Many particle aspects of a quantum wire

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The electronic systems in one dimension have features quite distinct from those in higher dimensions. The one-dimensional Fermi surface consists of two discrete points, while in higher dimensions it is continuous. The low-energy particle-hole pairs’ spectrum, which arises from momentum and energy conservations for wave vector between zero and twice of Fermi wave vector does not exist in one dimension. In higher dimensions, this region of missing states is filled in. The Fourier transform of one-dimensional Coulomb potential diverges. This led to widespread belief that one-dimensional interacting electronic systems should exhibit non-Fermi liquid (known as Luttinger liquid) behaviour. However, there is still no unambiguous evidence both for the real existence or non-existence of Luttinger liquid behaviour in a realistic quantum wire. This talk briefly reviews key aspects of Fermi liquid approach and Tomonaga-Luttinger liquid approach for a one-dimensional system. In this paper, structure factor, pair distribution function and electron-electron scattering, which exhibit similarity between two approaches in low energy and momentum regime, is reviewed.

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1 Introduction

Quantum wire (QW) is a system where electron motion is size-quantized in two directions (say y and z axes) and it is free along third direction (x-axis). The single electron wave function, \( \psi(x, y, z) \) can be taken as:

\[
\psi(x, y, z) = f(y)f(z)\exp(ikx).
\]

Where, \( f(y) \) and \( f(z) \) are localized functions. The \( k \) is wave vector component along x-axis. Several forms have been suggested for \( f(y) \) and \( f(z) \). One of the simpler ways is to choose a \( \delta \)-function type confinement along z-axis and an infinite potential well type confinement along y-axis, where electron wave function vanishes at boundaries of the QW across y-axis, at \( y=\pm a/2 \), where \( a \) is the width of a QW. One can take:

\[
f(y)=\sqrt{2/\pi}\sin((n+1/2)\pi a)/(a+1/2),
\]

where \( n \) is an integer. The single particle energy eigenvalue for \( n \)th sub-band of such a QW can then be given by

\[
E_n = \frac{\hbar^2 k^2}{2m^*} + \varepsilon_n,
\]

where \( m^* \) is effective electron band mass and \( \varepsilon_n \) is energy of \( n \)th sub-band. The phase space of one-dimensional electron gas (1DEG), which exists in a QW, is severely restricted. This results into various uncommon properties of 1DEG. The Fermi surface of 1DEG consists of two discrete points (\( \pm k_F \)), while it is continuous in higher dimensions, as can be seen from Fig. 1(a). The non-existence of low-energy particle-hole pairs, which complies with momentum and energy conservation laws, for \( 0<k<2k_F \), as is shown in Fig. 1(b). In higher dimensions this region of missing states is filled in. The existing literature on QW deals with almost all aspects. Broadly speaking, two types of approaches have been adopted to perform theoretical investigations on QW. They are commonly known as Fermi liquid approach and Luttinger liquid approach, respectively.

2 Salient features of Fermi Liquid and Tomonaga-Luttinger Liquid

The low-temperature electronic properties of simple metals can be qualitatively understood in terms of the simple Sommerfeld model. This model treats the conduction electrons as non-interacting fermions. Typical results are a specific heat linear in temperature and temperature-dependent spin susceptibility, which qualitatively agree with experimental measurements. Landau’s Fermi-liquid (FL) theory rests on the assumption of quasi-particles which are in a one-to-one correspondence to non-interacting fermions treated within Sommerfeld model. This leads to a linear specific heat and a constant spin susceptibility but involves renormalized
quantities like the effective mass and quasi-particle interaction parameters, which are difficult to calculate microscopically. The consistency of the approach was shown using perturbation theory to infinite order and more recently by renormalization group techniques\(^1\).

