TPMs.

This study presents development of resistance approach unit cell model with considering effect of primary and secondary parameters for predicting effective TC of suspension systems.
Experimental Section
Hexagon and Octagon Cylinders Model for Effective Thermal Conductivity Estimation of Suspension Systems

Development of resistance based unit cell model for estimating effective TC based on material micro and nano-structure is extremely important for thermal design and analysis of TPMs. Main feature of resistance based unit cell model is to assume one-dimensional heat conduction in a unit cell, which is divided into three parallel layers (solids, fluid and composite) normal to temperature gradient. Effective TC of TPMs is determined by considering equivalent electrical resistances of parallel and series in unit cell model. Medium geometry is considered as matrix of touching and non-touching in-line hexagon and octagon cylinders.

Hexagon Cylinder

Effective TC of suspension system can be estimated by considering a hexagon cylinder with cross-section ‘a x a’ with a connecting bar of width ‘c’ (Fig. 1a). Stagnant TC of two-dimensional periodic medium is finite contact between spheres by connecting plates with ‘c/a’ denoting contact parameter. Because of symmetry of plates, one fourth of square cross-section has been considered as a unit cell (Fig. 1b). Total resistance offered by hexagon cylinder in unit cell is given as

\[
R_{\text{total}} = \frac{k_s}{\alpha} + \left[ \frac{2k_i(\sqrt{3}-\lambda)}{[(\sqrt{3}-\lambda)+2\sqrt{3}]^{\frac{1}{a}} + \frac{k_s}{k_f} \cdot \frac{2k_i(1 + (1-\lambda))}{2\sqrt{3} \cdot e^{(\lambda-\frac{\phi}{\lambda})}} + \frac{k_s}{k_f} \cdot \frac{2k_i(1 + (1-\lambda))}{2\sqrt{3} \cdot e^{(\lambda-\frac{\phi}{\lambda})}} + \frac{k_s}{k_f} \cdot \frac{2k_i(1 + (1-\lambda))}{2\sqrt{3} \cdot e^{(\lambda-\frac{\phi}{\lambda})}} \right]
\]

\[\frac{k_s}{k_f} \left[ \frac{1}{\alpha} \left( \frac{k_s}{k_f} \right)^{1-\phi} \right] \]

\[\frac{1}{\alpha} \left( \frac{k_s}{k_f} \right)^{1-\phi} \]

\[\frac{k_s}{k_f} \left[ \frac{1}{\alpha} \left( \frac{k_s}{k_f} \right)^{1-\phi} \right] \]

\[\ldots (1)\]
where, conductivity ratio \( \alpha = k / k_s \), length ratio \( \varepsilon = a / l \), contact ratio \( \lambda = c / a \), and \( \varepsilon \lambda = c / l \).

Non-dimensional TC of two-dimensional hexagon cylinder is given as

\[
K_s = \frac{k_{eff}}{k_f} \left[ \frac{1}{\varepsilon^2} + \frac{1}{a^2} \left( \frac{k_{eff}}{k_f} \right)^2 \left( \frac{c}{a} \right) \right]^{-1} 
\]

\[ \frac{1}{\left( \frac{c}{a} \right)^2} \left( \frac{k_{eff}}{k_f} \right)^2 \left( \frac{c}{a} \right) \]

\[
K = \frac{k_{eff}}{k_f} \left[ \frac{2\sqrt{3}(\sqrt{3} - 1)}{1 - (\sqrt{3} - 1)^2} \right] \left[ \frac{2\sqrt{3}(\sqrt{3} - 1)}{1 - (\sqrt{3} - 1)^2} \right]^{-1} 
\]

\[
\left( \frac{1}{\varepsilon^2} \right) + \frac{1}{a^2} \left( \frac{k_{eff}}{k_f} \right)^2 \left( \frac{c}{a} \right) \left[ \frac{1}{\left( \frac{c}{a} \right)^2} \left( \frac{k_{eff}}{k_f} \right)^2 \left( \frac{c}{a} \right) \right]^{-1} 
\]

Octagon Cylinder

Effective TC of two dimensional medium can be estimated by considering an octagon cylinder with cross-section ‘a x a’ having a connecting bar width of ‘c’ (Fig. 1a). Total resistance of unit cell is given as

\[
R_{total} = \frac{\varepsilon^2}{a} \left( \frac{1}{1 + \sqrt{2}} \right)^2 \left( \frac{2\varepsilon}{k_f} \right) \left[ \frac{2\varepsilon}{k_f} \left( \frac{1 + \sqrt{2}}{1 + \sqrt{2}} \right) \right]^{-1} 
\]

\[
\left[ \frac{1}{\varepsilon^2} + \frac{1}{a^2} \left( \frac{k_{eff}}{k_f} \right)^2 \left( \frac{c}{a} \right) \right]^{-1} 
\]

\[
K = \frac{k_{eff}}{k_f} \left[ \frac{2\varepsilon}{k_f} \left( \frac{1 + \sqrt{2}}{1 + \sqrt{2}} \right) \right]^{-1} 
\]

\[
\left( \frac{1}{\varepsilon^2} \right) + \frac{1}{a^2} \left( \frac{k_{eff}}{k_f} \right)^2 \left( \frac{c}{a} \right) \left[ \frac{1}{\left( \frac{c}{a} \right)^2} \left( \frac{k_{eff}}{k_f} \right)^2 \left( \frac{c}{a} \right) \right]^{-1} 
\]

\[
\frac{1}{\left( \frac{c}{a} \right)^2} \left( \frac{k_{eff}}{k_f} \right)^2 \left( \frac{c}{a} \right) \]

