Irreversible modified complex Brayton cycle under maximum economic condition

Sudhir K Tyagi, Shengwei Wang & S C Kaushik*
Department of Building Services Engineering, The Hong Kong Polytechnic University, Hung Hom, Kowloon, Hong Kong
*Centre for Energy Studies, Indian Institute of Technology, Delhi, New Delhi 110 016

Received 27 January 2006; accepted 2 May 2006

An irreversible complex Brayton cycle under maximum economic conditions has been investigated. The economic function has been optimized with respect to the cycle temperatures, reheat and intercooling pressure ratios for a typical set of operating conditions. It is found that there are optimal values of the intercooling, reheat and cycle pressure ratios at which the cycle attains the maximum objective function, maximum power and maximum efficiency. The maximum economic point, maximum power point and the maximum efficiency point exist but the economic function corresponding to the maximum power and maximum efficiency is found to be lower than that can be attained and vice-versa. The optimal values of different parameters corresponding to the maximum economic function are different from those corresponding to the maximum efficiency and maximum power for the same set of operating conditions. Again, all the performance parameters further enhance as the heat capacitance rates of the external reservoirs and increased, while it is found to be reverse in the case of the working fluid heat capacitance rate and the inlet temperature of the conversing combustion chamber (CCC). The objective function and efficiency are found to be the decreasing functions of the reheat pressure ratio for lower values of CCC inlet temperature while there are optimal values of the reheat pressure ratio for higher values of the inlet temperature of the CCC.

Keywords: Complex Brayton cycle, Economic objective function, Efficiency regenerator, Turbine, Compressor

IPC Code: F01, F23L

1 Introduction

It is well known that reheating in gas turbines limits the extent to which an isothermal heat addition is approached. With respect to simple heat addition, when a compressible fluid with subsonic velocity flows through a frictionless constant area duct with heat addition, the temperature of the fluid increases along the duct. Also with respect to simple area change, when a compressible fluid with subsonic velocity flows through a frictionless adiabatic duct with decreasing area, the temperature of the fluid decreases along the duct. The idealized isothermal process consists of a compressible fluid with subsonic velocity flowing through a frictionless converging duct such that while heated all along the duct, any infinitesimal decrease in temperature due to simple area change is exactly compensated by the simple heat addition. It is noted that, since temperature of the fluid is constant during the isothermal heat addition, the kinetic energy of the fluid (and hence, the Mach number) must increase in order to satisfy the law of conservation of energy. Based on the nature of these flows, the simple heat addition (Rayleigh flow) and simple area change (isentropic flow) may be combined in such a way so as to yield an isothermal heat addition process1-7.

In recent decades, more efforts are being made for the improvement of the efficiency of a Brayton cycle using design modifications and/or extra parts. There are several ways to modify a Brayton cycle as reported in the literature, such as the use of two heat sources1-7, regenerator10-11, reheater11, intercooler 12-15 and/or their combinations 3-7,11,13-15. Vecchiarelli et al1 indicated that the modification of a gas turbine including two heat additions (rather than one) may result in some efficiency improvement with conventional cycles16-20. Some modifications and studies made by Göktun and Yavuz 2, Kaushik et al3, Tyagi4, Tyagi et al5, Erbay et al6 and Kaushik et al7, Sahin et al8, Cheng and Chen9, Wang et al10,11 and Tyagi et al12, showed that the efficiency of a Brayton cycle can be improved significantly by using the combinations of these above parts.
In this paper, we present a more general analysis based economic approach\textsuperscript{21-25} of an irreversible modified complex Brayton cycle with the non-isentropic processes for finite heat capacity of external reservoirs, following earlier researchers\textsuperscript{26-28}.

2 Cycle Analysis

Let \( Q_{h1} \) and \( Q_1 \) be the heat transfer rates to and from the heat engine respectively and \( Q_R \) the regenerative heat transfer rate then:

\[
Q_{h1} = (UA)_{h1} (LMTD)_{h1} = C_w (T_5 - T_{4R}) = C_{H1} (T_{H1} - T_{H2}) = C_{H1,\min} \epsilon_{H1} (T_{H1} - T_{4R}) \quad \ldots (1)
\]

\[
Q_{h2} = (UA)_{h2} (LMTD)_{h2} = \dot{m} (V_2 - V_3) / 2 = C_{H2} (T_{H3} - T_{H4}) = C_{H2,\min} \epsilon_{H2} (T_{H3} - T_{3}) \quad \ldots (2)
\]

