

Comparison of hexagon and octagon cylinder models with conventional models for effective thermal conductivity estimation of suspension systems

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This study presents hexagon and octagon cylinder models for effective thermal conductivity estimation of suspension system. Algebraic equations were derived based on isotherm approach for two dimensional spatially periodic medium. Model prediction was achieved with minimum ($\pm 10.83\%$) and maximum ($\pm 15.58\%$) deviation from experimental data for suspension systems. Model prediction was comparable with conventional models.

Keywords: Conventional models, Effective thermal conductivity, Isotherm, Resistance based unit cell model, Suspension systems

Introduction

Estimation of effective thermal conductivity (TC) of two-phase materials (TPMs) (ceramics, soils, foams, emulsion systems, porous and suspension systems, solid-solid mixtures, fiber reinforced materials and composites) is becoming increasingly important in microelectronic chip cooling, space craft structures, catalytic reactors, heat recovery process, heat exchangers, heat storage systems, petroleum refineries, solar collectors, and nuclear reactors. A resistance based unit cell approach has been applied for various TPMs to predict effective TC. Wiener¹ gave upper and lower limits to TC of TPMs based on parallel and series resistance. Most restrictive bounds^{2,3} (lower and upper) have been proposed for predicting effective TC of TPMs. Bruggeman⁴ extended Maxwell's result for lower concentration of dispersed phases to full range of concentration by assuming quasi-homogeneous mixture. Zehner & Schlunder⁵ proposed a model considering effect of primary and secondary parameters for cylindrical unit cell containing spherical inclusions. TC modeling⁶ was carried out based on unit cell approach with constant heat flux conditions for random distribution of spheres in a continuum of different

materials. Hsu *et al*⁷ obtained algebraic expressions for effective TCs of a number of porous media by applying lumped parameter method. Deisser & Boregli⁸ calculated TC of a saturated porous medium for a two-layer model representing as electrical resistance in an electrical circuit. A unit cell model⁹ was proposed consisting of spherical particles contacting each other with point to point. Hadley¹⁰ used empirically weighted averages of Hashin & Shtrikman³ bounds to model effective TC of several TPMs. Brailsford & Major¹¹ formulated solid or fluid component as a continuous phase, based on Maxwell's model. Lichteneker¹² proposed empirical models for effective TC of binary metallic mixture. Pande¹³ developed geometries for effective TC estimation of TPMs. Krupiczka¹⁴ carried out numerical study for effective TC based on a model made up of spheres in cubic lattice. Samantray *et al*¹⁵ proposed a comprehensive conductivity model by considering primary parameters based on unit cell and field solution approaches. Validity of model was extended to predict effective TC of various binary metallic mixtures¹⁶. Collocated parameter model¹⁷ was developed based on unit cell approach for predicting effective TC of TPMs.

This study presents development of resistance approach unit cell model with considering effect of primary and secondary parameters for predicting effective TC of suspension systems.

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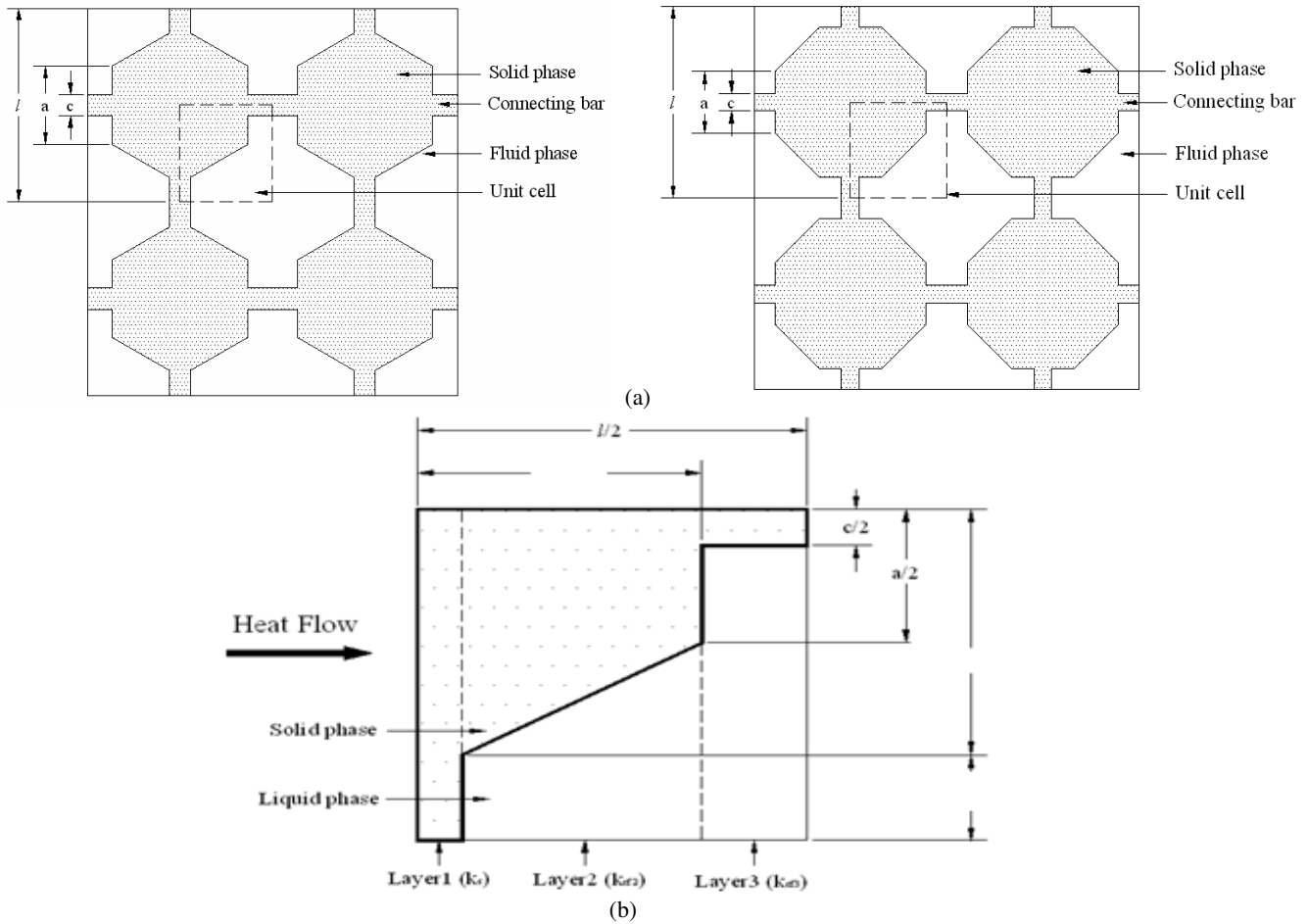


