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Determining the relation between effective coupling constant and quark mass in the parton model using massive nucleonic quarks in hypercentral potential

A A Rajabi

Physics Department, Shahrood University of Technology, PO Box 36155-316 Shahrood, Islamic Republic of Iran

Email: A.A.rajabi@shahrood.ac.ir

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The reliable values of effective strong coupling constants (α_s) for hadrons from various reactions have been found by investigating experimentally. So far, it has been proved that the α_s is not a fixed quantity. In this paper, we show that how the value of α_s depends on the constituent quark masses in hadrons. The effective values of α_s for different quark masses can be determined by using short-range attractive hyper-Coulombic potential, which would be in agreement with $\ln(q^2/m^2)$.

Keywords: Hypercentral, Three-body force, Valence quark, Coupling constant, Quark mass, CQM-Jacobi coordinates, H O potential

IPC Code: G01N15/00

1 Introduction

Constituent Quarks Model (CQM) has been recently widely used for description of the internal structure of baryons[1]. Based on the model used, the current quark masses are small (~ 4 to 10 MeV) while the constituent quark masses[2] are large (~ 100 ~ 350 MeV). In a non-relativistic quark model (or naive quark model: NQM), these values are 340 MeV approximately. In this paper, an intriguing connection between constituent quark masses and the effective strong coupling constant (α_s) has been investigating. These two parameters are directly related to each other. The internal quark motion is described by the Jacobi coordinates[3] ρ and λ. In order to describe the three-quark dynamics, it is convenient to introduce the hyperspherical coordinates, which are obtained by substituting the absolute values of ρ and λ in $x=\sqrt{\rho^2+\lambda^2}$ where x is the hyper radius. If it is assumed that the confining potential is hypercentral and hence depends only on x, there are 3 hypercentral potentials in this model, which lead to analytical solutions based on a suitable ansatz[4,5].

First, the six-dimension harmonic oscillator (h.o) potential, which has a two-body character, turns out to be exactly hypercentral since:

$$V_{h0}=\sum_{i<j}\frac{1}{2}k(r_i-r_j)^2=\frac{3}{2}kx^2=ax^2 \qquad \ldots(1)$$

The second one is the six-dimensional hyper Coulomb potential[3,4] which is attractive for small separations, originating from the colour charge: $V_{hyc}(x)=\frac{k\alpha_s}{x}=\frac{-c}{x}$ while at large separations a linear term gives rise to quark confinement[6] $V_{con}(x)=bx$ However, there have been some interesting attempts to interpolate[7-12] between V_{hyc} and V_{con}. In this case, all results for the three quarks problem can be obtained in closed analytic form and are thus rather transparent.

By determining α_s in our model, it is concluded that there is a reasonable consistency between the calculated values and the experimental results (Table 1).

Table 1— Comparing quark masses and the effective strong coupling constant (α_s) by author and experiment

Nucleon	Model by authors	Experiment (*)
P m_q	$(100\le m_u\le 313)$ MeV	(~100~350) MeV
α_s	$0.112\le(\alpha_s)^u\le 1.13$	$0.1\le\alpha_s\le 1$
Λ m_q	$(100\le m_u\le 313)$ MeV $(140\le m_s\le 460)$ MeV	(100~340) MeV (140 ~ 460) MeV
α_s	$0.125\le(\alpha_s)^u\le 1.120$ $0.127\le(\alpha_s)^s\le 1.120$	$0.1\le\alpha_s\le 1$

(*) The values are not directly measured, but inferred from experiment.

2 Hypercentral Relativistic Wave Function for Three Quarks in a Nucleon

The constituent quark model based on a hypercentral approach takes into account three body force effects and the standard two-body potential contributions.

Let's represent the quark wave function satisfying the Dirac equation by $\psi(\vec{x})$, so that:

$$[\gamma_0 \varepsilon + i\vec{\gamma}.\vec{\nabla} - (m + U(x))]\psi(\vec{x}) = 0 \quad \ldots(2)$$

The hypercentral potentials, which lead to analytical solution in our model, would be

$$U(x) = \frac{1}{2}(1 + e\gamma_0)A(x) \quad \ldots(3)$$

The parameter e can take any value[13-16]. In this investigation, it is taken as one. This case is important because it leads to an exact SU (2) symmetry, and hence to spin-orbit doublet degeneracy[12,16]. This is also studied by Bell and Ruegg[17].

