Kinetic theory of Shubnikov-de Hass oscillations in GaAs/AlGaAs

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A quantum theory is developed for the Shubnikov-de Haas oscillation in GaAs/AlGaAs. The envelope of the oscillation for the magnetoresistance is shown to arise from the thermal average of the sinusoidal density of states with the Fermi distribution function for the field-dressed electrons (guiding centers). It is proportional to $2 \sinh(2 \pi M^* k_B T / \hbar B)$, where $M^*$ is the magneto-transport mass. Comparison between theory and experiment directly gives the $M^*$-value: $M^* = 0.30 m^*$ which is 4.5 times the cyclotron mass $m^* = 0.067 m$. The carriers are likely to be the $c$-fermions, each electron with two flux quanta. The theory avoids the use of the Dingle temperature arising from the Landau level damping. The magneto-conductivity $\sigma$ for the center of oscillations is given by $\sigma = e^2 n / M^* \gamma$, where $n$ is the carrier density and $\gamma$ is the relaxation rate. This useful formula is obtained by applying kinetic theory (and the Boltzmann equation method) to the motion of the dressed electrons.

Keywords: Magnetoresistance, Shubnikov-de Hass oscillations, Quantum Hall effect, Magneto-transport mass

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1 Introduction

Traditionally the electron transport is treated, using kinetic theory or the Boltzmann equation method. In the presence of a static magnetic field, the classical electron orbit is curved. Then the basic kinetic theoretical picture in which the electron moves on a straight line, hits a scatterer (impurity), changes direction, and moves on to another straight line and breaks down. Besides, the Boltzmann collision terms containing the scattering cross-section cannot be written down. Fortunately, quantum theory can save the situation. When a magnetic field is applied slowly, the energy of the electron does not change but the cyclotron motion always acts so as to reduce the magnetic field. Hence, the total energy of the electron with its surrounding field is less than the electron energy plus its unperturbed field energy. In other words, the electron dressed with the field is stable against the break-up, and it is in a bound (negative energy) state. If an electric field is applied in the x-direction, the dressed electron whose position is the guiding center of circulation, preferentially jumps in the x-direction, and generates a current. Thus, we can apply kinetic theory to the guiding center motion. In particular, we obtain an expression for the electrical conductivity $\sigma$:

$$\sigma = e^2 n / (M^* \gamma), \quad \ldots \ (1)$$

where $n$ is the density of the dressed electrons, $e$ the charge, $M^*$ the magneto-transport (effective) mass different from the cyclotron mass $m^*$, and $\gamma$ is the relaxation rate.

In 2001, Zudov et al. observed two kinds of Shubnikov-de Haas (ShdH)-like oscillations in GaAs/AlGaAs subject to millimeterwave radiation, one kind periodic in $B^{-1}$ which appears on the high field side ($B \approx 0.4$ T) and the second kind also periodic in $B^{-1}$ appearing on the weak field side ($B \approx 0.2$ T) with a different period which exists only
with the radiation. Mani et al.² and Zudov et al.³ later found that the second kind of oscillations contain zero resistance states just like the Quantum Hall (QH) states⁴ occurring at higher fields (B~5 T). Fig. 1 represents the data after Mani⁵ for the diagonal resistance \( R_{xx} \) versus the reduced inverse magnetic field, \( B^{-1} \). The reduction is taken such that the two curves with (w/) and without (w/o) microwaves intersect at \( B^{-1} = 2, 3, 4 \) in Fig. 1. A set of the prominent ShdH oscillations occur for the sample without the radiation. We concentrate ourselves to this case. In 1982 Ando, Fowler and Stern⁶ surveyed experiments and theories of phenomena including the ShdH and de Haas-van Alphen (dHvA) oscillations in 2D electron systems. The prevalent thinking then and now is that the envelopes of the two oscillations can be characterized by the Dingle temperature ⁷ arising from the Landau Level (LL) damping. The susceptibility \( \chi \) is an equilibrium property, and hence, it should be calculated without considering the scattering (damping) mechanism. The purpose of the present work is first to resolve the mystery of the Dingle temperature and second to show that the magneto-transport mass \( M^* \) can be obtained directly from the envelopes of the oscillations.