Fig.1—Two-dimensional spatially periodic two-phase system: a) Touching hexagon/octagon cylinder; b) Unit cell of hexagon cylinder: $x, a\sqrt{3}/2; y_1, (a/2)+\{a/\sqrt{2}-c/2\sqrt{3}\}; y_2, \{(l/2)-[(a/2)+\{a/\sqrt{2}-c/2\sqrt{3}\}]\}$; Unit cell of octagon cylinder: $x, \{a/2+a/\sqrt{2}\}; y_1, \{a/2+a/\sqrt{2}\}; y_2, [(l/2)-\{a/2+a/\sqrt{2}\}]$

Experimental Section

Hexagon and Octagon Cylinders Model for Effective Thermal Conductivity Estimation of Suspension Systems

Development of resistance based unit cell model for estimating effective TC based on material micro and nano-structure is extremely important for thermal design and analysis of TPMs. Main feature of resistance based unit cell model is to assume one-dimensional heat conduction in a unit cell, which is divided into three parallel layers (solids, fluid and composite) normal to temperature gradient. Effective TC of TPMs is determined by considering equivalent electrical resistances of parallel and series in unit cell model. Medium geometry is considered as matrix of touching and non-touching in-line hexagon and octagon cylinders.

Hexagon Cylinder

Effective TC of suspension system can be estimated by considering a hexagon cylinder with cross-section

‘a x a’ with a connecting bar of width ‘c’ (Fig. 1a). Stagnant TC of two-dimensional periodic medium is finite contact between spheres by connecting plates with ‘c/a’ denoting contact parameter. Because of symmetry of plates, one fourth of square cross-section has been considered as a unit cell (Fig. 1b). Total resistance offered by hexagon cylinder in unit cell is given as

$$R_{total} = \frac{\epsilon\lambda}{\alpha} + \left[\frac{2\sqrt{3}\epsilon(\sqrt{3}-\lambda)}{[(\sqrt{3}-\lambda)+2\sqrt{3}] \times \frac{k_{sf}l}{k_f} \times 2\sqrt{3} \left(1 - \left(\epsilon + \left(\frac{\epsilon\lambda}{\sqrt{3}}\right)\right)\right)} + \epsilon(\sqrt{3}-\lambda) \right] \left[\frac{1}{[(\sqrt{3}-\lambda)+2\sqrt{3}] + \frac{k_{sf}l}{k_f} \times \frac{1}{\alpha} \times 2\sqrt{3} \left(1 - \left(\epsilon + \left(\frac{\epsilon\lambda}{\sqrt{3}}\right)\right)\right)} + \epsilon(\sqrt{3}-\lambda) \right] + \frac{(1-\epsilon\sqrt{3})}{\frac{k_{sf}l}{k_f} [\epsilon\lambda(1-\epsilon\lambda)]} + \left[\frac{1}{\epsilon\lambda} + \frac{1}{\left\{ \frac{1}{\alpha} \times \left(\frac{k_{sf}l}{k_f} \right) (1-\epsilon\lambda) \right\}} \right] \dots (1)$$

where, conductivity ratio (α) = k_s/k_f , length ratio (ϵ) = a/l , contact ratio (λ) = c/a , and $\epsilon \lambda = c/l$.

Non-dimensional TC of two-dimensional hexagon cylinder is given as

$$K = \frac{k_{eff}}{k_f} = \frac{\epsilon \lambda}{\alpha} + \frac{\left[\frac{2\sqrt{3}\epsilon(\sqrt{3}-\lambda)}{\left[\frac{(\sqrt{3}-\lambda)+2\sqrt{3}}{\alpha} \times \frac{k_{sf2}}{k_f} \times 2\sqrt{3} \left(1 - \left(\epsilon + \left(\epsilon - \frac{\epsilon \lambda}{\sqrt{3}} \right) \right) \right) \right] + \epsilon(\sqrt{3}-\lambda)} \right]}{\left[\frac{1}{(\sqrt{3}-\lambda)+2\sqrt{3}} + \frac{k_{sf2}}{k_f} \times \frac{1}{\alpha} \times 2\sqrt{3} \left(1 - \left(\epsilon + \left(\epsilon - \frac{\epsilon \lambda}{\sqrt{3}} \right) \right) \right) \right] + \epsilon(\sqrt{3}-\lambda)} \right]} + \left[\frac{\frac{(1-\epsilon\sqrt{3})}{\frac{k_{sf3}}{k_f} [\epsilon \lambda (1-\epsilon \lambda)]}}{\frac{1}{\epsilon \lambda} + \left\{ \frac{1}{\alpha} \times \left(\frac{k_{sf3}}{k_f} \right) (1-\epsilon \lambda) \right\}} \right]^{-1} \quad \dots(2)$$

Octagon Cylinder

Effective TC of two dimensional medium can be estimated by considering an octagon cylinder with cross-section ‘a x a’ having a connecting bar width of ‘c’ (Fig. 1a). Total resistance of unit cell is given as

$$R_{total} = \frac{\epsilon \lambda}{\alpha} + \left[\frac{\frac{1}{\alpha} \left[\left(\frac{(1-\lambda)}{1+\sqrt{2}} \right) + \frac{2\sqrt{2}}{2+\sqrt{2}} \right] \times \frac{2\epsilon([1-\lambda]+\sqrt{2})}{\frac{k_{sf2}}{k_f} \left[2(1-(\epsilon+\epsilon\sqrt{2})) \right] + [\epsilon(1-\lambda)+\sqrt{2}]} \right]}{\frac{1}{\alpha} \left[\left(\frac{(1-\lambda)}{1+\sqrt{2}} \right) + \frac{2\sqrt{2}}{2+\sqrt{2}} \right] + \frac{2\epsilon([1-\lambda]+\sqrt{2})}{\frac{k_{sf2}}{k_f} \left[2(1-(\epsilon+\epsilon\sqrt{2})) \right] + [\epsilon(1-\lambda)+\sqrt{2}]} \right]} + \left[\frac{[1-(\epsilon+\epsilon\sqrt{2})][\alpha+(1-\alpha)\epsilon \lambda]}{\alpha \{ (1-\epsilon \lambda) + \epsilon \lambda [\alpha+(1-\alpha)\epsilon \lambda] \}} \right]^{-1} \quad \dots(3)$$