\[
Q_{h3} = (UA)_{h3} (LMTD)_{h3} = C_w (T_8 - T_7) = C_{H3} (T_{H5} - T_{H6}) = C_{H3,\min} \epsilon_{H3} (T_{H6} - T_{7}) \quad \ldots (3)
\]

\[
Q_{l1} = (UA)_{l1} (LMTD)_{l1} = C_w (T_9 - T_1) = C_{L1} (T_{L2} - T_{L1}) = C_{L1,\min} \epsilon_{L1} (T_{R} - T_{L1}) \quad \ldots (4)
\]

\[
Q_{l2} = (UA)_{l2} (LMTD)_{l2} = C_w (T_2 - T_3) = C_{L2} (T_{C2} - T_{C1}) = C_{L2,\min} \epsilon_{L2} (T_{2} - T_{C1}) \quad \ldots (5)
\]

\[
Q = (UA)_{lR} (LMTD)_{lR} = C_w (T_9 - T_{4R}) = C_w (T_9 - T_4) = C_w \epsilon_{R1} (T_9 - T_4) \quad \ldots (6)
\]

where \( C_k \) (\( k = H1, H2, H3, L1, L2 \)) and \( C_w \) are the heat capacitance rates (mass flow rate times specific heat) of the external fluids and the working fluid, respectively. \((UA)_k\) and \((UA)_R\) are the overall heat transfer coefficient-area products on the different reservoirs and cycle and the regenerative heat exchanger, respectively. \( V_2 \) and \( V_3 \) are the velocities of the working fluid at state points 5 and 6, respectively. \( \dot{m} \) is the mass flow rate of the working fluid, \((LMTD)_i's\) are the Log Mean Temperature Differences between the external reservoirs and the cycle and \( \epsilon_j's\) (\( J = k, R \)) are the effectiveness of the various heat exchangers while \( C_{k,\min} \) and \( C_{k,\max} \) are defined as\textsuperscript{3-5,13-15}:

\[
C_{k,\min} = \min (C_k, C_W) \quad \text{and} \quad C_{k,\max} = \max (C_k, C_W) \quad \ldots (7)
\]

With the help of Fig. 1, the compressors and turbines efficiencies are given as:

\[
\eta_{c1} = \frac{(T_{2S} - T_1)}{(T_2 - T_1)}, \quad \eta_{c2} = \frac{(T_{4S} - T_3)}{(T_4 - T_3)}, \quad \eta_{t1} = \frac{(T_6 - T_7)}{(T_6 - T_{7S})}
\]

\[
\eta_{t2} = \frac{(T_8 - T_6)}{(T_8 - T_{8S})} \leq 1.0 \quad \ldots (8)
\]

Using Eqs (1-7), the temperature of different state points can be expressed in terms of \( T_4 \) and \( T_8 \) as given in Appendix. Again, using the second law of thermodynamics in this case of a modified complex Brayton cycle\textsuperscript{1-7,11-15}, we have:

\[
T_{2S} T_{4S} T_{7S} T_{9S} = R P t T_1 T_3 T_6 T_8 \quad \ldots (9)
\]

where \( R_p \) is the isothermal pressure drop ratio between process 5-6, \( k = 1 - 1/\gamma \) and \( \gamma \) is the specific heat ratio and solving Eq. (9) for \( T_4 \) (treating \( T_8 \) as independent for a given set of operating cycle parameters) yields:

\[
T_4 = \frac{-B + \sqrt{B - 4AC}}{2A} \quad \ldots (10)
\]

The power output and the thermal efficiency can be given as:

\[
P = Q_{H1} + Q_{H2} + Q_{H3} - Q_{l1} - Q_{l2} = K_1 - a_3 T_9 \quad \ldots (11)
\]

\[
\eta = \frac{P}{Q_{H1} + Q_{H2} + Q_{H3}} = \frac{K_1 - a_3 T_9 - b_3 T_9}{K_2 - c_3 T_9 - c_4 T_9} \quad \ldots (12)
\]

where the different parameters are given in the Appendix. The economic function which is defined as the power output divided by the total cost of the system\textsuperscript{21-25}, such as the capital and running costs\textsuperscript{21-24}, as well as the maintenance cost\textsuperscript{25} of the whole system, i.e.:

\[
F = \frac{P}{C_1 + C_E + C_M} = \frac{P}{a_4 (Q_{H1} + Q_{H2} + Q_{H3} + Q_{L1} + Q_{L2} + Q_{R}) + a_6 (Q_{H1} + Q_{H2} + Q_{H3}) + a_p P + a_4 P} \quad \ldots (13)
\]

where \( C_1, C_E \) and \( C_M \) refer to annual investment, energy consumption and maintenance costs, respectively. The investment cost was considered as the costs of the main system components that are the heat transfer coefficient-area products.
exchangers and the compression and expansion devices together. The investment cost of the heat exchangers is assumed to be proportional to the heat transfer capacity of the individual heat exchanger and the investment cost due to the compression and expansion devices is assumed to be proportional to the power output of the cycle. On the other hand, the energy consumption and the maintenance costs are assumed to be proportional to the energy input and power output of the cycle. The proportionality constants, $a_A$, $a_Q$, $a_P$, and $a_M$ are, the investment costs of the heat exchangers, compression and expansion devices, energy consumption and the power output, respectively and their unit is NCU/kW, while NCU refers to the National Currency Unit.

Re-arranging Eq. (13), yields:

$$aF = \frac{K_1 - a_T T - a_s T}{K_1 + K_2 T + K_3 T}$$  \hspace{1cm} \text{(14)}$$

where the different parameters are given in the Appendix. It is seen from Eqs (11)-(12) and (14), that the power output and efficiency economic function are functions of a single variable for a given set of fix cycle parameters. Thus, using Eqs (11), (12) and (14), their most extreme conditions i.e., $\frac{\partial P}{\partial T_T} = 0$, $\frac{\partial \eta}{\partial T_T} = 0$ and $\frac{\partial \alpha F}{\partial T_T} = 0$, along with Eq. (10), we can obtain the optimal values of $T_9$, for different performance parameters, as:

$$\left( T_{9,\text{opt}} \right)_{P_{\text{opt}}} = \frac{-B_1 + \sqrt{B_1^2 - A_1 C_1}}{A_1}$$ \hspace{1cm} \text{(15)}$$

$$\left( T_{9,\text{opt}} \right)_{\eta_{\text{opt}}} = \frac{-B_2 + \sqrt{B_2^2 - A_2 C_2}}{A_2}$$ \hspace{1cm} \text{(16)}$$

$$\left( T_{9,\text{opt}} \right)_{\alpha F_{\text{opt}}} = \frac{-B_3 + \sqrt{B_3^2 - A_3 C_3}}{A_3}$$ \hspace{1cm} \text{(17)}$$

where $\left( T_{9,\text{opt}} \right)_{P_{\text{opt}}}$, $\left( T_{9,\text{opt}} \right)_{\eta_{\text{opt}}}$ and $\left( T_{9,\text{opt}} \right)_{\alpha F_{\text{opt}}}$ are, respectively, the optimum values of the turbine outlet temperature corresponding to the maximum point of

Fig. 1—T-S diagram of an irreversible modified complex Brayton cycle.
the optimized power, efficiency and objective function at a given set of cycle parameters, while the other parameters are given in the Appendix.

3 Results and Discussion

Using the Eqs (15-17), we can calculate the maximum power output, the corresponding thermal efficiency as well as the maximum efficiency and the corresponding power and the other cycle parameters for a given set of operating condition such as: 
\[ a_d = 0.25, \quad a_h = a_l = 0.20, \quad a_p = 0.50, \quad a_m = 0.10, \]
\[ \varepsilon_j = \eta' s = 0.90, \quad T_{l_1} = T_{c_1} = 300 \, K, \quad T_{h_1} = 1200 \, K, \]
\[ R_{pt} = 0.85, \quad T_{h_3} = T_{h_5} = 1500 \, K, \quad C_h = 1.05 \, kW/K, \]
\[ C_w = 1.05 \, kW/K, \] and \[ R_{pt} = R_{ph} = 5.0. \] We have plotted the graphs for the optimized and optimal performance parameters against the intercooling, reheat, and cycle pressure ratios along with other cycle parameters as shown in Figs (2-6).