2 Theory

Let us take a dilute system of electrons moving in the plane. With a magnetic field \( \mathbf{B} \) perpendicular to the plane, each electron will be in the Landau state with the energy

\[
\varepsilon = (N_L + 1/2)\hbar \omega_c, \quad \omega_c = eB/m^*, \quad N_L = 0, 1, 2, \ldots, \quad \cdots \quad (2)
\]

where \( m^* \) is the cyclotron mass and \( \omega_c \) the cyclotron frequency. The degeneracy of the LL is

\[
eBA/(2\pi\hbar), \quad A = \text{sample area} \quad \cdots \quad (3)
\]

Thus, the weaker is the field the more LL’s, separated by \( \hbar \omega_c \), are occupied by the electrons. In this state, the electron can be viewed as circulating around the guiding center.

We consider first the case with no magnetic field. We assume a uniform distribution of impurities with the density \( n_i \). Solving the Boltzmann equation, we obtain the conductivity ⁸:

\[
\sigma = \frac{2e^2}{m^*(2\pi\hbar)^2} \int d^2p \varepsilon \left( \frac{dF}{d\varepsilon} \right) \quad \cdots \quad (4)
\]

where \( \varepsilon = p^2/(2m^*) \) and \( \Gamma \) is the energy (\( \varepsilon \))-dependent relaxation rate:

\[
\Gamma(\varepsilon) = n_i \int d\Omega(p/m^*) I(p, \theta)(1-\cos\theta) \quad \cdots \quad (5)
\]

with \( \theta = \text{scattering angle} \) and \( I(p, \theta) = \text{cross-section} \). The factor 2 is due to the spin degeneracy. The Fermi distribution function:

\[
F(\varepsilon) = \left[ \exp\{e\beta(\varepsilon - \mu)\} + 1 \right]^{-1} \quad \cdots \quad (6)
\]

with \( \beta = (k_B T)^{-1} \) and \( \mu = \text{chemical potential} \), is normalized such that:

\[
n = \frac{2}{(2\pi\hbar)^2} \int d^2p F(\varepsilon) \quad \cdots \quad (7)
\]

We introduce the density of states \( N(\varepsilon) \) and rewrite Eq. (4) as:

\[
\sigma = \frac{e^2}{m^*} \int_0^\infty d\varepsilon N(\varepsilon) \left( \frac{dF}{d\varepsilon} \right) \quad \cdots \quad (8)
\]

The function \( F(\varepsilon) \) drops steeply near \( \varepsilon = \mu \) at low temperatures: \( k_B T << \varepsilon_F \) (Fermi energy). In particular for a 2D quasi-free electron system, the density of states \( N(\varepsilon) = Am^*/\pi\hbar^2 \) is independent of the energy \( \varepsilon \). Then, the Dirac-delta-function replacement formula:

\[
\frac{dF}{d\varepsilon} = \delta(\varepsilon - \mu) \quad \cdots \quad (9)
\]
can be used. Assuming this formula and using:

\[ \int_0^\infty d\varepsilon (N\varepsilon) \left( \frac{dF}{d\varepsilon} \right) = \int_0^\infty d\varepsilon (N\varepsilon) F(\varepsilon) \]  \tag{10}

and comparing Eqs (1) and (8), we obtain:

\[ \frac{n}{\gamma} = \int_0^\infty d\varepsilon (N\varepsilon) \frac{1}{F(\varepsilon)} \]  \tag{11}

Next we consider the case with a magnetic field. We introduce kinetic momenta:

\[ \Pi_x = p_x + eA_x, \quad \Pi_y = p_y + eA_y \]  \tag{12}

The electron kinetic energy is:

\[ H = \frac{p_x^2 + p_y^2}{2m^*} = \frac{\Pi_x^2 + \Pi_y^2}{2m^*} \]  \tag{13}

The vector potential \( A = \frac{1}{2} B \times r \) can be written as \( A_x = -B_y, \quad A_y = \frac{1}{2} Bx, \quad A_z = 0 \). Using the quantum condition:

\[ \{x, p_x\} = \{y, p_y\} = \{x, p_y\} = 0 \]

we obtain:

\[ \{\Pi_x, \Pi_y\} = -(e\hbar/i)B \]  \tag{14}

If we introduce \( m^{*1/2} \Pi_x = P_x, \quad (eB)^{-1} m^{*1/2} \Pi_y = Q_y \), we obtain \( H = (P_x^2 + \omega^2 Q_y^2)/2 \) and \( [Q_y, P] = i\hbar \). Hence, the energy eigenvalues are given by \( (\mathcal{N}_L + 1/2) \hbar \omega_c \), confirming Eq. (2). After simple calculations, we obtain:

\[ dx \Pi_x dy \Pi_y = dp_x dy dp_y \]  \tag{15}

We can represent quantum states by quasi-phase space elements \( dx \Pi_x dy \Pi_y \). The Hamiltonian \( H \) in Eq. (13) does not depend on the position \((x, y)\). Assuming large normalization lengths \( (L_x, L_y) \), we can represent the Landau states by the concentric shells having the statistical weight:

\[ (2\pi)^2 L_x L_y (2\pi \hbar)^2 = A \omega_c (2\pi \hbar)^3 = eBA(2\pi \hbar)^2 \]  \tag{16}

with \( A = L_x L_y \) and \( \hbar \omega_c = \Delta (\Pi_x^2 / 2m^*) = PA \Delta \Pi / m^* \). Hence, the LL degeneracy is given by \( eBA/(2\pi \hbar) \) as stated in Eq. (3).

Let us consider the motion of the field-dressed electrons (guiding center). We assume that a dressed electron is a fermion with magneto-transport mass \( M^* \) and charge \( e \). The kinetic energy is represented by:

\[ H_k = (\Pi_x^2 + \Pi_y^2)/(2M^*) = \Pi^2 / (2M^*) \]  \tag{17}

According to Onsager's flux quantization hypothesis, the magnetic fluxes can be counted in units of \( \Phi_0 = e/h \) with \( h \) = Planck's constant. We assume that the dressed electron is composed of an electron and two elementary fluxes (fluxons). A further explanation of this model will be given later. We introduce a distribution function \( \phi(\Pi, t) \) in the \((\Pi_x, \Pi_y)\) space normalized such that:

\[ \frac{2}{(2\pi \hbar)^2} \int d^2 \Pi \phi(\Pi_x, \Pi_y, t) = \frac{N_A}{N} = n \]  \tag{18}

The Boltzmann equation for a homogeneous stationary system is

\[ \frac{\partial \phi}{\partial t} = -eV_B \frac{d}{d\Pi} \frac{\partial}{\partial \Pi} \phi(\Pi, \Pi_y, \theta) = \frac{N_A}{N} \phi(\Pi, \Pi_y, \theta) - \phi(\Pi, \Pi_y, \theta) \]  \tag{19}

In the actual experimental condition, the magnetic force term can be neglected. Assuming this condition, we obtain the same conductivity formula as given in Eq. (4) with \( m^* \) replaced by \( M^* \).

As the field \( B \) is raised, the LL separation \( \hbar \omega_c \) becomes greater and the quantum states are bunched together. The density of states should contain an oscillatory part in proportion to:

\[ \sin \left( \frac{2\pi \varepsilon'}{\hbar \omega_c} + \psi_0 \right), \quad \varepsilon' = \frac{\Pi^2}{2m^*} \]  \tag{20}

where \( \psi_0 \) is a phase. Since \( \varepsilon'/\hbar \omega_c \gg 1 \), the phase \( \psi_0 \) will be dropped hereafter. Physically, the sinusoidal variations in Eq. (20) come as follows. From the Heisenberg uncertainty principle and the Pauli exclusion principle, the Fermi energy \( \varepsilon_F \) remains approximately constant as the field \( B \) varies. The density of states is high when \( \varepsilon_F / \hbar \omega_c \) matches the \( \mathcal{N}_L \)-th level while it is small when \( \varepsilon_F \) falls between neighbouring LL’s. If the density of states oscillates violently in the drop of the Fermi distribution
function, one cannot use the delta-function formula given in Eq. (9). The width of \( \frac{dF}{d\epsilon} \) is of the order of \( k_B T \). The critical temperature \( T_c \) below which the oscillations can be observed is \( k_B T_c \sim \hbar \omega_c \). Below the critical temperature \( (T<T_c) \), we may proceed as follows. Let us consider the integral:

\[
I = \int_0^\infty d\epsilon F(\epsilon) \sin(2\pi \epsilon/\hbar \omega_c), \quad \epsilon = \frac{\pi^2}{(2M^*)}
\]

We introduce a new variable \( \xi = \beta(\epsilon - \mu) \), and extend the lower limit to \( -\infty \) (low temperature limit):

\[
\int_0^\infty d\xi \cdots \frac{1}{\exp{\{(\beta(\epsilon - \mu))\}}} = 1 \int_{-\infty}^\infty d\xi \cdots \frac{1}{\exp{\{\xi\}}} + 1
\]

\[
\rightarrow \beta^{-1} \int_{-\infty}^\infty d\xi \cdots \frac{1}{\exp{\{\xi\}}} + 1 \quad \text{(22)}
\]