A model for 1DEG was developed by Shin'ichiro Tomonaga and Joaquin Luttinger\(^2\) and it has been advanced by many other theoreticians. In a pioneering paper Tomonaga\(^2\) treated the case of a two-body interaction which is long ranged in real space and showed that the low-energy excitations of the non-interacting as well as the interacting system can be described in terms of non-interacting bosons. The idea used was that a long-range interaction in real space is short ranged in momentum space, only particles and holes in the vicinity of the Fermi points are involved in the interacting ground state and the states with low excitation energy. To obtain his results, Tomonaga\(^2\) linearized the energy dispersion around the two Fermi points, \(\pm k_F\). Later on, Luttinger\(^3\) used a model with strictly linear energy dispersions and extended it to incorporate unphysical states of excited positrons, as is shown in Fig. 2. He presented the exact result for the mean occupation number \(n_k\). In the two intermediate regions around the two Fermi points at \(\pm k_F\), with particle-hole pairs present, the single particle energy dispersion, \(\varepsilon_k\), linearized in order to apply Bloch’s sound wave method, given by:

\[
\varepsilon_k = \varepsilon_F \pm v_F (k \mp k_F),
\]

where \(\varepsilon_F\) and \(v_F\) are Fermi energy and Fermi velocity, respectively. The single particle spectrum and the particle-hole spectrum for free Fermi gas within Luttinger model is exhibited in Fig. 2(a & b), respectively\(^4\). Elementary excitations in Tomonaga-Luttinger (TL) model are quanta of charge density and spin density. The characteristic properties of a Luttinger liquid (LL) are; (i) a conserved charge, (ii) a characteristic Kohn anomaly.

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\(\varepsilon_F\) and \(v_F\) in the text.

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**Fig. 1**—(a) Single particle spectrum of free electron gas and (b) particle-hole pair spectrum.

**Fig. 2**—Diagrams that correspond to Fig. 1 are drawn in Luttinger model.
wave vector, $2k_F$, varying linearly with charge density, (iii) persistent currents at low temperatures, quantized in units that carry momentum $2k_F$ (iv) a spectrum of collective density wave elementary excitations, with a dispersion linear in $k$ at long wavelengths that defines a sound velocity (v) power-law decay of correlation functions at temperature, $T=0$, with coupling-strength-dependent. The exponents of the anomalous power-law, decay of various correlation functions are determined by the anomalous dimension, calculated explicitly for the TL model. The zero temperature $n_k$ in TL liquids does not has the usual discontinuity at $k=k_F$ relating to the existence of Fermi surface, but has a power law behaviour $n(k-k_F)\sim 1-2k_F/k\sigma^{1/2}$, where the so-called Luttinger exponent, $\sigma$ is related to the interaction strength and it is non-universal\textsuperscript{1,5}. Also, the single-particle density of states near $k_F$ is expected to follow a power-law; $\nu(E) \propto E^{\alpha}$, which leads to a power-law behaviour of the electron tunneling. The transport properties of a Luttinger liquid along the chain direction depend on the scattering mechanism and the band filling. The $dc$ and frequency-dependent transport characteristics of a LL significantly deviate from those of a FL. Drastic effect of a single impurity on the transport properties of a LL are theoretically expected. In particular, in the case of repulsive interactions the conductance is predicted to vanish as a power of the temperature. Apart from QW, the LL approach can possibly be applied to organic conductors like the Bechgaard salts as well as inorganic materials and carbon nanotubes. Though TL liquid model has become the state-of-the-art to describe correlated 1DEG, still there is no unambiguous evidence both for the real existence or non-existence of TL liquid behaviour in a QW. The experimental data obtained on QW structures from measurements on photocuminescence, inelastic light scattering (Raman scattering), far infrared spectroscopy, capacitance studies, etc, exhibits no obvious sign of TL liquid behaviour.\textsuperscript{6,7} For instance, luminescence experiments show large Fermi edge singularities, which mean that $n_k$, is not continuous. This is contrary to the TL model analysis that asks for a continuous $n_k$ through $k_F$. On the other hand, experimental results can be successfully explained on the basis of normal 1D-FL theory. The anomaly between the TL theoretical projections and experiments can be ascribed to two apparent reasons: (i) The TL model is based on artificial assumptions that two completely linear dispersing bands of electrons with an infinite bandwidth, are populated by infinite density of electrons and a short ranged interaction. (ii) A realistic QW is much different from the theoretical zero temperature ideal 1DEG, on which the TL model is based, because of finite temperature, finite size and scattering which may serve to stabilize FL behaviour in semiconductor QW. Also, the theoretical prediction that the presence of any disorder in 1DEG localizes all electronic states (Disorder-induced Anderson localization) becomes redundant because in the state-of-the-art semiconductor QW, prepared using modulation doping techniques, the typical localization lengths exceeds the physical length of QW and therefore the electrons may be considered to be extended for all practical purposes. There are still arguments in favour of the ordinary FL theory to be valid in presence of disorder. A realistic QW can never be free from electron-impurity/disorder scattering, which causes a rapid decay of excitations.