Figs 2(a-c) show the variations of the optimized and optimal performance parameters against the intercooling pressure ratios for a typical set of operating conditions. All the parameters first increase and then decrease as the intercooling and reheat pressure ratios are increased (Fig. 2). It shows clearly that there exists an optimal value of the intercooling pressure ratio at which these parameters attain their maximum values. But the optimal values of the intercooling pressure ratio are different for different parameters. Also the optimal values of the intercooling ratio corresponding to the optimized power are much higher than those of the optimal values of these pressure ratios corresponding to the optimized objective function and efficiency. Thus, according to Figs 2 (a-c), the rational range of the optimal values of the intercooling ratio from the point of view of the economics as well as of thermodynamics can be determined by the following relation:

\[
\left( R_{pi, opt} \right)_{P_{opt}} \geq \left( R_{pi, opt} \right)_{aF_{opt}} \geq \left( R_{pi, opt} \right)_{\eta_{opt}} \quad \ldots (18)
\]

where \( \left( R_{pi, opt} \right)_{P_{opt}} \), \( \left( R_{pi, opt} \right)_{aF_{opt}} \) and \( \left( R_{pi, opt} \right)_{\eta_{opt}} \) are the optimal values of the intercooling pressure ratio corresponding to the optimized power, objective function and efficiency, respectively. Again, the optimized value of each of the performance parameters is higher than those of their optimal values and the trend is different for different performance parameters. Thus, based on Figs 2(a-c), it can be given as:

![Fig. 2—Variations of the (a) optimized and (b) optimal powers and (c) the efficiencies, with respect to the intercooling pressure ratio. The parameters are $\varepsilon_j = \eta' s = 0.90$, $T_{l_1} = T_{c_1} = 300 \, K$, $T_{h_1} = 1200 \, K$, $C_h = 1.05 \, kW/K$, $R_{pt} = 0.85$, $C_w = 1.05 \, kW/K$ and $R_{pt} = R_{ph} = 5.0$.](image-url)
\[
(P_{\text{opt}}^*) \geq (P_m^*)_{\text{opt}} \geq (P_m^*)_{\eta_{\text{opt}}} \quad \ldots \quad (19)
\]
\[
(aF_{\text{opt}}) \geq (aF_m)_{\text{opt}} \geq (aF_m)_{\eta_{\text{opt}}} \quad \ldots \quad (20)
\]
\[
(\eta_{\text{opt}})_{P_{\text{opt}}} \geq (\eta_m)_{aF_{\text{opt}}} \geq (\eta_{\text{opt}})_{P_{\text{opt}}} \quad \ldots \quad (21)
\]

where \(P_{\text{opt}}^*, \eta_{\text{opt}}\) and \(aF_{\text{opt}}\) are the maximum values of the optimized power output, efficiency and the objective function respectively. Similarly, \(P_m^*, \eta_m\) and \(aF_m\) are the maximum values of the optimal power output, efficiency and the objective function, respectively, while the different subscripts are mentioned clearly in the nomenclature.

Also the optimal values of these pressure ratios corresponding to the optimized power are lesser than those corresponding to the optimal power. Similarly, the optimal values of these pressure ratios at optimized efficiency are lesser than those corresponding to the optimal efficiency, for the same set of operating parameters, as can be seen clearly from Figs 2 (a-c).

Figure 3 and Table 1 show the effects of reheat pressure ratio of the optimized and optimal performance parameters for different values of inlet temperature the converging combustion chamber \((T_{\text{H1}})\). It is found that there are optimal values of reheat pressure ratio at which the optimized and optimal powers output attain their maximum values for different values of \(T_{\text{H1}}\). But there are some different results for the optimized and optimal objective function and efficiency. In other words, the optimized and optimal objective function and efficiency are decreasing functions of the reheat pressure ratio for lower values of \(T_{\text{H1}}\) while there are optimal values of the reheat pressure ratio at which the optimized and optimal objective function and efficiency attain their maxima for higher values of \(T_{\text{H1}}\). Again, all the performance parameters go down as \(T_{\text{H1}}\) is increased. Thus, we can conclude that the overall cycle performance goes down as \(T_{\text{H1}}\) is increased but the objective function and efficiency do not attain any optima against the reheat pressure ratio, while it is reverse for the power output. Thus the reheat pressure ratio and \(T_{\text{H1}}\) are the most important parameter and should be checked carefully for the optimum performance of the cycle. These conclusions mentioned here may provide some important
theoretical instructions for the optimal design and operation of an irreversible modified complex Brayton Cycle.