Non-dimensional thermal conductivity of two-dimensional octagon cylinder is given as

$$K = \frac{k_{eff}}{k_f} = \frac{\epsilon \lambda}{\alpha} + \left[\frac{\frac{1}{\alpha} \left[\left(\frac{(1-\lambda)}{1+\sqrt{2}} \right) + \frac{2\sqrt{2}}{2+\sqrt{2}} \right] \times \frac{2\epsilon([1-\lambda]+\sqrt{2})}{\frac{k_{sf2}}{k_f} \left[2(1-(\epsilon+\epsilon\sqrt{2})) \right] + [\epsilon(1-\lambda)+\sqrt{2}]} \right]}{\frac{1}{\alpha} \left[\left(\frac{(1-\lambda)}{1+\sqrt{2}} \right) + \frac{2\sqrt{2}}{2+\sqrt{2}} \right] + \frac{2\epsilon([1-\lambda]+\sqrt{2})}{\frac{k_{sf2}}{k_f} \left[2(1-(\epsilon+\epsilon\sqrt{2})) \right] + [\epsilon(1-\lambda)+\sqrt{2}]} \right]} + \left[\frac{[1-(\epsilon+\epsilon\sqrt{2})][\alpha+(1-\alpha)\epsilon \lambda]}{\alpha \{ (1-\epsilon \lambda) + \epsilon \lambda [\alpha+(1-\alpha)\epsilon \lambda] \}} \right]^{-1} \quad \dots(4)$$

Results and Discussion

Effective TC of TPMs mainly depends on characteristics of constituent phase, including TCs of solid and fluid phases, size, shape and thermal contact between solid-solid and solid-fluid interface. Effect of concentration (ν) on non-dimensional TC of two-dimensional hexagon and octagon cylinders for conductivity ratio ($\alpha = 20$) have been investigated. Present model (Fig. 2) with hexagon and octagon cylinders lies between parallel and series lines for conductivity ratio ($\alpha = 20$) and contact ratio ($\lambda = 0-0.2$). Contact ratio is having major influence on effective TC of TPMs when contact ratio varying between 0 to 0.2, but there is no influence beyond 0.2. For octagon cylinder, present correlation is applicable for concentration varying from 0 to 0.7. For further increment in concentration, non-dimensional TC is increasing beyond upper bound. Similarly, for hexagon cylinder, present correlation is applicable, if concentration varying from 0 to 0.8. Both models are not applicable for concentration beyond 0.8, because of limitations in model shapes.

Predicted non-dimensional TC increases with conductivity ratio and contact ratios for hexagon and octagon cylinders. For lower (0.3) concentrations (Fig. 3), deviation between all models is almost same. For higher ($\nu = 0.8$) concentration (Fig. 4) and higher conductivity ratios, deviation is more within models. Effect of conductivity ratio for low (0.3) and high (0.8) concentrations on conventional models are also shown (Fig. 5). Contact ratio (λ) is found deterministic parameter when conductivity ratio (α) is high whereas concentration is deterministic parameter when α is approaching to 1. Similarly, for lower conductivity ratios ($\alpha < 1$), non-dimensional TC is insensitive to contact ratios, but it is sensitive to higher conductivity ratios ($\alpha > 1$). From iso-conductance point ($\alpha = 1$), non-dimensional TC approaches to 1 for all models with same slope. Present model shows a good trend for concentrations 0.3 and 0.8. For low values of α , TC estimations of all models are comparable, but they deviate when conductivity ratio approaches to 100.

Comparison of present TC models (hexagon and octagon cylinders) with experimental data for suspension systems and conventional models were carried out (Tables 1-2). A theoretical value of effective TC is determined for each case and it is compared with well established conventional models^{4-6,10-13}. Experimental data for suspension systems were taken from reported