3 Structure Factor and Pair Distribution Function

The electron-electron interactions play very important role in determination of properties involving many body aspects such structure factor, pair distribution function and electron-electron scattering rate of a low dimensional system. The phenomenon of screening and the many body aspects that involve screening are the simplest and most important manifestations of electron-electron interactions and the screening of the electron-electron interaction is one of the most important effects that make possible the use of free electron or quasiparticle model in describing the properties of 1DEG (Ref. 8). The pair distribution function, $g(r)$ is an average distribution of electrons about any electron, in a solid and it is defined as the probability that another particle is at position, $r$ if there is already one at $r=0$. The structure factor, $S(q)$ can be defined as\textsuperscript{1,9};

$$S(q) = \frac{-1}{n^2} \int d\omega \text{Im}[\chi(q,\omega)]$$

\text{... (1)}

The $g(r)$ can be calculated from $S(q)$ through Fourier transform as follows\textsuperscript{1,9};

$$g(r) = 1 + \frac{1}{n^2} \int dq \cos(qr) [S(q) - 1]$$

\text{... (2)}
The quantity of central importance in studying $s(q)$ and $g(r)$ is $\chi(q,\omega)$, which is defined by

$$\chi(q,\omega)=\frac{\chi_0(q,\omega)}{\varepsilon(q,\omega)}$$  \quad \ldots \quad (3)$$

where dielectric function, $\varepsilon(q,\omega)$ is defined as

$$\varepsilon(q,\omega)=1-V^{\text{eff}}(q)\chi_0(q,\omega)$$  \quad \ldots \quad (4)$$

$\chi_0(q,\omega)$ is the 1D irreducible polarizability function and the effective electron-electron interaction, $V^{\text{eff}}(q)$ is given by

$$V^{\text{eff}}(q)=V^0(q)[1-G(q)]$$  \quad \ldots \quad (5)$$

where, $V^0(q)$ is bare coulomb potential and $G(q)$ is the static local field correction (LFC) term. The $V^0(q)$ for QW can be given by

$$V^0(q) = \frac{2e}{m^*} \int_0^\infty \frac{dt}{t} \frac{H(\lambda)}{\lambda}$$  \quad \ldots \quad (6)$$

where, $\lambda=\sqrt{q^2+k_F^2}$. The structural function, $H(\lambda)$ is the expectation value of $\exp(-\lambda[y-y'])$ with the use of localized functions $f(y)$ and $f(y')$. There have been several ways to calculate $G(q)$ for 1DEG. The Hubbard approximation is the simplest way to incorporate the static LFC. The $G(q)$ within Hubbard approximation is given by

$$G(q) = \frac{1}{2} \frac{V^0(\sqrt{q^2+k_F^2})}{V^0(q)}$$  \quad \ldots \quad (7)$$

The $V^0(\sqrt{q^2+k_F^2})$ is obtained from $V^0(q)$ on replacing $q$ by $\sqrt{q^2+k_F^2}$.

The $\chi_0(q,\omega)$ for the case of negligibly small amount of disorder (impurity) and Fermi energy much smaller than the inter-subband energy difference can be calculated in single subband approximation. The real and imaginary parts of $\chi_0(q,\omega)$ are given by

$$\chi_{01}(q,\omega) = \frac{m^*}{\pi q} \log \left[ \frac{\omega^2-(E_q-qv_F)^2}{\omega^2-(E_q+qv_F)^2} \right]$$  \quad \ldots \quad (8a)$$

and

$$\chi_{02}(q,\omega) = \frac{m^*}{\hbar^2 q}, \quad \text{when} \quad \left| E_q-qv_F \right| \leq \omega \leq (E_q+qv_F)$$