Based on the basic principle of a complex Brayton cycle, it is also important to note the fact that the intercooling and reheat pressure ratios are always larger than unity and less than the cycle pressure ratio, i.e.:

\[ 1 < R_{pi} < R_p \quad \text{and} \quad 1 < R_{ph} < R_p \quad \ldots \quad (22) \]

According to Eq. (21) and Figs (2 and 3), we also can give the optimal criteria for the cycle pressure ratio based on the optimized and optimal performance parameters. Using Eqs (15)-(17) and (18)-(22), we can also give the optimum criteria for other cycle parameters such as the various state points temperature, heat transfer rates to and from the cycle and so on. The criteria will be helpful for engineers to optimally design and operate an irreversible modified complex Brayton cycle, for a typical set of operating parameters.

The physical meaning and explanations about the results obtained so far can be discussed in different ways, based on different reasons attached with the system, such as, the economics, energy consumption, irreversibility, and so on. Based on Fig. 1 and Eq. (22), we can see that if cycle pressure ratio tends to unity, the heat will flow directly from the source to the sink, so no real cycle is performed. Again, if cycle pressure ratio tends to the reservoirs temperature ratio \( \left( T_{in}/T_{is} \right)^{(p-h^o)} \), the heat transfer rates to and from the cycle tends to zero, again no real cycle is performed. Thus at the above mentioned limits, the significant cycle cannot be formed, so the power output and hence, the efficiency tends to zero, which is not the case in real practice. Thus, there are some optimal values of cycle pressure ratio (and hence for the intercooling and reheat ratios) between unity and \( \left( T_{in}/T_{is} \right)^{(p-h^o)} \), at which the cycle attains the optimum performance for a typical set of operating parameters.

Again as the inlet temperature \( T_{in} \) of the converging combustion chamber increases, the overall cycle performance goes down but the utility of the reheat exists while it is reverse otherwise. From Fig. 1, we can see that if \( T_{in} \) is much higher or nearly equal to \( T_{in} \), then the cycle temperature at state point 5 (\( T_5 \))
attains a higher value and the input energy during process 5-6 decreases as a result, the cycle performance goes down. Again, if \( T_{H1} \) is lower then \( T_5 \) is also lower and the cycle gains more energy during process 5-6 and hence, the cycle performance enhances.

The effects of various heat capacitance rates on the performance parameters are shown Figs (4-5). It is found that for \( C_k > 1.00 \text{ kW/K} \) and \( C_w < 1.00 \text{ kW/K} \) the optimal objective function \([ (aF_m)_{\eta} ]\) is an increasing function of the intercooling pressure ratio. Except these cases, the optimal and optimized objective function and efficiency first increase and then decrease as the intercooling pressure ratio is increased. Again, all the performance parameters enhance as the heat capacitance rates of the external fluids is increased, while it is found to be reverse in the case of working fluid heat capacitance rate.

Since, the higher are the heat capacitance rates of the external fluids, the lesser will be the temperature difference between the cycle and the external reservoirs and hence, lesser will be the external irreversibility associated with the cycle, as a result, the better performance of the cycle. Similarly, as heat capacitance rate of the working fluid is increased, the temperature differences between the cycle and the external reservoirs decreases and hence, the external irreversibility associated with the cycle again decreases. But the power consumption due to higher mass flow rate of the working fluid (as the specific heat does not change significantly) and hence, the internal dissipation (due to different reasons) enhances, as a result, the performance of the cycle goes down.

### 3.1 Objective function versus power output

The variation of the objective function against the power is shown in Fig. 6. It is found that the objective function first increases, attains its maximum point and then decreases as the power is increased. Thus, the maximum point of the objective function and the maximum point of the power exist but at a different point. Also there exists a point at which the objective function and the power attain equal values called as the point of intersection. Except the point of intersection, there are two values of the objective function for a single value of power output. One belongs to higher value of the objective function while the other belongs to the lower value of the objective function. Thus, the optimum operating

#### Table 1 — Optimized and optimal objective function and efficiency against the reheat pressure ratio for different values of \( T_{H1} \), while other parameters are same as given in the text