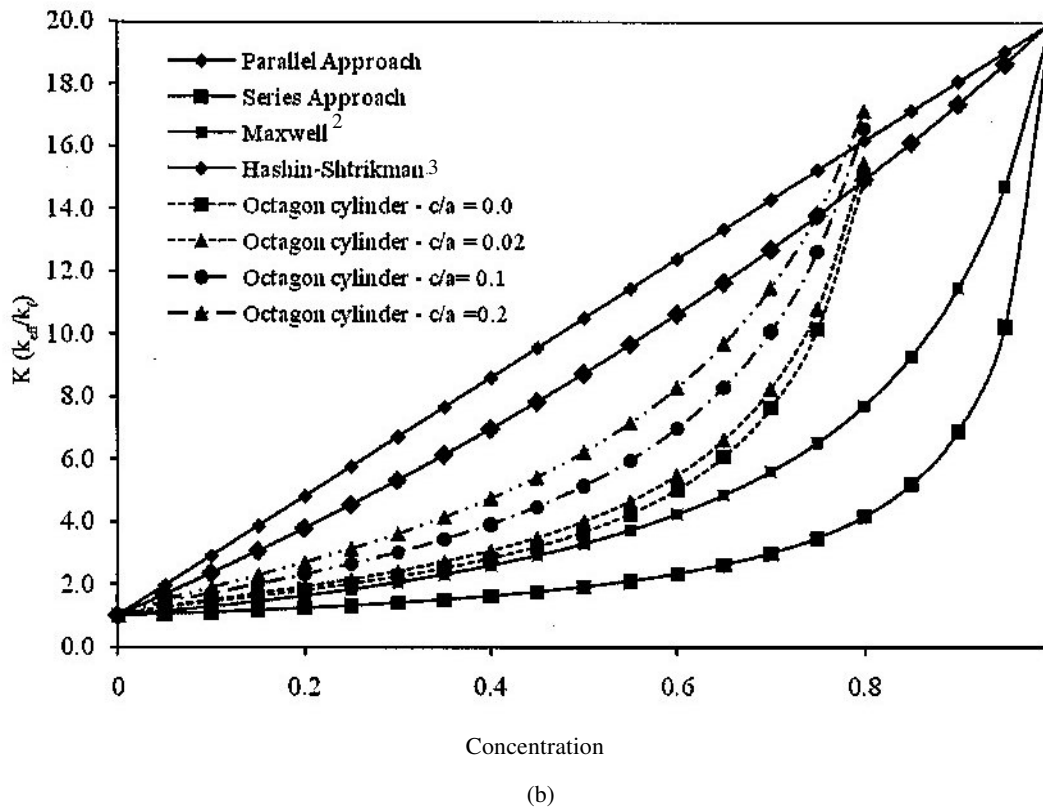
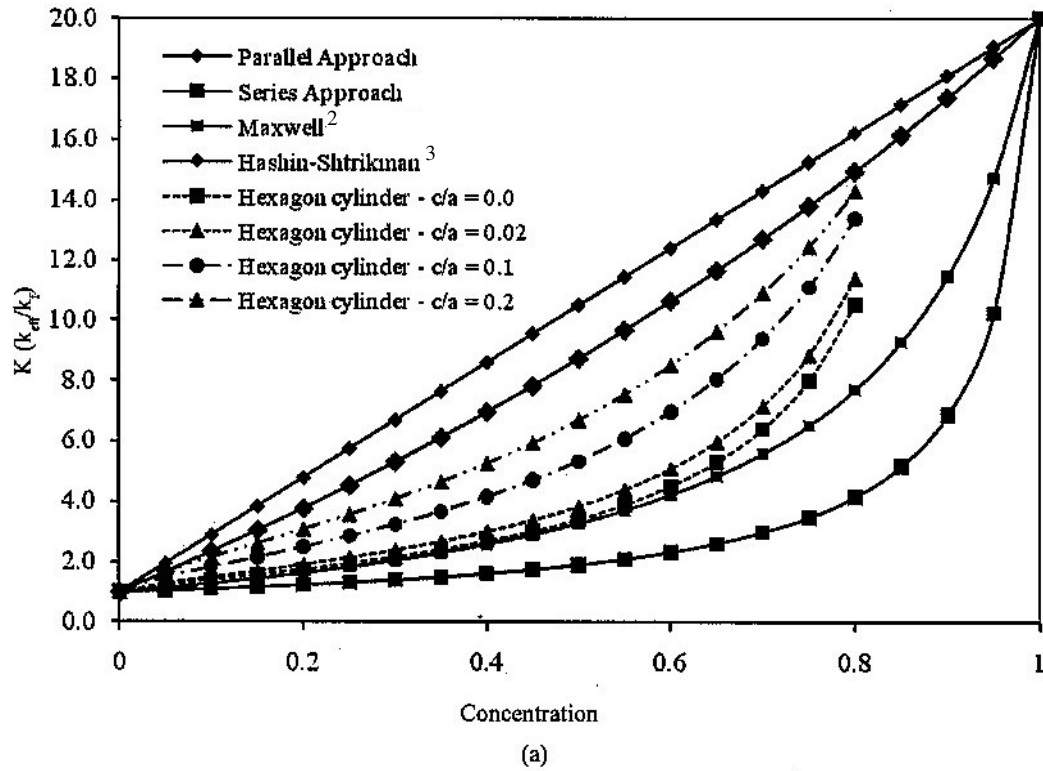


Fig. 2—Variation of non-dimensional thermal conductivity with concentration of 2-dimensional spatially periodic two-phase systems for $\alpha = 20$: a) hexagon cylinder; b) octagon cylinder