Hence, from Eqs (1-3), the potential can be taken as

$$A(x) = ax^2 + bx - \frac{c}{x}. \quad \ldots(4)$$

This potential has interesting properties and yields good physical results. The solution of Dirac equation can be worked out analytically. The quark potential, $U(x)$ in Eq. (3) is assumed to depend on the hyper radius x only. The Dirac equation may transform in various ways under a Lorentz transformation. The form is in common use for scalar hypercentral potential $(U_0(x))$ and vector hypercentral potential $(V_0(x))$ are often taken as follows:

$$-i\vec{\alpha}.\vec{\nabla}\psi(x) + \beta[m + U_0(x)]\psi(x) + V_0(x)\psi(x) = \varepsilon\psi(x) \quad \ldots(5)$$

From Eqs (2,3,5)

$$V_0(x) = U_0(x) = \frac{1}{2}A(x). \quad \ldots(6)$$

The solution given in Eq. (5) reduces to:

$$\psi_{jj_3}(r) = \begin{bmatrix} \phi \\ \chi \end{bmatrix} = N \begin{bmatrix} g_\gamma(x)\, Y_{il}^{j_3}(x) \\ if_\gamma(x) Y_{il'}^{j'_3}(x) \end{bmatrix}. \quad \ldots(7)$$

Now, we have to solve these coupled equations.

The scalar potential $U_0(x)$ is bracketed with the mass and the vector potential $V_0(x)$ goes with the energy (ε) in the Dirac equation.

The eigenspinor of Eq. (5) denoted by $\psi_{jj_3}(x)$ is rewritten as:

$$\begin{aligned}(\sigma.P)\chi + (m + U_0(x) + V_0(x))\varphi = \varepsilon\varphi \\ (\sigma.P)\varphi - (m + U_0(x) - V_0(x))\chi = \varepsilon\chi\end{aligned} \quad \ldots(8)$$

Now, we combine these two equations for Dirac upper component, and from Eqs (4-6) we have:

$$\frac{P^2 g(x)}{m+\varepsilon} + (m - \varepsilon + A(x))g(x) = 0 \quad \ldots(9)$$

The internal quark motion is usually described by means of the Jacobi relative coordinates as given in Eq. (2). After separating the common motion, the P^2 operator of a quark, in the $3q$ system becomes ($\hbar = c = 1$) (Eq. 2)

$$P^2 = -(\nabla_\rho^2 + \nabla_\lambda^2) = -(\frac{d^2}{dx^2} + \frac{5}{x}\frac{d}{dx} + \frac{L^2(\Omega)}{x^2}) \quad \ldots(10)$$

Hence

$$\begin{aligned}g''_\gamma(x) + \frac{5}{x}g'(x) + \frac{L^2 g(x)}{x^2} + \\ (\varepsilon^2 - m^2 - (\varepsilon + m)A(x))g(x) = 0\end{aligned} \quad \ldots(11)$$

with A(x) given in Eq. (4), where $L^2(\Omega) = -\gamma(\gamma+4)$ is the grand orbital operator and γ is the grand angular quantum number given by $\gamma = 2n + l_\rho + l_\lambda$. Here, l_ρ and l_λ are the angular momenta associated with the ρ and λ variables, respectively and n is a non-negative integer number (n = 0, 1, 2).