<table>
<thead>
<tr>
<th>( R_{ph} )</th>
<th>( T_{H1} = 1200\text{K} )</th>
<th>&amp;</th>
<th>( T_{H1} = 1350\text{K} )</th>
<th>&amp;</th>
<th>( T_{H1} = 1500\text{K} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \eta_{opt} )</td>
<td>( aF_{m1} )</td>
<td>( \eta_{i,2d} )</td>
<td>( aF_{opt} )</td>
<td>( \eta_{opt} )</td>
<td>( aF_{m1} )</td>
</tr>
<tr>
<td>1.00</td>
<td>0.702</td>
<td>0.657</td>
<td>0.670</td>
<td>0.723</td>
<td>0.658</td>
</tr>
<tr>
<td>1.15</td>
<td>0.700</td>
<td>0.657</td>
<td>0.669</td>
<td>0.722</td>
<td>0.658</td>
</tr>
<tr>
<td>1.30</td>
<td>0.699</td>
<td>0.657</td>
<td>0.668</td>
<td>0.722</td>
<td>0.657</td>
</tr>
<tr>
<td>1.45</td>
<td>0.697</td>
<td>0.656</td>
<td>0.666</td>
<td>0.721</td>
<td>0.656</td>
</tr>
<tr>
<td>1.60</td>
<td>0.695</td>
<td>0.655</td>
<td>0.665</td>
<td>0.720</td>
<td>0.653</td>
</tr>
<tr>
<td>1.75</td>
<td>0.693</td>
<td>0.655</td>
<td>0.663</td>
<td>0.719</td>
<td>0.651</td>
</tr>
<tr>
<td>1.90</td>
<td>0.692</td>
<td>0.654</td>
<td>0.662</td>
<td>0.718</td>
<td>0.649</td>
</tr>
<tr>
<td>2.05</td>
<td>0.690</td>
<td>0.653</td>
<td>0.660</td>
<td>0.716</td>
<td>0.646</td>
</tr>
<tr>
<td>2.20</td>
<td>0.688</td>
<td>0.652</td>
<td>0.658</td>
<td>0.715</td>
<td>0.643</td>
</tr>
<tr>
<td>2.35</td>
<td>0.686</td>
<td>0.651</td>
<td>0.657</td>
<td>0.714</td>
<td>0.641</td>
</tr>
<tr>
<td>2.50</td>
<td>0.684</td>
<td>0.650</td>
<td>0.655</td>
<td>0.713</td>
<td>0.638</td>
</tr>
<tr>
<td>2.65</td>
<td>0.683</td>
<td>0.649</td>
<td>0.654</td>
<td>0.712</td>
<td>0.635</td>
</tr>
<tr>
<td>2.80</td>
<td>0.681</td>
<td>0.648</td>
<td>0.652</td>
<td>0.710</td>
<td>0.633</td>
</tr>
<tr>
<td>2.95</td>
<td>0.679</td>
<td>0.647</td>
<td>0.650</td>
<td>0.709</td>
<td>0.630</td>
</tr>
<tr>
<td>3.10</td>
<td>0.677</td>
<td>0.646</td>
<td>0.649</td>
<td>0.708</td>
<td>0.628</td>
</tr>
<tr>
<td>3.25</td>
<td>0.676</td>
<td>0.645</td>
<td>0.647</td>
<td>0.707</td>
<td>0.625</td>
</tr>
<tr>
<td>3.40</td>
<td>0.674</td>
<td>0.644</td>
<td>0.646</td>
<td>0.706</td>
<td>0.623</td>
</tr>
<tr>
<td>3.55</td>
<td>0.672</td>
<td>0.644</td>
<td>0.644</td>
<td>0.705</td>
<td>0.621</td>
</tr>
<tr>
<td>3.70</td>
<td>0.671</td>
<td>0.643</td>
<td>0.643</td>
<td>0.703</td>
<td>0.618</td>
</tr>
<tr>
<td>3.85</td>
<td>0.669</td>
<td>0.642</td>
<td>0.641</td>
<td>0.702</td>
<td>0.616</td>
</tr>
<tr>
<td>4.00</td>
<td>0.667</td>
<td>0.641</td>
<td>0.640</td>
<td>0.701</td>
<td>0.614</td>
</tr>
</tbody>
</table>
region should be chosen carefully in order to attain the maximum performance of the cycle from the point of view of economics as well as from the point of view of thermodynamics.

So far we have given some optimal criteria of an irreversible modified complex Brayton cycle under maximum economic conditions. It is also important to note that the present cycle model is general and the results obtained by earlier researchers\textsuperscript{1-7,8,12,13-18} are the special cases of this cycle model. For example, if $R_{\text{pi}} = R_{\text{ph}} = 1.0$, and $\eta_C = \eta_T < 1.0$, are chosen, the results obtained in this paper are identical to those obtained in Refs (1-7). If $R_{\text{pi}} = R_{\text{ph}} = 1.0$ and $\eta_C = \eta_T < 1.0$, are chosen, the results are identical to those obtained in Refs (8-10, 18-20). Again, if $R_{\text{pi}} = R_{\text{ph}} = 1.0$ and $\eta_C = \eta_T \leq 1.0$, are chosen, the results are identical to those obtained in Refs (12-15) and so on.