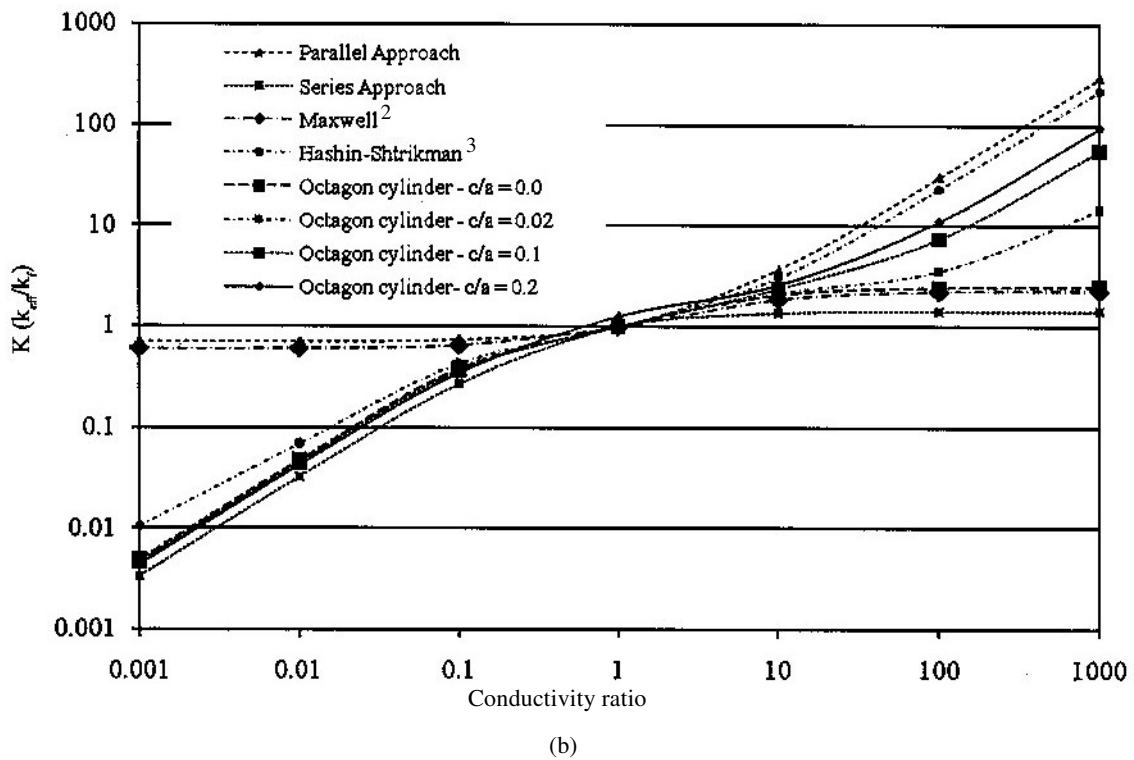
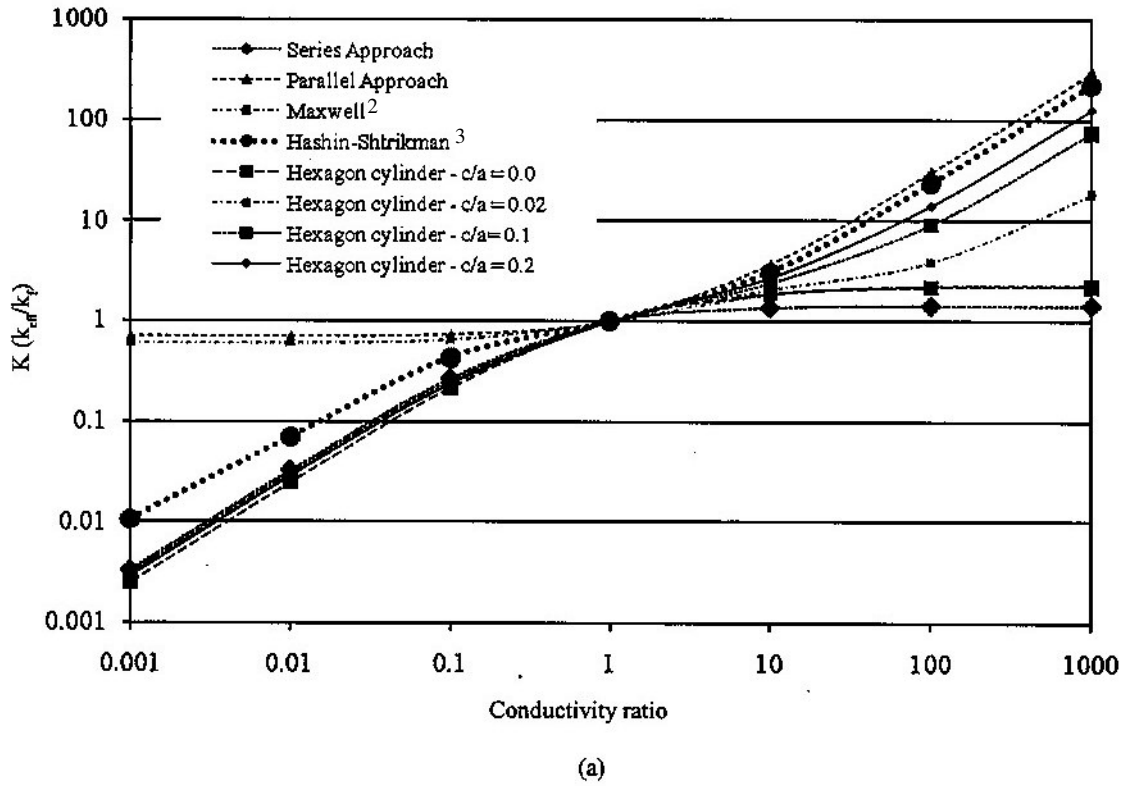
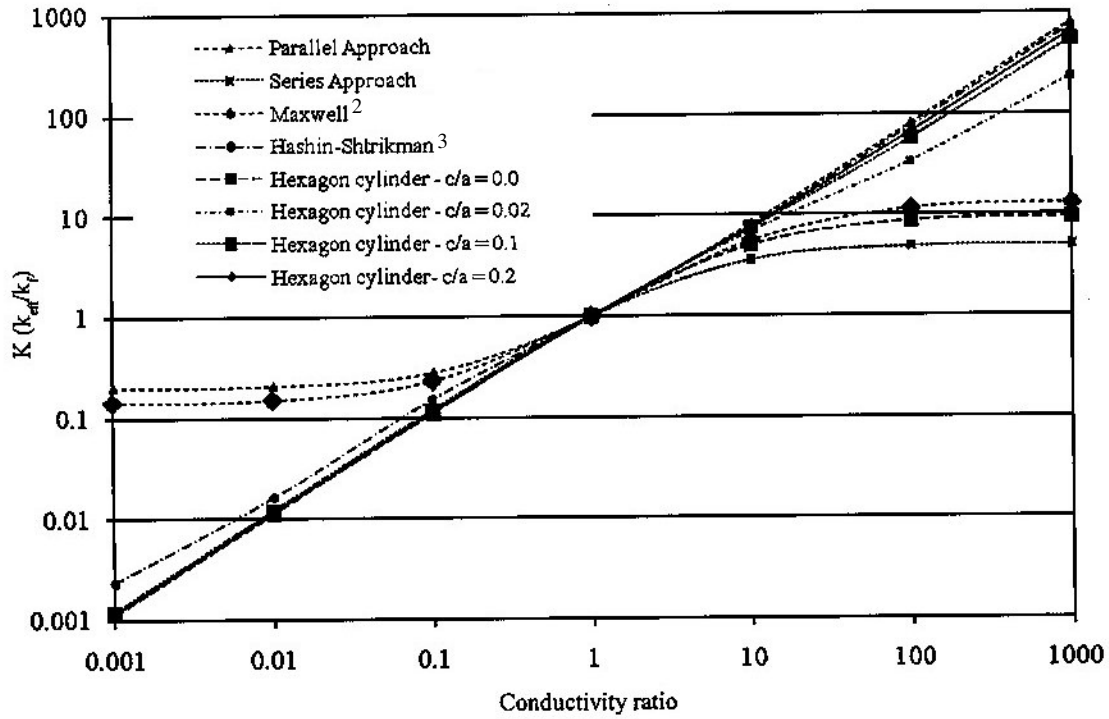
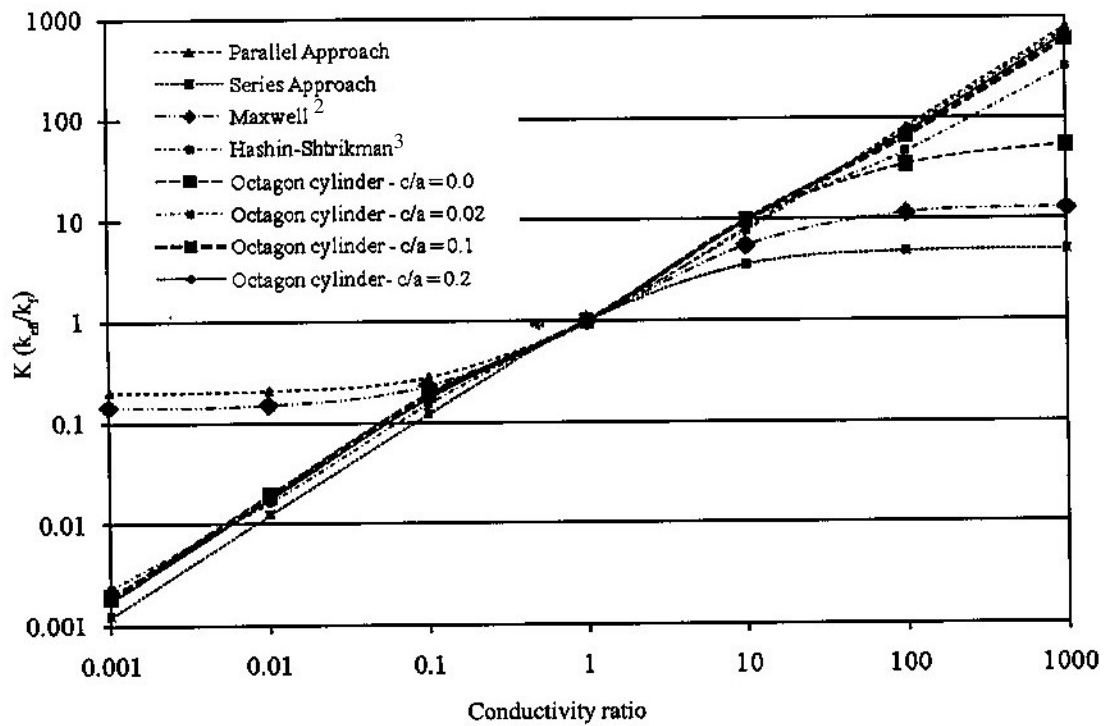


