Generation of kinetic Alfven wave by velocity shear in the plasma sheet boundary layer during substorm

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The kinetic Alfven wave (KAW) driven by velocity shear in the plasma sheet boundary layer (PSBL) has been investigated. Expressions for the dispersion relation and growth rate of the kinetic Alfven wave have been derived for the two regimes separately i.e., for the case of weak shear and strong shear. It is found that for both weak and strong shear regimes the resonant electrons act as the main energy source whereas; the ion longitudinal motion suppresses the instability in case of the strong shear. The results explain the generation of energetic KAWs in the PSBL during the substorm onset by the weak velocity shear. As the shear becomes strong during the substorm the energy of the KAWs is transferred to the plasma particles through the Landau damping of the wave, which may then lead to the parallel energisation of the electrons observed recently by Polar satellite. The loss of the Poynting flux, as observed by Polar and Akebono satellite, is also explained in terms of the Landau damping of the KAW by the strong shear of substorm. The temperature anisotropy and density of the plasma particles enhance the growth rate of shear driven KAW at the onset of the substorm.

Keywords: Kinetic Alfven wave, Plasma sheet boundary layer, Substorm event, Weak and strong velocity shear, Landau damping, Loss of poynting flux

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1 Introduction

Recently, the kinetic Alfven wave (KAW) has received much attention in space plasma and in the laboratory plasmas. These waves play an important role in the energy transport, particle acceleration, auroral currents and in ultra low frequency (ULF) emission in earth’s magnetosphere1-3. The importance of the KAWs in the substorm process has been pointed out by Samson et al.4, who used ground based observations in the auroral zone and showed that strong ULF (1-10mHz) Alfven wave activity was present at both boundaries of the auroral region4. Recently, the most energetic KAWs have been observed by the Polar satellite in the plasma sheet boundary layer (PSBL) during the substorm period5. Keiling et al.5 have reported substorm related, globally excited Alfven waves on a temporal scale of 6 to 300s at geocentric distance between 5-6 $R_E$ ($R_E$ is earth’s radius).

Wygant et al.6,7 have also reported the evidence of small-scale, large amplitude KAWs at the PSBL at altitudes of 4-6 $R_E$. The small-scale spikes of the duration 250 ms to 1s have electric field amplitudes up to 300mV/m and associated magnetic field variations between 0.5 to 5 nT. The large scale KAWs have periods of 20-60s. These waves carry sufficient poynting flux to provide the energy necessary to produce auroral displays and have been observed to accelerate electrons locally. Similar wave activity has been seen from Cluster satellite and it is verified with direct multipoint measurements that these waves had scales perpendicular to the magnetic field of the order of the electron inertial length8.

That, Alfven waves are an integral part of the substorms, has become clear since their prediction9,10. The question of what generates the KAW has been partially answered. Milkhailovskii11 has shown that KAW can be excited by the density gradient in inhomogeneous plasma. Recently Dwivedi et al12 have shown the generation of KAW during the substorms as the consequence of the parallel electric field and Shrivastava and Tiwari13 by the parallel electric field alongwith the temperature anisotropy in the auroral acceleration region. There is considerable evidence that the compressional MHD waves mode couple into kinetic Alfven waves in the PSBL to cause the aurora1-14. Allan and Wright15 have modeled the propagation of fast mode waves in the tail lobe, which converts into shear Alfven waves at the PSBL. Another possible model suggests that these waves may be directly produced by reconnection processes in the tail16 or indirectly through linear or non-linear
mode conversion or by localized plasma flows in the tail\textsuperscript{17}, but the mechanism that creates the largest Alfv\"{e}n wave, that those occur in the PSBL during the substorms expansion phase, is still open to speculation.

Sheared velocity flows exist widely in space plasmas such as those in the magnetopause, PSBL, Polar cusp, comet tails and solar wind streams\textsuperscript{18}. These flows are important free energy sources of various macro instabilities and micro instabilities including the K-H instabilities, Alfv\"{e}n wave, the magneto acoustic instability and the KAW\textsuperscript{18-20}.