4 Conclusions

An irreversible modified complex Brayton cycle model including external and internal irreversibilities for the finite heat capacities of external reservoirs has been studied in detail. The economic function, power output and efficiency are optimized with respect to the cycle temperatures, intercooling and reheat pressure ratios and the optimum operating parameters are calculated for different cases. It is found that there exist the optimal values of the intercooling, reheat and cycle pressure ratios as well as the turbine outlet temperature at which it attains the maximum performance. All the performance parameters change as any of the cycle parameters is changed. The conclusions can be summarized in the following points.

• There are optimal values of the intercooling pressure ratio at which the optimized and optimal performance parameters attain their maximum values. Also the optimized performance parameters are found to be higher then the corresponding optimal parameters for the same set of operating conditions. But it is found to be reverse in the case of the intercooling pressure ratio. In other words, the optimal values of the intercooling pressure ratio corresponding to the optimized performance parameters are found to be lower than those of the intercooling pressure ratio corresponding to the optimal performance parameters.

• Also there are optimal values of the reheat pressure ratio at which the optimized and optimal power output attain their maximum values for all values of $T_{\text{HI}}$. But the objective function and efficiency are found to be decreasing function of the reheat pressure ratio for lower values of $T_{\text{HI}}$. On the other hand, for higher values of $T_{\text{HI}}$, there exist the optimal values of the reheat pressure ratio at which the objective function and efficiency attain their maximum values. Again all the performance parameters go down as $T_{\text{HI}}$ is increased. Hence, the utility of the reheat process is a matter of careful consideration and should be studied in deep before making any decision in this regard. The optimal values of intercooling reheat and cycle pressure ratios are different for different performance parameters and change as any of the cycle parameters is changed.

• The optimized and optimal objective function and efficiency are found to be increasing functions of the heat capacitance rates of the external reservoirs while it is found reverse in the case of the working fluid heat capacitance rate. Also the effects of these heat capacitance rates are found to be more in the case of the optimal performance parameters than those of the optimized performance parameters. Again, the effect of working fluid heat capacitance rate is found to be more effective than those of the other heat capacitance rates on all the performance parameters, for the same set of operating conditions.

The results obtained in this cycle model will be useful to understand and improve the design and performance of an irreversible modified complex Brayton cycle. It is also important to note that the results obtained by earlier researchers and available in the literature are the special cases of the present cycle model.

References

5 Tyagi S K, Kaushik S C & Tyagi B K, Thermodynamic evaluation regenerative closed cycle Brayton heat engine with isothermal heat addition, NREC-2000, Nov 30 to Dec 2, 2000 at IIT, Bombay (India), pp 419-424
The different parameters used so far are as below:

\[ x_i = C_{i,\text{ini}}^2 \left[ \frac{E_i}{W} \right] (i = 1, 2, 3), \]
\[ y_j = C_{i,\text{ini}}^3 \left[ \frac{E_i}{W} \right] (j = 1, 2), a' = (1 + (R_k^2 - 1) \eta_{c,1}), \]
\[ b' = (1 - \eta_{c,1} - (R_k^2 - 1) \eta_{c,1}), R_{p_1} = P_2 / P_1, R_{p_2} = P_3 / P_1, \]
\[ R_{p_3} = P_4 / P_1, k = 1 - 1 / \gamma, a_1 = \varepsilon_R (1 - \eta_{c,1}), \]
\[ b_1 = (1 - \varepsilon_R) (1 - \eta_{c,1}), c_1 = \eta_T c_1, a_2 = a' (1 - \eta_{c,1}), a_1, \]
\[ b_2 = a'(1 - \eta_{c,1}) b_1, c_2 = a'(1 - \eta_{c,1}) c_1 + y_1 T_1 c_1, \]
\[ a_3 = (1 - \eta_{c,2}) a_2 + \eta c_2, b_3 = (1 - \eta_{c,2}) b_2, c_3 = (1 - \eta_{c,2}) c_2, \]
\[ a_4 = (1 - \varepsilon_R) (1 - x_1), b_4 = \varepsilon_R (1 - x_1), c_4 = x_1 T_{H1}, \]
\[ a_5 = b' (1 - x_1) a_4, b_5 = b' (1 - x_1) b_4, \]
\[ c_5 = b' (1 - x_1) c_4 + x_3 T_{H5}, a_6 = (1 - \eta_{T2}^{-1}) a_5, \]
\[ b_6 = (1 - \eta_{T2}^{-1}) b_5, c_6 = (1 - \eta_{T2}^{-1}) c_5, \]
\[ A = a_6 a_6 - a a_2 a_5, B = a_7 T_4 + b_7, \]
\[ C = a_8 T_4^2 + b_8 T_4 + c_7, \]
\[ a_7 = a_9 b_9 + a_9 b_9 + \alpha(a_9 b_9 + a_9 b_9), \]
\[ b_7 = a_9 c_9 - a(a_9 c_9 + a_9 c_9), \]
\[ a_8 = b_9 c_9 - \alpha(b_9 c_9 + a_9 c_9), b_8 = b_9 c_9 + b_9 c_9 + \alpha(b_9 c_9 + b_9 c_9), \]
\[ c_7 = c_9 c_9 - \alpha(c_9 c_9 + a_9 c_9), \]
\[ K_i = C_w \left[ x_i (1 - \varepsilon_R) + y_2 x_2 + x_2 y_2 b' a_4 \right], \]
\[ a_9 = C_w \left[ x_i (1 - \varepsilon_R) + y_2 x_2 + x_2 y_2 b' a_4 \right], \]
\[ b_9 = C_w \left[ x_i (1 - \varepsilon_R) + y_2 x_2 + x_2 y_2 b' a_4 \right], \]
\[ c_9 = C_w \left[ x_i (1 - \varepsilon_R) + y_2 x_2 + x_2 y_2 b' a_4 \right] \]
Nomenclature

\[ A \] = Area (m²)
\[ aF \] = Dimensionless objective function
\[ aF^* \] = Dimensionless optimal objective function
\[ aF^*_{\text{opt}} \] = Dimensionless optimized objective function
\[ C \] = Heat Capacitance rates (kW/K)
\[ m \] = Mass flow rate (kg/s)
\[ M \] = Mach number
\[ N \] = Number of heat transfer units
\[ P \] = Power (kW)
\[ P^* \] = Dimensionless power
\[ P^*_{\text{opt}} \] = Dimensionless optimized power
\[ R_p \] = Cycle pressure ratio
\[ R_{pi} \] = Intercooling pressure ratio
\[ R_{ph} \] = Reheat pressure ratio
\[ R_{pt} \] = Isothermal pressure drop
\[ Q \] = Heat transfer rates (kW)
\[ R_0 \] = Gas constant (kJ/kmol-K)
\[ S \] = Isentropic
\[ T \] = Temperature (K)
\[ U \] = Overall heat transfer coefficient (kW/m²/K)
\[ V \] = Velocity (m/s)
\[ V_s \] = Velocity of sound (m/s)
\[ J \] = \( k, R \)
\[ k \] = H1, H2, H3, L1, L2
\[ 1-9 \] = State points

Greek Letters

\[ \eta \] = Efficiency
\[ \eta_{\text{opt}} \] = Optimized efficiency
\[ \eta_{\text{m}} \] = Optimum efficiency
\[ \varepsilon \] = Effectiveness
\[ \alpha \] = \( \left( \frac{R_p R_{pi}}{R_{ph}} \right)^{1/\gamma} \)
\[ \gamma \] = Specific heat ratio

Subscripts

\[ aF^*_{\text{opt}} \] = Corresponding to the maximum point of the optimized objective function
\[ aF^*_{\text{opt}} \] = Corresponding to the maximum point of the optimized objective function
\[ \eta_{\text{m}} \] = Corresponding to the maximum point of the optimum efficiency
\[ \eta_{\text{opt}} \] = Corresponding to the maximum point of the optimized efficiency
\[ P_{\text{m}} \] = Corresponding to the maximum point of the optimal power
\[ P_{\text{opt}} \] = Corresponding to the maximum point of the optimized power
\[ C \] = Compressor
\[ H \] = Hot-side/heat-source
\[ i \] = Intercooler-side/Intercooling
\[ h \] = Re-heater/reheat-side
\[ L \] = Heat-sink/cold-side
\[ \text{max} \] = Maximum
\[ m \] = Optimum
\[ \text{opt} \] = Optimized
\[ R \] = Regenerator
\[ S \] = Ideal/isentropic
\[ T \] = Turbine
\[ W \] = Working fluid