Fig. 3—Variation of non-dimensional thermal conductivity with conductivity and contact ratios for lower concentration ($\nu = 0.3$) two-phase systems: a) hexagon cylinder; b) octagon cylinder



(a)



(b)

Fig. 4—Variation of non-dimensional thermal conductivity with conductivity and contact ratios for higher concentration ($\nu = 0.8$) two-phase systems: a) hexagon cylinder; b) octagon cylinder

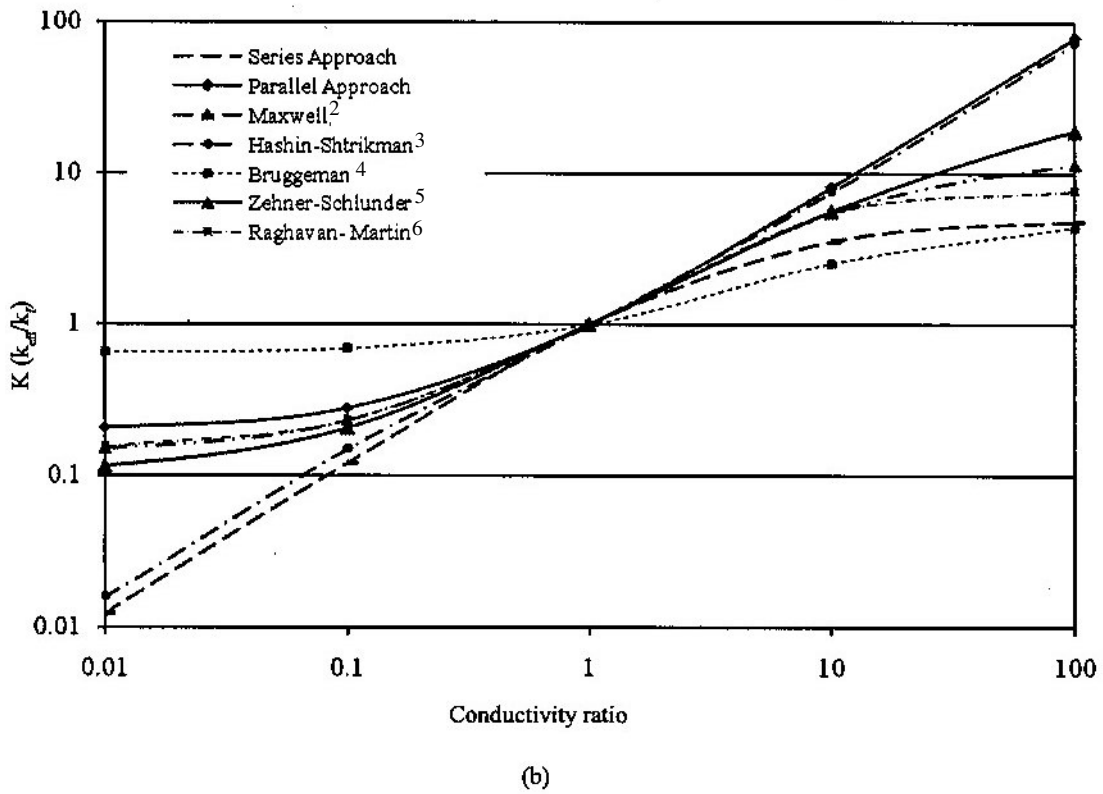
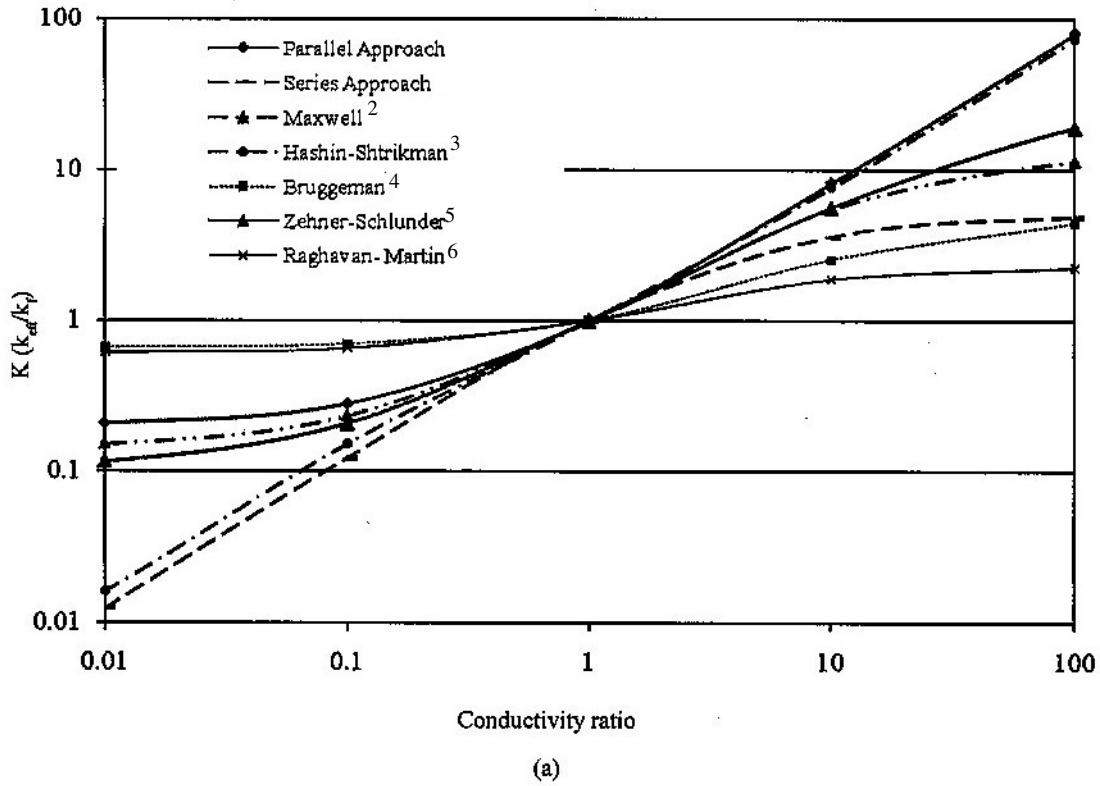