Using the method used by Znojil[16,17], we find the upper component $g_\gamma(x)$ of the Dirac hypercentral spinor. Now, for the eigenfunction $g_\gamma(x)$, we make an ansatz (Refs 4, 18, 19, 20, 21).

$$g_\gamma(x) = \exp(h(x)) \quad \ldots(12)$$

with $h(x)$ as:

$$h(x) = -\frac{1}{2}\alpha x^2 + \beta x + \delta \ln x \quad \ldots(13)$$

This implies

$$g''_\gamma(x) + \frac{5}{x}g'_\gamma(x) = \left[h''(x) + h'^2(x) + \frac{5h'}{x}\right]g_\gamma(x) \quad \ldots(14)$$

Then our purpose is to find the fraction of the power of (x) to the one on the left hand side. A comparison of Eq (14) and Eq (11) yield, α, β and γ:

$$\begin{cases} \alpha = \sqrt{a_1} = \sqrt{a(\varepsilon+m)} \\ \beta = -\left[2\alpha(3+\gamma)-(\varepsilon^2-m^2)^{\frac{1}{2}}\right] \\ \beta = \dfrac{-c(\varepsilon+m)}{2\gamma+5} = \dfrac{-b(\varepsilon+m)}{2\alpha} \\ \delta = \gamma, -\gamma-4 \end{cases} \quad \ldots(15)$$

Taking $\delta = \gamma$ leads to the wave function, which is well behaved at the origin.

The constraints between the potential parameters a, b and c are as follows:

$$b = \frac{2c\sqrt{a}(\varepsilon+m)}{2\gamma+5} \quad \ldots(16)$$

$$\varepsilon^2 - m^2 = 2[a(\varepsilon+m)]^{\frac{1}{2}}(3+\gamma) - \frac{c^2(\varepsilon+m)^2}{(2\gamma+5)^2} \quad \ldots(17)$$

We try to solve this problem in the presence of the center of mass correction, where we have

$$M' = M + E_{cm} = 3\varepsilon \quad \ldots(18)$$

Here, M' is the corrected nucleon mass with center of mass energy ε_{cm}.

For three quarks with energy ε and mass m, from Eq. (9), we have:

$$\left[\sum_{i=1}^{3}\frac{d^2}{dr_i^2} + \sum_{i=1}^{3}(\varepsilon+m)A(r_i) - 3(\varepsilon^2+m^2)\right]\prod_{i=1}^{3}\varphi_i = 0 \quad \ldots(19)$$

From Jacobin coordinates, this separates into three equations for ρ, λ and R where one of them determine the center of mass

$$\vec{R} = \frac{1}{3}(\vec{r}_1 + \vec{r}_2 + \vec{r}_3), \quad \ldots(20)$$

and the other two equations, for ρ and λ, have been combined as a hypercentral equations which were discussed previously.

Let $\eta = \sqrt{3}R$. Then

$$\left[-\frac{d^2}{d\eta^2} + A_1(\eta) - (\varepsilon^2 - m^2)\right]\varphi(\eta) = 0 \quad \ldots(21)$$

Now, it is obvious that the center of mass energy is:

$$E_{cm} = (\varepsilon^2 - m^2)^{\frac{1}{2}} \quad \ldots(22)$$

From Eq. (18), and using Bogoliuabv's assumption: $M' = 3\varepsilon$, and assuming $\zeta = \frac{m}{\varepsilon}$, then

$$M' = M + E_{cm} = 3\varepsilon = \frac{M}{3-\sqrt{1-\xi^2}} \quad \ldots(23)$$

From $m = \varepsilon\xi$, Eq. (23) leads to:

$$\varepsilon + m = \frac{(1+\xi)M}{3(3-\sqrt{1-\xi^2})} \quad \ldots(24)$$

and

$$(\varepsilon - m) = \frac{(1-\xi)}{3(3-\sqrt{1-\xi^2})} \quad \ldots(25)$$

And Eqs (16) and (17), with the center of mass correction, would become:

$$b = \frac{2c\sqrt{a}(1+\xi)M}{3(2\gamma+5)(3-\sqrt{1-\xi^2})} \quad \ldots(26)$$

$$c = (2\gamma+5)\left[2a^{\frac{1}{2}}\left(\frac{3-(3\sqrt{1-\xi^2})}{(1-\xi)M}\right)^{\frac{3}{2}}(3+\gamma) - \frac{(1-\xi)}{(1+\xi)}\right]^{\frac{1}{2}} \quad \ldots(27)$$

Eq. (27) is a relationship between energy and the nucleon mass M and the parameter a and c.