The purpose of this paper is to explain the generation of the recently observed\textsuperscript{5-7} KAW in the PSBL by the sheared velocity flow at the onset of the substorm. In the second section, the expression for the dispersion relation and growth rate of the KAW are derived in the presence of the weak and strong shear separately. The analysis is based on the kinetic model of the shear driven Alfv\"{e}n wave developed by Wang \textit{et al}.\textsuperscript{18}, so that finite ions Larmour radius correction and wave particle resonance are properly treated. In the model of Wang \textit{et al}.\textsuperscript{18} there is an assumption that the plasma beta (\(\beta\)) is low and the disturbance of magnetic field is very small and neglected. But in the substorm the disturbance of magnetic field is large which is considered as the magnetic field perturbation associated to the kinetic Alfv\"{e}n wave. Thus, the disturbance of magnetic field during substorm is considered in this paper as the transient model of magnetosphere-ionosphere coupling as the wave associated magnetic field. In the result section, the expressions for the dispersion relation and growth rate of the wave are graphically analysed using recent satellite data and then the conclusions are presented.

\section{Assumptions}

The basic assumptions are same as that by Wang \textit{et al}.\textsuperscript{18}. The ambient magnetic field and flow velocity are along the z-direction and the velocity spatial variation is only in the x direction i.e., \(B = B_0 \hat{z}, \ V_0 = V_0(x) \hat{z}\). The perturbed quantities are assumed to vary as \(\exp[-i(k_x x + k_z z - \omega t)]\). The unperturbed distribution function is given by

\[ F_{\alpha}^{(0)} = \frac{n_0}{\pi v_{nz}} \sqrt{\pi v_{nz}} \exp \left[ -\frac{v^2}{2v_{nz}^2} - \frac{(v_z - V_0(x))^2}{2v_{nz}^2} \right] \]  \hspace{1cm} (1)

where \(n_0\) is the density of the plasma particles.

\[ \alpha = i, e; \ v^2 = v_x^2 + v_z^2, \ V = x + \left( v_y / \omega_{ci} \right) \] are the constants of motion. \(v_{ix}, v_{iz}\) and \(v_{nx}, v_{nz}\) are the perpendicular and parallel thermal velocity respectively. \(\omega_{ci}\) is the cyclotron frequency.

Expanding at \(x = 0\) gives\textsuperscript{18}

\[ F_{\alpha}^{(0)} = F_{\alpha}^{(0)} \left( 1 - \left( \frac{v_z - V_0}{v_{nz}} \right)^2 \left( \frac{v_y}{\omega_{ci}} \right) \right), \]  \hspace{1cm} (2)

where \(\kappa_v = \frac{1}{v_0(x)} \frac{dv_0(x)}{dx} \)  \hspace{1cm} (3)

so that

\[ \frac{\partial F_{\alpha}^{(0)}}{\partial x} = - \left( \frac{(v_z - V_0)V_0K_v}{v_{nz}^2} \right) f_{\alpha}^{(0)} \]  \hspace{1cm} (4)

The perturbation are considered in a plasma where only the low frequency oscillations (\(\omega << \omega_{ci}\)) with the wave vector \(k\) approximatively perpendicular to \(B_0\), i.e., \(\cos \theta << 1\) is important\textsuperscript{18,21}.

\section{Dispersion Relation}

The dispersion relation for the velocity shear driven KAW as derived by Wang \textit{et al}.\textsuperscript{18} (1998) is given by:

\[ \left( \varepsilon^2 \omega^2 - k_x^2 v_A^2 - \frac{\lambda_i}{\Pi_0} - k_x \frac{k_x V_0 T_i}{m_i} \right) \left[ 1 - \left( \frac{k_x V_0 K_v}{k_z \omega_{ke}} - 1 \right) \right] \]

\[ \times \left[ 1 + \frac{k_x V_0 K_v}{k_z \omega_{ke}} \right] \left( 1 + i \left( \frac{\pi \omega}{2 k_z v_{ke}} \right) \right) \]

\[ - \Pi_0 \left( 1 - \frac{k_x V_0 K_v}{k_z \omega_{ke}} \right) \left( \frac{k_x^2 v_{ke}^2 T_e}{m_e \omega^2} - \frac{T_i}{T_e} \right) - \frac{T_i}{T_e} \lambda_i k_x^2 v_A^2 = 0 \]  \hspace{1cm} (5)

where \(\varepsilon = \frac{T_i}{T_{zi}}\) is the ion temperature anisotropy. \(\bar{\omega}\) is the Doppler shifted frequency of the wave, \(v_A\) is the Alfv\"{e}n velocity, \(k_x\) and \(k_z\) are the perpendicular and parallel wave vectors, \(m_i\) and \(m_e\) are the ion and electron mass, \(T_i\) and \(T_e\) are the ion and electron temperature.