Fig. 5— Variation of non-dimensional thermal conductivity with conductivity ratio for conventional models for two-phase systems with: a) lower concentration ($v = 0.3$); and b) higher concentration ($v = 0.8$)

Table.1—Comparison of present resistance based unit cell model with experimental data for Suspension systems

S.No	Sample (solid/fluid phase)	k_s	k_f	$\alpha (k_s/k_f)$	ν	k_{exp}	$\lambda (c/a)$	k_{octa}	% Devi	k_{hex}	% Devi
1	Graphite/Water ¹⁸	160.03	0.66	241.01	0.05	0.001	0.83	0.90	7.84	0.90	8.17
2	Graphite/Water ¹⁸	160.03	0.66	241.01	0.11	0.001	1.13	1.07	4.96	1.07	5.19
3	Graphite/Water ¹⁸	160.03	0.66	241.01	0.17	0.001	1.44	1.26	12.50	1.25	12.64
4	Graphite/Water ¹⁸	160.03	0.66	241.01	0.24	0.001	1.92	1.50	21.93	1.60	16.39
5	Selenium/polypropylene glycol ¹⁹	5.19	0.14	37.09	0.10	0.001	0.18	0.21	14.72	0.20	13.63
6	Selenium/polypropylene glycol ¹⁹	5.19	0.14	37.09	0.20	0.001	0.22	0.26	19.54	0.26	17.50
7	Selenium/polypropylene glycol ¹⁹	5.19	0.14	37.09	0.30	0.001	0.32	0.33	4.71	0.32	1.82
8	Selenium/polypropylene glycol ¹⁹	5.19	0.14	37.09	0.40	0.001	0.42	0.42	0.14	0.40	4.16
9	Aluminum/water ²⁰	204.24	0.66	310.86	0.06	0.001	0.76	0.91	19.14	0.91	19.92
10	Aluminum/water ²⁰	204.24	0.66	310.86	0.12	0.001	0.97	1.09	12.42	1.09	12.89
11	Aluminum/water ²⁰	204.24	0.66	310.86	0.18	0.001	1.40	1.28	8.95	1.28	8.71
12	Aluminum/water ²⁰	204.24	0.66	310.86	0.21	0.001	1.81	1.40	22.70	1.40	22.66
13	Graphite/Water ²⁰	160.94	0.56	286.37	0.16	0.001	1.19	1.05	11.79	1.05	11.41
14	Zinc sulphate/lard ²⁰	0.61	0.20	3.11	0.38	0.001	0.31	0.38	23.68	0.32	2.64
15	Zinc sulphate/lard ²⁰	0.61	0.20	3.11	0.56	0.001	0.35	0.51	45.65	0.39	10.23
16	Marble/Vaselene ²⁰	2.98	0.19	16.09	0.60	0.001	0.75	0.89	18.63	0.79	5.25
Average deviation, %									15.58		10.83

Devi, Deviation; exp, experimental; hex, hexagon; octa, octagon

Table.2— Comparison of experimental data with conventional models for suspension systems

S.No	k_{exp}	k_{Brug}	% Devi	k_{Z-S}	% Devi	k_{R-M}	% Devi	k_{Had}	% Devi	k_{B-M}	% Devi	k_{Lid}	% Devi	k_{Pand}	% Devi
1	0.83	0.97	17.22	0.76	8.16	0.70	15.37	0.78	5.70	0.77	7.50	0.87	5.20	0.20	76.30
2	1.13	1.40	24.05	0.99	12.19	0.77	32.22	0.94	16.50	0.91	19.70	1.21	7.50	0.79	29.90
3	1.44	1.89	31.73	1.31	8.57	0.85	40.60	1.13	21.60	1.07	25.70	1.69	17.60	1.42	1.20
4	1.92	2.53	32.15	1.80	5.89	1.00	48.01	1.37	28.30	1.28	33.10	2.48	29.20	2.20	14.80
5	0.18	0.25	40.01	0.18	0.49	0.16	12.09	0.19	4.70	0.18	1.50	0.20	11.60	0.17	5.50
6	0.22	0.39	75.88	0.25	16.35	0.19	13.67	0.25	13.20	0.24	7.40	0.29	31.70	0.24	7.70
7	0.32	0.53	69.63	0.36	13.11	0.24	23.72	0.32	2.20	0.30	4.50	0.41	31.40	0.31	1.80
8	0.42	0.69	63.41	0.49	15.89	0.33	22.88	0.42	1.00	0.39	8.50	0.59	40.80	0.40	5.90
9	0.76	1.00	31.23	0.77	1.70	0.70	8.09	0.79	3.50	0.77	1.30	0.90	18.40	0.14	81.90
10	0.97	1.43	47.74	1.02	5.09	0.76	21.21	0.95	2.20	0.91	6.10	1.27	31.20	0.82	15.10
11	1.40	1.93	37.47	1.36	3.00	0.85	39.21	1.13	19.30	1.07	23.70	1.79	27.90	1.54	10.10
12	1.81	2.24	24.24	1.60	11.16	0.92	49.12	1.25	30.70	1.18	35.00	2.19	21.40	1.99	9.90
13	1.19	1.54	30.12	1.08	8.47	0.71	40.17	0.93	21.60	0.88	25.60	1.40	17.90	1.16	2.50
14	0.31	0.33	5.81	0.38	22.41	0.29	6.56	0.31	0.30	0.31	1.20	0.30	2.40	0.26	14.70
15	0.35	0.39	11.00	0.51	45.36	0.36	3.65	0.38	7.80	0.37	6.20	0.37	5.40	0.28	20.40
16	0.75	0.96	27.95	0.26	65.82	0.77	2.67	0.79	6.30	0.74	0.80	0.98	31.20	0.38	49.70
Average deviation, %			35.60		15.23		23.70		11.56		12.98		20.66		21.71