Eqs (12, 13, 15, 24, 25) are used to find $g_\gamma(x)$ of the nucleon with mass M and the parameter ξ as follows:

$$g_\gamma(x) = x^\gamma e^{E_{cm}x}\exp\left[-\frac{1}{2}\alpha x^2 - 2\alpha(3+\gamma)x\right] \quad \ldots(28)$$

The lower component $f_\gamma(x)$ of the Dirac hypercentral spinor can be found from Eqs (9) and (28). The normalized spin $\frac{1}{2}$ positive parity solution of the quark under standard hyperspherical potential given in Eq. (4) is introduced by the following form:

$$\psi_\gamma^0(x) = \begin{bmatrix} g_\gamma(x) \\ \dfrac{-i\vec{\sigma}.\hat{x}(3-\sqrt{1-\xi^2})}{M(1+\xi)}\left(g'_\gamma(x) - \dfrac{J^2 - L^2 - \frac{3}{4}}{x^2}g_\gamma(x)\right) \end{bmatrix} \quad \ldots(29)$$

Table 2—Effective coupling constant α_s and effective colour charge (c) and strength of confinement (b) and *h o* potential (*a*) indicating for proton with (M = 938), $\gamma = 0$

ξ	$m_q\,(f_m^{-1})$	(α_s)	**c**	$a(fm^{-3})$	**b** (fm^{-2})
0.387	0.613	0.112	0.075	0.055	0.068
0.412	0.65	0.321	0.214	0.044	0.065
0.437	0.692	0.399	0.332	0.033	0.062
0.487	0.770	0.541	0.366	0.019	0.035
0.537	0.849	0.699	0.466	0.012	0.023
0.587	0.928	0.718	0.479	0.005	0.015
0.637	1.007	0.815	0.543	0.003	0.011
0.687	1.009	0.832	0.554	0.001	0.009
0.737	1.165	0.94	0.627	0.42×10^{-3}	0.006
0.787	1.244	1.004	0.669	0.13×10^{-3}	0.005
0.837	1.323	1.102	0.736	0.03×10^{-4}	0.003
0.887	1.402	1.121	0.747	0.56×10^{-5}	0.002
0.937	1.481	1.125	0.750	0.36×10^{-6}	0.0009
0.987	1.560	1.132	0.755	0.28×10^{-6}	0.2×10^{-4}
1	1.581	1.135	0.757	0.19×10^{-6}	0.12×10^{-4}

Table 3 — Indicating proton, $\gamma = 1$

ξ	$m_q\,(fm^{-1})$	$(\alpha_s)_u$	**c**	$a(fm^{-3})$	**b** (fm^{-2})
0.542	0.86	0.145	0.096	0.080	0.118
0.567	0.900	0.201	0.142	0.062	0.112
0.592	0.937	0.354	0.118	0.046	0.107
0.642	1.018	0.498	0.332	0.026	0.103
0.692	1.094	0.552	0.368	0.014	0.09
0.742	1.170	0.609	0.406	0.007	0.082
0.792	1.259	0.732	0.488	0.003	0.075
0.842	1.332	0.752	0.501	0.001	0.072
0.952	1.510	0.910	0.606	0.49×10^{-3}	0.065
1	1.581	0.934	0.623	0.15×10^{-3}	0.036

If the nucleon mass M and the phenomenological quark mass $100mev \le m_q \le 350mev$ and as input of the Eq. (29) contain unknown parameter *a* only. In order to find this parameter for different values of $(\gamma = 0, 1, 2, ...)$ another constraint is introduced, $\frac{g_A}{g_V} = 1.26$, as an input.

$$\frac{g_A}{g_V} = \frac{5}{3}(1-2\delta) = \frac{5}{3}(1-2 < \psi_\gamma | l_z | \psi_\gamma >) \qquad \text{...(30)}$$

This has been performed for the first time by Golwich[22]. Let first assume $\gamma = 0$. The potential parameter from Eqs (30, 27, 26) can be extract for proton with $M = 938\,mev$ and $m_q = 100\,mev$ as follows: $a = 0.055\ fm^{-3}$, $b = 0.68\ fm^{-2}$, $c = 0.075\ fm^{-3}$.