Using \( \sin(A + B) = \sin A \cos B + \cos A \sin B \) and

\[
\int_{-\infty}^\infty d\xi \frac{\exp(i\pi \xi)}{\exp{\{\xi\}}} = \frac{\pi}{\sinh{\pi \alpha}} \quad \text{(23)}
\]

we obtain

\[
I = \pi k_B T \frac{\cos(2\pi \epsilon_f/\hbar \omega_c)}{\sinh(2\pi^2 M^* k_B T/\hbar \epsilon_f)} \quad \text{(24)}
\]

Here we used \( M^* \mu(T=0) = m^* \epsilon_f \), which arises from the fact that the Fermi momentum is the same for both the dressed and undressed electrons. For very low fields, the oscillation number in the range \( k_B T \) becomes more. Hence, the sinusoidal contributions must cancel out. This is represented by the factor \( \sinh(2\pi^2 M^* k_B T/\hbar \epsilon_f) \).

We start calculation with Eq. (11). For the field-free case, we may use Eqs (9) and (10) and obtain:

\[
\frac{n}{\gamma_0} = \frac{N (\epsilon_f') \epsilon_f'}{AF(\epsilon_f')}, \quad \epsilon_f' = \frac{p^2}{2m^*}
\]

For a finite \( B \) the non-oscillatory part (background) contributes a similar amount:

\[
\frac{n}{\gamma} = \frac{N (\epsilon_f') \epsilon_f'}{AF(\epsilon_f')}, \quad \epsilon_f' = \frac{\pi^2}{2M^*}
\]

Calculated for the dressed electrons. The oscillatory part can be calculated by using the integration formula \( I \) in Eqs (21) and (24). This part is much smaller than \( N \epsilon_f/\gamma F \) since the contribution is limited to the small energy range \( (k_B T) \). It is also small by the sinusoidal cancellation. In summary:

\[
\frac{n}{\gamma} = \frac{N (\epsilon_f) \epsilon_f}{AF(\epsilon_f)} = \frac{n}{\gamma F(\epsilon_f)} \quad \text{(25)}
\]

\[
\phi \equiv \frac{\pi k_B T}{\epsilon_f} \frac{\cos(2\pi \epsilon_f/\hbar \omega_c)}{\sinh(2\pi^2 M^* k_B T/\hbar \epsilon_f)} 
\]

The resistivity \( \rho \equiv \sigma^{-1} \) can then be expressed as:

\[
\rho = \frac{M^*}{\epsilon_f} (1-\phi) \quad \text{(28)}
\]

where we used the approximation \( (1+\phi) \approx 1 - \phi \).

If the decay rate \( \delta \) is defined as:

\[
\sinh(\delta/B) = \sinh(2\pi^2 M^* k_B T/\hbar \epsilon_f) \quad \text{(29)}
\]

which is measured carefully, the effective mass \( M^* \) can be obtained directly by \( M^* = e \hbar \delta/(2\pi^2 k_B T) \). This is quite remarkable. The relaxation rate \( \gamma \) can now be obtained by Eq. (1) with the measured magnetoconductivity. All electrons, not just those excited electrons near the Fermi surface, are subject to the electric field. Hence, the carrier density \( n \) appearing in Eq. (1) is the total density of the dressed electrons. This \( n \) also appears in the classical Hall resistivity \( (\rho_H) \) expression:

\[
\rho_H = E_H / j = v_d B/(\epsilon n) = B/(en) \quad \text{(30)}
\]

where the Hall field \( E_H = v_d B \), the current density \( j = \epsilon n v_d \), \( v_d \) = drift velocity, were used. Hence, the density \( n \) can be obtained precisely by the Hall effect measurements. All (dressed) electrons are subject to the magnetic field, and hence the magnetic susceptibility \( \chi \) is proportional to the electron density \( n \), ensuring the similarity between the ShdH and DHvA oscillations. In the present theory, the two masses \( m^* \) and \( M^* \) are introduced naturally corresponding to the two physical processes, the cyclotron motion of the electron and the magnetotransport motion of the dressed electron. The dressed particles are there whether the system is...
probed in equilibrium or in non-equilibrium as long as the system is subjected to a magnetic field.