$$=0 \quad \text{otherwise} \quad \ldots \quad (8b)$$

where, $E_q=\hbar^2 q^2/2m^*$

The above formalism has been applied to a GaAs-QW, which is parameterized in terms of $m^*=0.068 m_e$ and $\varepsilon_0=12.0$ (Ref. 9). An analytical solution of Eq. (1) with the use of Eqs (5-8) is not possible. However, the analytical results that retain the essential physics could be obtained within what is known as plasma pole approximation (PPA). Equation (1) has been solved numerically with the use of Eqs (5-8) as well as analytically within PPA. The PPA ignores the particle-hole excitations and it assigns whole spectral weight, dictated by the $f$-sum rule, to an effective collective plasma excitation, which is assumed to be a real pole of the response function. The phase space restriction on particle-hole excitations increases the spectral weight of the plasma excitation over a wide range of wave vectors and the collective plasma excitation plays a more prominent role in a 1DEG, in contrast to 2D and 3D systems. The PPA indeed works extremely well for QW structures. The long-wavelength IRPA plasmon dispersion has been shown to be identical to the exact result in TL model. The real part of dielectric function in PPA can be written as:

$$\varepsilon_1(q,\omega) \cong 1-\omega_p^2(q)/\omega^2$$  \quad \ldots \quad (9)$$

where,

$$\omega_p^2(q)=\left( E_q+qv_F \right)^2+4E_qqv_F/(A-1)$$  \quad \ldots \quad (10)$$

with $A=\exp[\pi q \hbar^2 V^{\text{eff}}(q)]$. To show explicit connection between FL and TL approaches, $\omega_p(q)$ can be expanded to get

$$\omega_p^2(q)=\left[ t_F/q \left( 1+2V^{\text{eff}}(q)/\pi t_F \right)^{1/2} \right]^2$$  \quad \ldots \quad (11)$$

which is linear in $q$. The elementary excitations used in TL also exhibit linear dispersion. The computed $\omega_p(q)$ from Eq. (10) is plotted in Fig. 3. The
integration over $\omega$ in Eq. (1) after using PPA value of $e(q,\omega)$ gives

$$S(z) = \frac{2z^2 + bh_0^2(2+t)}{t^2} \frac{\log \left( \frac{((y_1^2+y_0^2-y_1r)(y_1^2+y_0^2+y_1r)}{(y_1^2-y_0^2-y_1r)(y_1^2+y_0^2+y_2r)} \right)}{2\sqrt{\Delta^3}} \left[ \tan^{-1} \left( \frac{2y_3+r}{\sqrt{\Delta}} \right) + \tan^{-1} \left( \frac{2y_3-r}{\sqrt{\Delta}} \right) \right] \cdots \text{(12)}$$

$z = q/k_F$, $b = m^*V_{\text{eff}}/zh_F^2$, $y_2 = z(z+2)$, $y_1 = z(z-2)$,

$t = \sqrt{1+b^2}$, $r = \sqrt{2y_0(t+1)}$, $\Delta = \sqrt{4y_0^2t-r^2}$,

$y_0^2 = \frac{8z^3}{A-1} + (z^2+2z)^2$

Computed results on $S(z)$ for a GaAs-QW for $k_F=5\times10^5$ cm$^{-1}$ from Eq. (12) and from Eq. (1) with the use of Eqs (5-8) are displayed in Fig. 4. As can be noted from the Fig. 4, simple analytical result using PPA shows very good agreement with the detailed numerical result, for $z \leq 3/2$. It is because of two reasons; (i) in a QW, the prohibition of particle-hole excitations from a large portion of the low energy phase space increases the dominance of plasma excitations and (ii) oscillator strength of the plasma excitations in a QW extends well into the range of large wave vector and it decreases slowly with wave vector. We further notice from Fig. 4, $S(q)$ declines on reducing the width of QW, at all values of $q$, however increase in $S(q)$ is not proportionate to $a$ and it tends to saturate for larger values of $a$, at a given $q$-value. $S(q)$ also declines on increasing the carrier density, for given values of $q$ and $a$. The prior reported calculations on $S(q)$ had been made by solving numerically the simultaneous equations for $S(q)$, $G(q)$ and $\chi(q,\omega)$, as their definitions are interdependent in Singwi-Tosi-Land-Sjölander (STLS) approximation. However, in our approach $S(q)$, $G(q)$ and $\chi(q,\omega)$ are defined and computed independently of each other. This greatly simplified computation task and also enabled us to provide analytical results on $S(q)$ within PPA. Our results show very good agreement with the prior reported work.