\[ \lambda_i = \frac{k_x^2 v_{ke}^2 T_e}{m_e \omega^2} \]  \hspace{1cm} (6)

\[ \Pi_0 = \exp(-\lambda_i) I_0(\lambda_i) \]  \hspace{1cm} (7)

and

\[ \Pi_0 = \exp(-\lambda_i) I_0(\lambda_i) \]  \hspace{1cm} (8)
with \( I_0(\lambda_i) \) as the modified Bessel’s function.

The small imaginary term of the order of \( \frac{\omega}{k_z v_{\|z}} \) in the dispersion relation is due to the resonant electron and the small term of the order of \( \frac{k_z^2 v_{\|z}^2}{\omega^2} \) is due to the ion longitudinal motion.

**A. Case of weak shear**

For the case of weak shear the boundary condition is\(^{18}\)

\[
1 < \alpha_i < 2/\beta_{i\|} \quad \text{…(9)}
\]

where \( \alpha_i = \frac{k_z V_0 \beta_i}{k_z \omega_{ci} \rho_{ci}} \) \( \text{…(10)} \)

with \( \beta_v = \kappa_c \rho_{ci} \) \( \text{…(11)} \)

\( \kappa_c \) is given by Eq. (3) and \( \rho_{ci} \) is the ion gyroradius.

\[
\beta_{i\|} = \frac{v_{i\|}}{v_A^2} \quad \text{…(12)}
\]

Considering the imaginary part of the Eq.(5) and using:

\[
\frac{\lambda_i}{1 - \Pi_0} \approx 1
\]

The dispersion relation for the KAW driven by weak shear is given by

\[
v_{pe1}^2 = \frac{\omega^2}{k_z^2} = \frac{v_A^2}{\varepsilon^2} \left( 1 - \frac{k_z dV_0}{dx} \right) \frac{v_{j\|}^2}{k_z \omega_{ci} \rho_{ci}^2} \quad \text{…(13)}
\]

which gives the parallel phase velocity of the shear driven KAW in the boundary \( 1 < \alpha_i < 2/\beta_{i\|} \) and is same as derived by Wang et al.\(^{18}\).

**B. Case of strong shear**

For the case of strong shear the boundary condition is:

\[
\alpha_i > 2/\beta_{i\|} \quad \text{…(14)}
\]

where \( \alpha_i \) and \( \beta_{i\|} \) are given by Eqs (10) and (12).

Again, considering the imaginary part of Eq. (5) and applying condition given in Eq. (14) gives the dispersion relation of the KAW driven by strong shear as:

\[
v_{pe2}^2 = \frac{\omega^2}{k_z^2} \left[ 1 + \frac{m_i}{m_e} \left( \frac{\alpha_i \beta_{i\|}}{2} \right) \right] \quad \text{…(15)}
\]

which is the same as derived by Wang et al.\(^{18}\) and it gives the parallel phase velocity of the shear driven KAW in the regime \( \alpha_i > 2/\beta_{i\|} \).

Eqs (11) and (13) show that the dispersion relation of the shear driven KAW is affected by the temperature anisotropy and density of the plasma particle.

**4 Growth Rate of the Wave**

**A. Case of weak shear**

Growth rate of the KAW for the case of weak shear is derived from:

\[
\gamma = \frac{R_D(\omega,k)}{\frac{\partial}{\partial \omega} I_m D(\omega,k)} \quad \text{…(16)}
\]

where \( D(\omega,k) \) is given by Eq. (5) and \( v_{pe1} \) by Eq. (13):

The growth rate comes out to be:

\[
\gamma = \left( \frac{T_i}{T_i} \right) \frac{\lambda_i k_z v_{\|z}}{2\sqrt{2\pi} \left( 1 - \frac{\alpha_i \beta_{i\|}}{2} \right) \left( 1 + \alpha_i \frac{m_i}{m_e} \right) \left( 1 - \Pi_0 \right)} \quad \text{…(17)}
\]

where \( \lambda_i \) is given by Eq. (7) and \( \alpha_i \) and \( \beta_{i\|} \) by Eqs (10 & 12). Eq. (17) is same as derived by Wang et al.\(^{18}\).