The effective α_s were chosen and shown in Tables 2 and 3. Calculations for different values of γ = 1, 2, … can be repeated in the same way.

Similarly, calculations made for proton can be repeated for other nucleons such as Λ, *N*, …

Taking Λ as another example, the quark masses of s and u in Λ can be calculated from $M'_\Lambda = 2\varepsilon_u + \varepsilon_s$.

Here, the difference between *u-quark* and *s-quark* masses must be considered using the ratio $\frac{m_s}{m_u} = 1.46$ of chiral symmetry[23].

Parameters *a*, *b* and *c* for quarks in Λ were obtained using the method, which are shown in Tables 3 and 4.

3 Effective Strong Coupling Constant α_s for *u* and *s* quark

In section (2), the parameter *c* i.e. the strength of the hyper central Coulomb potential with regard to nucleon mass was calculated using the evaluated results, and the effective strong coupling constant (α_s) was then found. The short hyper Coulomb like term in our hypercentral potential given in Eq. (4) can be written as $\left(\frac{k\alpha_s}{r}\right)$ where *k* is the colour factor, which is $-\frac{2}{3}$ for the nucleon assuming 3 flavours for quarks. In Tables 2 and 3, α_s has been found for quarks. In Tables 2 and 3 α_s has been found for different values of the γ and quark masses. If we compare the hyper Coulomb potential parameter $-\frac{2}{3}\alpha_s$, then the values for proton and Λ quarks were calculated and presented in Tables 2 and 3. The results fall in the following ranges for α_s.

For *P* quarks, Tables 2 and 3

$$\begin{cases} 0.112 \le (\alpha_s)_u \le 1.130 \\ 0.145 \le (\alpha_s)_u \le 0.934 \end{cases}$$

For Λ quarks, Tables 4 and 5 are as follows:

$$\begin{cases} 0.125 \le (\alpha_s)_u \le 1.120 \\ 0.127 \le (\alpha_s)_s \le 1.120 \end{cases}$$

The results of Tables 4 and 5 show that α_s for strange quarks change in the same range as for the non-strange quarks. When we fit the above formula for hadron masses the best result is obtained when the coupling constant α_s is taken as a function of the effective quark masses, i.e phenomenologically α_s terns not to be a constant but a slowly varying function. The best fit for this function is in logarithm in agreement with $\ln\frac{q^2}{m^2}$ of the QCD coupling if the mass parameter is identified with the effective quark masses. This is not a surprising although in the Dirac Equation. We have assumed α_s to be a constant; actually it is in reality a slowly varying function of the mass scale. The mass scale being small such as quark mass scale is a reflective of long or low energy

Table 4 — For *u* quarks in Λ ($M_\Lambda = 1115.6$ MeV), $\gamma = 0$

ξ	m_u (fm^{-1})	(α_s)	c	a (fm^{-3})	**b** (fm^{-2})
0.402	0.603	0.089	0.059	0.080	0.120
0.452	0.734	0.195	0.130	0.046	0.114
0.502	0.821	0.459	0.306	0.026	0.110
0.552	0.902	0.518	0.354	0.171	0.094
0.602	0.983	0.612	0.408	0.007	0.0806
0.652	1.064	0.632	0.421	0.003	0.079
0.702	1.145	0.754	0.497	0.002	0.065
0.752	1.226	0.815	0.543	5×10^{-4}	0.057
0.802	1.307	0.895	0.597	2×10^{-4}	0.046
0.852	1.393	0.938	0.625	3.8×10^{-5}	0.028
0.902	1.469	0.968	0.645	4.4×10^{-6}	0.016
0.952	1.512	1.012	0.675	2.19×10^{-7}	0.005
1	1.631	1.111	0.741	1.41×10^{-7}	0.003