Computed $g(r)$ from Eq. (2) with the use of; (a) numerically computed $S(q)$ for two values of $a$ (20 and 10) nm and (b) analytical $S(q)$ for $a=20$ nm at $k_F=5\times10^5$ cm$^{-1}$ is plotted as a function of $k_Fr$ in Fig. 5. As is seen from Fig. 5, $g(r)$ using PPA-$S(q)$ shows good agreement with that using numerically computed $S(q)$. Therefore it can be concluded that many body effects in a QW are well
described by simple analytical result within PPA. For small $q$, PPA approach is almost identical to TL approach and exact solutions are also possible within FL approach, in low energy excitation regime. The FL approach, which is favoured by experimental findings seems to be a better approach to QW. We further notice that our computed results on $g(r)$ suggest that on decreasing the width of QW for fixed $k_F$, the average distribution of electrons about an electron enhances at larger $r$-values and it becomes negative at smaller values of $r$. The IRPA fails to describe $g(r)$ when $g(r)$ becomes negative. This has also been observed in the calculations of $g(r)$ using STLS approximation\textsuperscript{12}. We also computed $g(r)$ as function of $r$ for different values of $k_F$ and found that an increase in carrier density leads to the reduction at larger $r$-values and enhancement at smaller values of $r$, in the average distribution of electrons about an electron in a QW of fixed $a$.

4 Electron-electron Scattering Rate

Many-body theoretical description of a system of electrons involves an important concept of quasi-particle lifetime. The life of an electron in a given state is determined by the scattering rate, which is largely contributed by electron-electron scattering in
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The case of an interacting 1DEG. The quasiparticle description of electrons in an interacting system is based on the assumption that the inverse lifetime of an electronic excitation is small as compared to its excitation energy \( \epsilon \) above Fermi level. Quasiparticle description may breakdown in 1D and quasi-1D systems, in which a strong electron-electron interaction results into a rapid decay of electronic excitations. However, there has been a viewpoint that the scattering of electrons by random impurity/disorder potential could restore the Fermi-surface and a Fermi-liquid quasiparticle description can be applied to these systems. The disorder effects are specific to a confined system that exhibits the strong deviations from bulk behaviour, depending on the ratio of system size and the disorder correlation length. To find answer to these questions, we performed a theoretical study of inelastic scattering rate for electron-electron scattering, \( 1/\tau_{ee} \) in presence of disorder potential in QW. Another aim has been to see whether or not a satisfactory description of electron-electron scattering in 1D and quasi-1D systems consisting of significant random impurity scattering centers can be provided beyond TL model. Apart from the construction of many-body theory, decay time of an electronic excitation plays an important role in a number of physical processes such as damping of collective excitations, dephasing of localized electrons, optical transitions, Hall effect, etc. Within RPA, the \( 1/\tau_{ee} \) is given by

\[
\frac{1}{\tau_{ee}(\epsilon,\tau)} = \frac{2}{\pi \nu} \int \omega d\omega \int \frac{dq}{(2\pi)} |V(q,\omega)|^2 \left[ \text{Re} \left\{ \nu \xi(q,\omega) \right\} \right]^2 \left[ 1 - \nu \xi(q,\omega) \right] \quad \cdots (13)
\]

The screened electron-electron interaction is defined as;

\[
V(q,\omega) = \frac{V^0(q)}{1 + V^0(q)\nu(q,\omega)} \quad \cdots (14)
\]

\( \xi(q,\omega) \) is a response function that involves; (i) electron Green function averaged over impurity configuration, (ii) electron-impurity scattering rate \( (1/\tau) \) and (iii) density of states \( (\nu) \). In case of a QW, major contributions to \( 1/\tau_{ee}(\epsilon,\tau) \), when \( \epsilon \lesssim \epsilon_F \), come from virtual plasmons.