**B. Case of strong shear**

Growth rate for the KAW driven by strong shear is derived from:

\[
\gamma = \left( \frac{T_i}{T_i} \right) \frac{\lambda_i k_z v_{\|z}}{\sqrt{2\pi} \left( 1 - \frac{\alpha_i \beta_{i\|}}{2} \right) \left( 1 + \alpha_i \frac{m_i}{m_e} \right) \left( 1 - \Pi_0 \right)} \quad \text{…(17)}
\]

Here \( v_{pe2} \) is given by Eq. (15).
The growth rate is then obtained as:

\[
\gamma = \left[ e^2 v_{pz2}^2 - v_A^2 \left( 1 - \frac{\alpha_c \beta_{\perp}}{2} \right) \right] (1 + \alpha_c) \sqrt{\frac{\pi}{2}} \frac{k_z}{v_{tez}} \times \Pi_0 (2 - \alpha \beta_{\perp}) (1 - \alpha_c) \left( \frac{v_{A,tez}^2}{v_{pz2}^2} \right)^2 \times \frac{T_e}{T_i} - 2\alpha^2 (1 + \alpha_c) \right] \quad \ldots (19)
\]

which is the same as derived by Wang et al.\(^\text{18}\) where \(v_{pz2}\) is given by Eq. (15), \(\alpha_c = \frac{k_z dV_x}{d\rho} \), \(\epsilon\), \(\Pi_0\), \(\alpha\) and \(\beta_{\perp}\) are given by Eqs (6), (8), (10) and (12), respectively.

The expression for the growth rate show that the resonant electrons, associated with the term \(z e^2 \omega^2 \varepsilon \omega \beta_{\perp} \left( 1 - \frac{\alpha_c \beta_{\perp}}{2} \right) \right] (1 + \alpha_c) \sqrt{\frac{\pi}{2}} \frac{k_z}{v_{tez}} \times \Pi_0 (2 - \alpha \beta_{\perp}) (1 - \alpha_c) \left( \frac{v_{A,tez}^2}{v_{pz2}^2} \right)^2 \times \frac{T_e}{T_i} - 2\alpha^2 (1 + \alpha_c) \right] \quad \ldots (19)

For the values of the various parameters given in the result section, the threshold value of the strong shear for the damping of the KAW is obtained to be 0.329.

6 Results
A graphical analysis of the results is shown in Fig. (1-12). Parameters appropriate to the PSBL are used\(^\text{5-7}\).

\(B_0 = 400 \text{ nT, } \omega_{ci} = 38.33 \text{ Hz, } n_o = 1 \text{ cm}^{-3}, v_A = 8720 \text{ km/sec, } T_{ze} = 25 - 50 \text{ eV, } T_{ce} = 2 \text{ keV, } T_{id} = 4 \text{ keV, } T_{id} = 1 - 2 \text{ keV, } T_e \text{ (plasma sheet) } \)

\(0.1 - 1 \text{ keV, } \lambda_{\perp} = 20 - 120 \text{ km for } \frac{T_e}{T_i} = 1, \rho_{ci} = 2.285 \times 10^6 \text{ cm} = 22.85 \text{ km, } k_z = 1 \times 10^{-10} \text{ cm}^{-1}, V_0 = 400 \text{ km/s} .

Figures (1-6) show the effect of sheared velocity, temperature anisotropy and density on the wave frequency and growth rate of the KAW in case of weak shear.