Table 5 — For *s* quarks in Λ (M_Λ = 1115.6 MeV), $\gamma = 0$

ξ	$m_s\,(fm^{-1})$	$(\alpha_s)_s$	**c**	$a(fm^{-3})$	**b** (fm^{-2})
0.402	0.959	0.091	0.060	0.071	0.119
0.452	1.108	0.254	0.169	0.052	0.115
0.502	1.196	0.498	0.332	0.029	0.112
0.552	1.317	0.612	0.408	0.184	0.096
0.602	1.434	0.749	0.499	0.009	0.092
0.652	1.555	0.807	0.538	0.004	0.079
0.702	1.672	0.871	0.580	0.003	0.068
0.752	1.748	0.933	0.622	6×10^{-4}	0.062
0.802	1.915	1.072	0.714	3×10^{-4}	0.056
0.852	2.32	1.142	0.761	4.2×10^{-5}	0.036
0.902	2.153	1.165	0.776	4.8×10^{-6}	0.013
0.952	2.270	1.190	0.793	3.14×10^{-7}	0.009
1	2.386	1.195	0.796	3.82×10^{-7}	0.0002

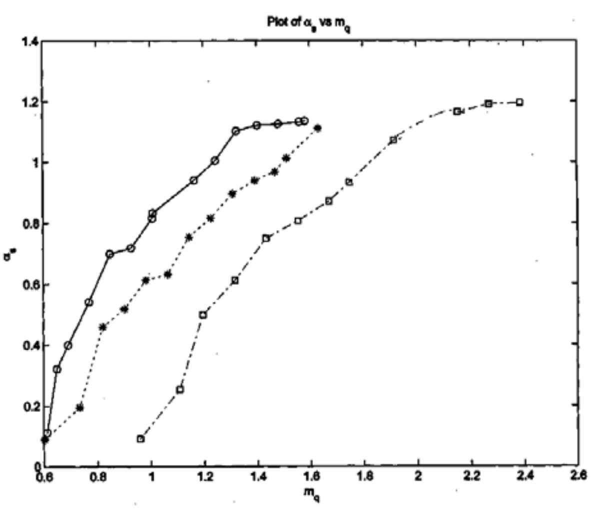

Fig. 1 — Variation of effective α_s as a function of quark masses that shows α_s is the same for each *u* and *s* flavor

behaviour of colour forces, which are effectively the binding force of the hadrons[24].

4 Conclusions

In our parton model, we have quarks and gluons, which arrange themselves into bound state, namely, hadrons. Here, we have considered α_s for hadrons. (Physical quantities usually depend on α_s and some mass parameter (*m*)). In our model, we have found that *m* is the constituent quark masses, which are in the range (~100~312 MeV) for proton. Hence, α_s explicitly and directly depend on them. For a constant q $(q > m)$, Fig. (1) shows that α_s as a function of m_q` agrees (well with $\alpha_s \sim g^2(q^2)\ln(\frac{q^2}{m^2})$.

This has non-trivial implications for the quark model and leads to further improvements. All this lead us to the fact that the quark masses may well depend on α_s (Fig. 1). The results of Tables 4 and 5, for Λ, show that α_s for strange quarks change in the same range as for the non-strange quarks.

In Tables (2-5), α_s of the proton and the Λ for different quark masses are presented.

This particular quark mass, scale within the same range, which often appears in the quark model phenomenologically. Using these Tables (2-5) and comparing α_s for *u* and *s* it is concluded that α_s is the same for each *u* and *s* flavour, therefore α_s is independent of quark flavours.

In Fig. 1, the variation of α_s as a function of quark masses is shown. These curves are established using the data of Tables (2-5). So far, there has been no direct experimental and theoretical evidences that show α_s vary with quark masses.

According to Tables (2-5), as the masses of the valence quarks increase the effect of Coulomb potential dominates and the effective h o potential decreases. In this case, the Coulomb like potential parameter, $c >>$ other potential parameters that agrees with the work of Ref. 5 that takes the potential of h o, confinement potentials and some other potentials as perturbation.

This model is also acceptable for all mesons with flavour SU (3). For mesons, we have antiquarks, and the vector potential has opposite sign. The antiparticle energy eigenvalues are also negative. Hence, the eigenspinor will have upper and lower component interchanged in as given in Ref. 24.

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