The \( 1/\tau_{ee}(\epsilon,\tau) \) has been computed numerically after evaluating correct values of \( V(q,\omega) \) and \( \xi(q,\omega) \) in diffusive limit for GaAs-QW. Analytical results on \( 1/\tau_{ee}(\epsilon,\tau) \) have also been obtained for

\[
\text{Fig 6—Dimensionless electron-electron scattering rate as a function of } x = \epsilon/\epsilon_F \text{ for a quantum wire at } \epsilon_F = 5.6 \text{ meV, } \epsilon_0 = 12.5 \text{ and } k_F = 10^6 \text{ cm}^{-1}. \text{ Analytical results from equation (15); curve-A when } a = 50 \text{ Å and } h/\tau_F = 0.1, \text{ curve-D when } a = 50 \text{ Å and } h/\tau_F = 0.4. \text{ Numerically computed results from Eq. (13); curve-B for } a = 20 \text{ Å and } h/\tau_F = 0.1, \text{ curve-C when } a = 50 \text{ Å and } h/\tau_F = 0.1, \text{ curve-E for } a = 50 \text{ Å and } h/\tau_F = 0.4.
\]
the case of both $\varepsilon$ and $1/\tau << \varepsilon_F$ and $V(q,\omega)$ is fairly independent of $q$ and $\omega$. A constant effective electron-electron interaction potential and a linear $q$-dependent energy difference between two electronic states have been used in obtaining analytical results. These simplifying approximations to obtain analytical results make our approach similar to the treatment within TL-model. The simple analytical results are:

$$\frac{\hbar}{\tau_{ee}(x,S)\varepsilon_F} = \frac{x}{\pi} + \frac{S^2}{2\pi \sqrt{S^2 + x^2}} \ln \left[ \frac{\sqrt{S^2 + x^2} - x}{\sqrt{S^2 + x^2} + x} \right]$$

... (15)

$s = \hbar/\tau_{ee}\varepsilon_F$, $x = \varepsilon/\varepsilon_F$

Numerically computed $1/\tau_{ee}(\varepsilon,\tau)$ from Eq. (13) and that from Eq. (15) are plotted as a function of $\varepsilon/\varepsilon_F$ in Fig. 6. As can be seen from the figure, the agreement between analytical and numerically computed results is very good when $x << 1$ and it is fairly good for rest of the values of $x$. It can also be noted from the Fig. 6 that the inclusion of $q$-dependence in screened potential brings down the magnitude and changes the $x$-dependence of $\hbar/\tau_{ee}(x,S)\varepsilon_F$, more obviously at larger values of $x$ ($>0.2$) when $S$ is small ($<0.2$). The computed curves D and E varies as $\sqrt{x}$-dependent type behaviour supporting the earlier findings\textsuperscript{14}. Further, the computed curve-E can almost be fitted to $\hbar/\tau_{ee}(x,S)\varepsilon_F = \alpha \sqrt{x} + \beta x$, where $\alpha$ and $\beta$ depend on $h/\tau$ and $k_F$. It can also be noticed that the magnitude of $\hbar/\tau_{ee}(x,S)\varepsilon_F$ declines on increasing $a$, at all values of $x$ and $S$.

Numerically computed $1/\tau_{ee}(\varepsilon,\tau)$ from Eq. (13) and that from Eq. (15) is plotted as function of $S$ in Fig. 7. Analytical results shows very good agreement with numerically computed results when $x << 1$. The agreement between simple analytical results and the detailed numerical results is good even at higher values of $S$ and $x$. For smaller values of $S (<0.1)$ numerically computed $\hbar/\tau_{ee}(x,S)\varepsilon_F$, declines on decreasing $S$, the range of $S$ in which $\hbar/\tau_{ee}(x,S)\varepsilon_F$ increases on increasing $S$ narrows down on reducing $x$. In this region of $x$-$S$, RPA fails to provide a satisfactory description of electron-electron scattering in a quantum wire. The electron-electron scattering in presence of significant electron-impurity scattering in a QW can very well be described within a quasiparticle FL treatment by taking into account correct diffusive behaviour of the electron gas.

5 Conclusions

The results presented in this paper clearly suggest that FL approach using RPA or IRPA gives satisfactory description of many particle aspects of a
QW consisting impurity potential. Further, for small wave vector and energy limiting cases of FL approach, one obtains exact analytical results and theoretical treatments in FL approach become almost analogous to those of TL approach. The plasmon energy varies linearly with wave vector, in low energy and wave vector regime. In this energy and wave vector regime, plasmons are nothing but the quasi-particles used in TL model.

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