B-M, Brailsford and Major; Brug, Bruggeman; Devi, Deviation; eff, Effective; exp, Experimental; Had, Hadley; hex, Hexagon; Lid, Litchnecker; octa, Octagon; Pand, Pande; R-M, Raghavan-Martin; Z-S, Zehner - Schlunder

studies¹⁸⁻²¹. Suspension systems were considered for various concentrations (0.05-0.60) and for lower and higher values of conductivity ratio. For suspension systems (solid/liquid phase), hexagon cylinder predicted values were within range of $\pm 10.83\%$ deviation from experimental data's. All other geometries showed a good agreement with experimental values within the range of $\pm 15.58\%$ maximum deviation. Similarly, for the same range, conventional models deviated from experimental data within minimum ($\pm 11.56\%$) and maximum

($\pm 35.60\%$) average deviation, because all experimental values were low and medium concentration with higher conductivity ratio. Thus, proposed models are good for predicting effective TC of wide variety of suspension systems.

Conclusions

Resistance based unit cell model is developed with the effect of hexagon and octagon cylinders for estimating effective TC of two-phase systems. Present models

predicted effective TC (maximum deviation, $\pm 15.58\%$) from experimental data for suspension systems.

Nomenclature

a	Length of hexagon and octagon cylinders
c	Width of connecting plate in hexagon and octagon cylinders
k_{eff}	Effective thermal conductivity of two-phase materials, W/mK
k_r	Fluid or continuous thermal conductivity, W/mK
k_s	Solid or dispersed thermal conductivity, W/mK
k_{sf}	Equivalent thermal conductivity of a composite layer, W/mK
R	Thermal resistance, $\text{m}^2\text{K}/\text{W}$
l	Length of the unit cell, m

References

- Wiener O, Lamellare doppelbrechung, *Phys Z*, **5** (1904) 332-338.
- Maxwell J C, *A Treatise on Electricity and Magnetism* (Clarendon Press, Oxford, U K) 1873, 365.
- Hashin Z & Shtrikman S, A variational approach to the theory of the effective magnetic permeability of multiphase materials, *J Appl Phys*, **33** (1962) 3125-3131.
- Bruggeman DAG, Dielectric constant and conductivity of mixtures of isotropic materials, *Ann Phys*, **24** (1935) 636-679.
- Zehner P & Schlunder E U, On the effective heat conductivity in packed beds with flowing fluid at medium and high temperatures, *Chem Engg Technol*, **42** (1970) 933-941.
- Raghavan V R & Martin H, Modeling of two-phase thermal conductivity, *Chem Engg Process*, **34** (1995) 439-446.
- Hsu C T, Cheng P & Wong K W, A lumped parameter model for stagnant thermal conductivity of spatially periodic porous media, *J Heat Transfer*, **117** (1995) 264-269.
- Deisser R G & Boregli J S, An investigation of effective thermal conductivities of powders in various gases, *ASME Trans*, **80** (1958) 1417-1425.
- Kunii D & Smith JM, Heat transfer characteristics in porous rocks, *Amer Inst Chem Engg J*, **6** (1960) 71-78.
- Hadley G R, Thermal conductivity of packed metal powders, *Int J Heat Mass Transfer*, **29** (1986) 909-920.
- Brailsford A D & Major K G, The thermal conductivity of aggregates of several phases including porous materials, *J Appl Phys*, **15** (1964) 313-319.
- Litchnecker K, The electrical conductivity of periodic and random aggregates, *Physik Z*, **27** (1926) 115.
- Pande R N, Dependence of effective thermal conductivity on source geometry, *Int J Pure Appl Phys*, **26** (1988) 691-695.
- Krupiczka R, Analysis of thermal conductivity in granular materials, *Int Chem Engg*, **7** (1967) 122-144.
- Samantray P K, Karthikeyan P & Reddy K S, Estimating effective thermal conductivity of two-phase materials, *Int J Heat Mass Transfer*, **49** (2006) 4209-4219.
- Karthikeyan P & Reddy K S, Effective conductivity estimation of binary metallic mixtures, *Int J Therm Sci*, **46** (2007) 419-425.
- Reddy K S & Karthikeyan P, Estimation of effective thermal conductivity of two-phase materials using collocated parameter model, *Heat Transfer Engg*, **30** (2009) 1-14.
- Sugawara A & Hamada A, Thermal conductivity of dispersed systems, in *10th Thermal Conductivity Conf* (Massachusetts, USA) 1970, 7.
- Baxley A L & Couper J R, Thermal conductivity of two-phase systems. Part-IV Thermal conductivity of suspensions, *Research Report Series No.8* (University of Arkansas, Eng. Exp.Station, Arkansas, USA).
- Jonson F A, The thermal conductivity of aqueous thoria suspensions, in *Atomic Energy Res Estab, AERE RIR* (Great Britain) 1958, 2578-2584.
- Lees C H, *Proc R SIX Land*, **A191** (1898) 339.