Figure (1) shows the variation of wave frequency \(\omega / \omega_{ci} \) with perpendicular wave vector \(k_{\perp} \rho_{ci} \) for different values of weak velocity shear \(\beta_v = \kappa_v \rho_{ci} \text{, } \kappa_v = \frac{1}{V_0} \frac{dV_x}{dx} \). It is observed that the wave frequency \(\omega / \omega_{ci} \) decreases with the \(k_{\perp} \rho_{ci} \) and the effect of the shear is to further decrease the wave.
frequency. The effect is prominent towards \( k_{\perp} \rho_i \sim 1 \). The observed frequency of the KAW is in agreement with the reported values\(^5,6\) of 1-10 mHz in the PSBL. Figure (2) shows the variation of the growth rate \( \left( \frac{\gamma}{\omega_{ci}} \right) \) with perpendicular wave vector \( (k_{\perp} \rho_i) \) for different values of weak velocity shear (\( \beta_v \)). It is observed that for each value of \( \beta_v \) the growth rate increases with \( k_{\perp} \rho_i \). The growth of KAW in the presence of weak shear may be due to the inverse Landau damping of the wave by the resonant wave–particle interaction. This is in agreement with previous result of Milkhaivlovs\(^{21}\). Further, the effect of the increasing values of weak velocity shear is to enhance the growth rate. Thus, the growth rate of the KAW may be enhanced at the onset of the substorm.

Figures 3 and 4 show the effect of temperature anisotropy on the frequency and growth rate of the weak-shear driven KAW. It is observed that the effect of the increasing temperature anisotropy is to lower the wave frequency of KAW in the PSBL (Fig. 3). The effect of the temperature anisotropy is to enhance the growth rate of weak shear driven KAW in the PSBL by coupling the free energy of the anistropic plasma. This is also in agreement with previous results of Wang et al\(^{18}\). Thus, the temperature anisotropy of the plasma particle in the PSBL destabilizes the low-frequency KAW at the onset of the substorm.

Figures 5 and 6 depict the effect of the plasma density on the weak shear driven KAW. The frequency of the KAW driven by weak shear is observed to decrease with the increasing density of
the plasma particles and the growth rate of the wave is observed to increase with the density. The reason may be the inverse Landau damping of the wave with the increased number of resonant particles. From Figs 1-6, the growth rate of the KAW is positive and increases with the sheared velocity, temperature anisotropy and density increase, respectively, but the frequency decreases. In general, the growth rate of the wave is considered as the small imaginary part of the wave frequency and does not affect the wave frequency. In our consideration, it is assumed that the growth rate is not affecting the wave frequency in the linear analysis. The plotted Figs1-6 represent the real part of the frequency.

Thus, the increased density of the plasma particles may lead to the generation of low frequency KAW with enhanced growth rate in the PSBL at the substorm onset. Figures 7-12 show the variation of frequency and growth rate of the KAW driven by strong shear.

Figure 7 shows the variation of the wave frequency $\frac{\omega}{\omega_0}$ with perpendicular wave vector $(k_z \rho_{ci})$ for different values of strong velocity shear $(\beta_v)$. It is observed that the wave frequency increases with perpendicular wave vector, which is the characteristic of the KAW. The effect of the increasing values of the strong shear is to enhance the wave frequency. Thus, the KAW observed during the substorm phase may have increased wave frequency.

Figure 8 shows the variation of the damping rate of the KAW with $(k_z \rho_{ci})$ for different values of strong velocity shear. It is observed that the wave is damped
in the presence of the strong shear, which may be due to the Landau damping of the KAW by the resonant wave-particle interaction. This is in agreement with previous work of Lysak and Song.\textsuperscript{17} The loss in the energy of the wave, due to its damping by strong shear at the time of substorm, may be then transferred to the resonant particles. Thus, the loss of Poynting flux recently reported by Keiling et al.\textsuperscript{5} and Wygant et al.\textsuperscript{6,7} may be due to the effect of strong shear on kinetic Alfvén wave in the energy transfer process from PSBL to the auroral ionosphere by KAW accelerating auroral particles through its Landau damping.

Analytical results derived here also show that in case of strong shear, the ion longitudinal motion, which is associated with the term $v_{\parallel}^2$ in Eq. (19) will suppress the instability.\textsuperscript{18} The effect of $\beta_v$ is to further increase the damping. Thus, the energy of the KAW in the PSBL may be dissipated to energies the electrons along the auroral field lines at the substorm.