3 Comparison with Experiment

Fig. 1 indicates that the resistance $R_{xx}$ linearly decreases with $B^{-1}$ in the low field limit. This behaviour can be explained by using Eq. (1) as follows. For high purity samples at very low temperature (~0.7 K) the impurity and phonon scatterings are negligible. By energy-time uncertainty principle, the dressed electron can spend a short time at an upper LL and come back to the ground LL with a different guiding center, thus causing a guiding center jump. We assume that the relaxation rate $\gamma$ is the natural linewidth arising from the LL separation divided by $h$, that is, the cyclotron frequency $\omega_c$:

$$\gamma = \omega_c$$

This generates the desired $B^{-1}$-dependence.

We fitted Mani’s data in Fig. 1 with

$$R = A + B x + \frac{\{E \cos(2\pi C x - \pi/4) + F\} x}{\sinh(D x)}$$

where $A = 2.3$, $B = -0.18$, $C = 23$, $D = 3.1$, $E = 22.0$ and $F = 7.0$. The fits agree with the data within the experimental errors. Using $D x = \delta/B$, we obtain:

$$M' = 0.30 \, \text{m}$$

This value is 4.5 times greater than the cyclotron mass $m^* = 0.067 \, m$.

The field-dressed electron is the same entity as the composite (c-) particle having an electron and fluxons in the theory of the QH states. The c-boson (fermion) was introduced by Zhang, Hansson and Kivelson$^9$ (Jain$^{10}$). The statistics with respect to the center-of-mass motion follows the Ehrenfest-Oppenheimer-Bethe’s rule$^{11}$ that the composite is bosonic (fermionic) if it contains an even (odd) number of elementary fermions. Thus, the dressed electron with one (two) fluxons, is bosonic$^{12}$ (fermionic). The c-particle is often viewed as the electron attached with Chern-Simons (CS) gauge objects (vortices). This is not satisfactory since the CS object is neither boson nor fermion, and hence, the statistics of the c-particle cannot be determined.

Mani$^5$ measured the resistance in high purity samples for fields of up to 6 T. Mani’s data in Fig. 2 indicate the ShdH oscillations below 1 T and the QH states above 1 T. The resistivity vanishes at filling factor $\nu = 3$ ($B = 4.5$ T) and $T = 0.5$ K, where the Hall resistivity shows a plateau. A similar feature is observed at $\nu = 4$ ($B = 3.5$ T). These features are caused by the Bose-Einstein Condensation (BEC) of the c-bosons, each with one fluxon. Below $B = 3$ T the resistivity curve oscillates with the minima at $\nu = 5, 6, \cdots$ and the maxima at $\nu = 11/2, 13/2, \cdots$. These maxima are caused by the c-fermions, each with two fluxons. The curve’s oscillations become a ShdH-like for higher $\nu$ (small fields). The binding energy and the entropy are greater for the c-fermions than for the c-bosons. Hence, from the free energy consideration the c-fermions are dominant at the lowest fields and lowest temperatures. By logical extension, the carriers in the ShdH oscillations are the c-fermions, each with two fluxons.

Jain$^{10}$ introduced the effective magnetic field:

$$B' \equiv B - B_v = B - (1/\nu) n_e (h/e)$$

relative to the standard field for the c-fermion at the even denominator fraction. The c-fermion moves field-free at the exact fraction, where all fluxons in the system are attached to the c-particles. The excess (or deficit) of the magnetic field is simply the effective magnetic field $B'$. This $B'$ should be inserted in the magnetic force term in Eq. (19). The magnitude of the effective field $B'$ is less than half the distance (field) of the neighbouring minima:

$$|B'| \leq \frac{1}{2} (B_N - B_{N-1})$$
Hence, the magnetic force term is negligible for the lowest fields compared with the electric force term. The effective mass estimated from the QH state at $\nu=1/2$ has the values$^{13}$: $(0.8 \sim 1.0) m$. We may assign $0.30 m$ to the majority carrier mass. The minority carrier has a heavier mass. A further experimental study is needed here.

4 Conclusions
In summary, the envelope of the oscillations

$$[\sinh(2\pi^2 M^*k_g T/\hbar e B)]^{-1}$$

arises from the sinusoidal density of states averaged over the Fermi distribution of the dressed electrons. No impurities played a role in the average process. The theory may be checked by varying the impurity density. Our theory predicts little change in the clearly defined envelope arising from the impurity scattering. The scattering will change the relaxation rate $\gamma$ and the magneto-conductivity for the center of the oscillations through Eq. (1).

References