Results and Discussion

Effective TC of TPMs mainly depends on characteristics of constituent phase, including TCs of solid and fluid phases, size, shape and thermal contact between solid-solid and solid-fluid interface. Effect of concentration \( (\upsilon) \) on non-dimensional TC of two-dimensional hexagon and octagon cylinders for conductivity ratio \( (\alpha = 20) \) have been investigated. Present model (Fig. 2) with hexagon and octagon cylinders lies between parallel and series lines for conductivity ratio \( (\alpha = 20) \) and contact ratio \( (\lambda = 0-0.2) \). Contact ratio is having major influence on effective TC of TPMs when contact ratio varying between 0 to 0.2, but there is no influence beyond 0.2. For octagon cylinder, present correlation is applicable for concentration varying from 0 to 0.7. For further increment in concentration, non-dimensional TC is increasing beyond upper bound. Similarly, for hexagon cylinder, present correlation is applicable, if concentration varying from 0 to 0.8. Both models are not applicable for concentration beyond 0.8, because of limitations in model shapes.

Predicted non-dimensional TC increases with conductivity ratio and contact ratios for hexagon and octagon cylinders. For lower \((0.3)\) concentrations (Fig. 3), deviation between all models is almost same. For higher \((\upsilon = 0.8)\) concentration (Fig. 4) and higher conductivity ratios, deviation is more within models. Effect of conductivity ratio for low \((0.3)\) and high \((0.8)\) concentrations on conventional models are also shown (Fig. 5). Contact ratio \( (\lambda) \) is found deterministic parameter when conductivity ratio \( (\alpha) \) is high whereas concentration is deterministic parameter when \( \alpha \) is approaching to 1. Similarly, for lower conductivity ratios \((\alpha < 1)\), non-dimensional TC is insensitive to contact ratios, but it is sensitive to higher conductivity ratios \((\alpha >1)\). From iso-conductance point \((\alpha = 1)\), non-dimensional TC approaches to 1 for all models with same slope. Present model shows a good trend for concentrations 0.3 and 0.8. For low values of \( \alpha \), TC estimations of all models are comparable, but they deviate when conductivity ratio approaches to 100.

Comparison of present TC models (hexagon and octagon cylinders) with experimental data for suspension systems and conventional models were carried out (Tables 1-2). A theoretical value of effective TC is determined for each case and it is compared with well established conventional models. Experimental data for suspension systems were taken from reported
Fig. 2—Variation of non-dimensional thermal conductivity with concentration of 2-dimensional spatially periodic two-phase systems for \( \alpha = 20 \): a) hexagon cylinder; b) octagon cylinder
Fig. 3—Variation of non-dimensional thermal conductivity with conductivity and contact ratios for lower concentration ($\nu = 0.3$) two-phase systems: a) hexagon cylinder; b) octagon cylinder
Fig. 4—Variation of non-dimensional thermal conductivity with conductivity and contact ratios for higher concentration (\(\nu = 0.8\)) two-phase systems: a) hexagon cylinder; b) octagon cylinder
Fig. 5—Variation of non-dimensional thermal conductivity with conductivity ratio for conventional models for two-phase systems with: 
a) lower concentration ($\nu = 0.3$); and b) higher concentration ($\nu = 0.8$)
studies\textsuperscript{18-21}. Suspension systems were considered for various concentrations (0.05-0.60) and for lower and higher values of conductivity ratio. For suspension systems (solid/liquid phase), hexagon cylinder predicted values were within range of ±10.83\% deviation from experimental data's. All other geometries showed a good agreement with experimental values within the range of ±15.58\% maximum deviation. Similarly, for the same range, conventional models deviated from experimental data within minimum (±11.56\%) and maximum (±35.60\%) average deviation, because all experimental values were low and medium concentration with higher conductivity ratio. Thus, proposed models are good for predicting effective TC of wide variety of suspension systems.

**Conclusions**

Resistance based unit cell model is developed with the effect of hexagon and octagon cylinders for estimating effective TC of two-phase systems. Present models...
predicted effective TC (maximum deviation, ± 15.58 %) from experimental data for suspension systems.

Nomenclature

- \( a \): Length of hexagon and octagon cylinders
- \( c \): Width of connecting plate in hexagon and octagon cylinders
- \( k_{\text{eff}} \): Effective thermal conductivity of two-phase materials, \( \text{W/mK} \)
- \( k_f \): Fluid or continuous thermal conductivity, \( \text{W/mK} \)
- \( k_s \): Solid or dispersed thermal conductivity, \( \text{W/mK} \)
- \( k_{sf} \): Equivalent thermal conductivity of a composite layer, \( \text{W/mK} \)
- \( R \): Thermal resistance, \( \text{m}^2\text{K}/\text{W} \)
- \( l \): Length of the unit cell, \( \text{m} \)

References