Fig. 6 — Variation of growth rate $\frac{\gamma}{\omega_{ci}}$ with perpendicular wave vector ($k_{\perp} \rho_{ci}$) for different values of plasma density ($n_0$) and fixed value of weak shear ($\beta_v$=0.06)

Fig. 7 — Variation of wave frequency $\frac{\omega}{\omega_{ci}}$ with perpendicular wave vector ($k_{\perp} \rho_{ci}$) for different values of strong velocity shear ($\beta_v$)

Figures 9 and 10 show the effect of temperature anisotropy on the frequency and growth rate of the strong shear driven KAW. It is observed that the temperature anisotropy lowers the wave frequency and the damping rate of the KAW driven by strong velocity shear in the PSBL.

Figures 11 and 12 show the effect of density of plasma particles on the KAW. It is observed that the increasing density lowers the frequency of the KAW driven by strong velocity shear and that it enhances the damping rate of the KAW, which may be due to the Landau damping the wave. In Figures 1 and 7, the plots of the wave frequency all start with the same value (about 7.2) as $\beta_v$ is not effective at zero $k_{\perp} \rho_{ci}$. However, in Figs 3, 5, 9 and 11 there is at least one plot which starts at same value of frequency (about 7.2) as the parameters correspond to that of
Figs 1 and 7. The other curves must differ according to the parameters selected as represented by the expressions of frequency (Eqs 13 and 15) in the case of weak and strong shear, respectively. Thus, the density of plasma particles of the PSBL may add to the damping of the KAW during the substorm period.

7 Discussion

Thus, the results have shown the generation of the energetic KAW, recently observed by the Polar and Cluster satellites in the PSBL, by the weak velocity shear at the onset of the substorm. The results have further shown that during the time of substorm when the velocity shear becomes strong, the KAW gets damped. The damping of the wave may then give rise to the observed parallel electron energisation. The temperature anisotropy and the density of the plasma particles at the PSBL add to these effects. The observed frequency of the KAW is in agreement with the reported values of 1-10 mHz in the PSBL.

Loss of Poynting flux in the auroral acceleration region has been reported by Keiling et al. They have reported that the magnitude of the Poynting flux observed with Freja satellite at altitude below 8000 km, which is about 4 ergs cm$^{-2}$s$^{-1}$ (when mapped to 100 km altitude) was much smaller than the Polar observations of Poynting flux of 125 ergs cm$^{-2}$s$^{-1}$ (when mapped to 100 km altitude) at altitudes of 4-7 $R_E$ supporting the scenario where the high altitude Poynting flux is dissipated in the acceleration of electrons in the auroral acceleration region.

The results of the present study have shown that at the time of the substorm when the shear is strong the KAW gets damped. Thus, observed loss of Poynting...
The resonant electrons, associated with the small imaginary term \( -\frac{T_i}{2k_i v_{Te}} \) in Eqs (14) and (15), are found to be an essential energy source of the instability. However, in case of strong shear, the ion longitudinal motion associated with the term \( v_{iz}^2 \) suppress the instability. Recent observations at altitudes of 4-6 \( R_E \) in the PSBL and deeper within the plasma sheet show that large amount of energy is transferred earthward by the Poynting flux carried by the Alfven waves. Simultaneously measured electron velocity space distribution functions from the Polar hydra instrument include parallel electron heating features and earthward electron beams indicating strong parallel energisation. This enhanced Alfvenic Poynting flux dominates other forms of energy flux along plasma sheet magnetic field lines and coincides with magnetically conjugate intense aurora.

The shear driven KAW during the substorm, as reported in the present study, may carry significant electro magnetic energy from the reconnection region to the auroral regions. Their Landau damping by the strong shear may then lead to the observed parallel electron energisation. Lysak and Chaston et al. have also reported that once the KAW reach the auroral region, they are one strong candidate for the acceleration of the electrons that cause aurora. It has also been suggested by Keiling et al. that the large scale shear Alfven waves (presumably those reported in the PSBL) become KAW in the small scale limit, which can provide the parallel electric field necessary for auroral electron acceleration.
The KAW with enhanced growth rate in the PSBL generated at the onset of the substorm, as reported in the present work, may also play an important role in the local heating of the plasma sheet and may also produce transversely heated ions besides the earthward, magnetic field aligned electron beams. The results may also be applicable to laboratory plasma with sheared flows.

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