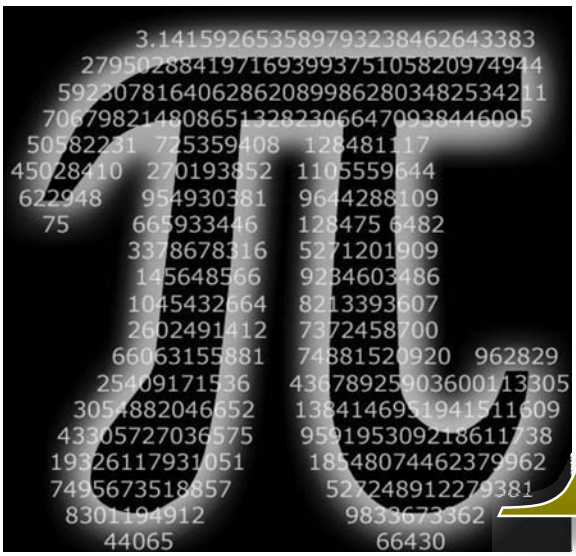
RAMDAS IYER & BHARAT BHUSHAN SRIVASTAVA



Unending Journey of Pi

Throughout the history of mathematics, there have been many efforts to determine Pi more accurately and to understand its nature; fascination with the number has even carried over into non-mathematical culture. Many countries in the world even celebrate Pi Day and Pi Approximation Day to commemorate the significance of the value of Pi.

Pi has been known for almost 4000 years—but even if we calculate the number of seconds in those 4000 years and calculate *pi* to that number of places, we would still only be approximating its actual value. It is very much rational to talk about this irrational number, which means that its value cannot be expressed exactly as a fraction m/n, where m and n are integers. Consequently, its decimal representation never ends or repeats. It is also a transcendental number, which implies, among other things, that no finite sequence of algebraic operations on integers (powers, roots, sums, etc.) can be equal to its value.

π (sometimes written pi) is a mathematical constant whose value is the ratio of any circle's circumference to its diameter; this is the same value as the ratio of a circle's area to the square of its radius. It is approximately equal to 3.14159 in the usual decimal notation. π is one of the most important mathematical and physical constants: many formulae from mathematics, science, and engineering involve π. It also appears in many different formulas that have nothing to do with circles.

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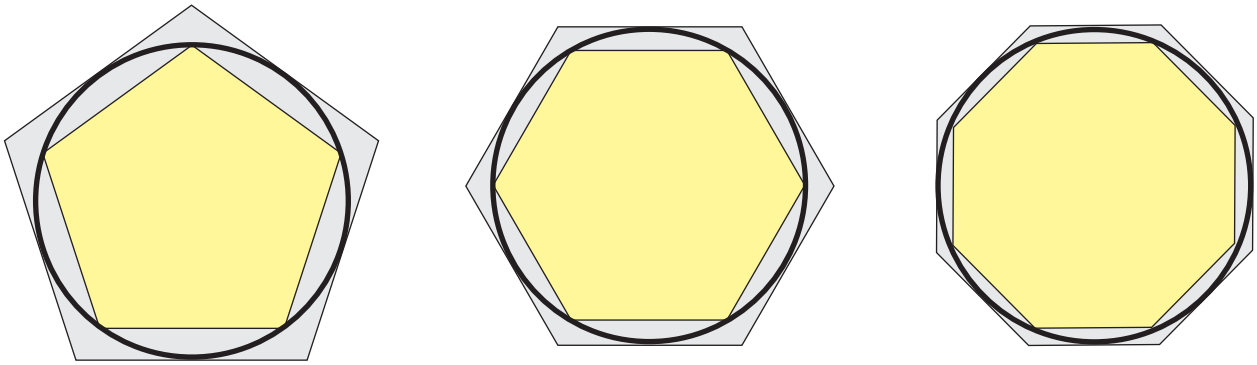
History of Pi

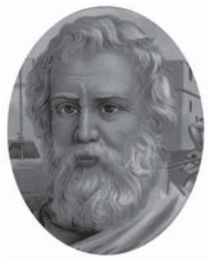
Mathematicians began using the Greek letter π in the 1700s. Introduced by William Jones in 1706, use of the symbol was popularized by Euler, who adopted it in 1737. Historians estimate that by 2000 B.C. humans had noticed that the ratio of circumference to diameter was the same for all circles. This discovery hinged on the idea of proportion – in this case humans noticed that if you double the distance "across" a circle, then you double the distance "around" it. In today's algebraic notation this implied the formula

$$\pi = \frac{\text{Circumference}}{\text{Diameter}}$$

where Pi is constant.

The significance of this discovery is clear. Circles are everywhere – in the sun, the moon, the pupils of our eyes, the most basic religious rituals and the earliest man-





Archimedes (left) approximated the area of a circle by using the Pythagorean Theorem to find the areas of two regular polygons (top)

made structures. Achieving a greater mathematical understanding of Pi would lead to scientific and technological advances that would further the development of civilization, as well as create some very interesting problems in pure mathematics.

But one problem remained – what was the numerical value of Pi?

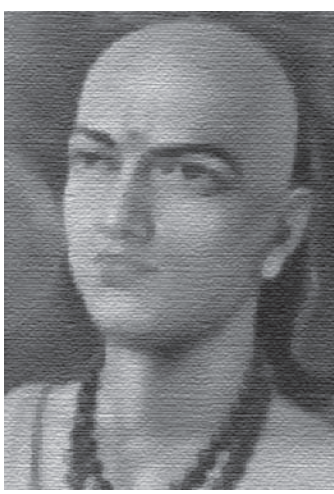
The ancient Babylonians calculated the area of a circle by taking three times the square of its radius, which gave a value of *pi* = 3. One Babylonian tablet (ca. 1900–1680 BC) indicates a value of 3.125 for *pi*, which is a closer approximation. In the Egyptian *Rhind Papyrus* (ca.1650 BC), there is evidence that the Egyptians calculated the area of a circle by a formula that gave the approximate value of 3.1605 for *pi*.

Euclid of Alexandria (325-265 BC) is the one who proved that the ratio of circumference over diameter is always the same, regardless of the size of the circle. He did it by inscribing regular polygons (i.e., an octagon or eight-sided figure) inside circles of different sizes. He was able to show that the perimeter of the polygon was proportional to the radius (which is half of the diameter), regardless of its size. He then increased the number of sides of the polygon, realizing that as he increased them the perimeter of the polygon got closer and closer to that of the circle. Therefore, he was able to prove that the perimeter of the circle, or circumference, is proportional to the radius and also to the diameter.

The ancient cultures mentioned above found their approximations by measurement. The first calculation of *pi* was done by Archimedes of Syracuse (287–212 BC), one of the greatest mathematicians of the ancient world. Archimedes approximated the area of a circle by using the Pythagorean Theorem to find the areas of two regular polygons: the polygon *inscribed within the circle* and the polygon *within which the circle was circumscribed*. Since the actual area of the circle lies between the areas of the inscribed and circumscribed polygons, the areas of the polygons gave upper and lower bounds for the area of the circle. Archimedes knew that he had not found the value of *pi* but only an approximation within those limits. In this way, Archimedes showed that *pi* is between 3 1/7 and 3 10/71.

A similar approach was used by Zu Chongzhi (429–501), a brilliant Chinese mathematician and astronomer. Zu Chongzhi would not have been familiar with Archimedes' method—but because his book has been lost, little is known of his work. He calculated the value of the ratio of the circumference of a circle to its diameter to be 355/113. To compute this accuracy for *pi*, he might have started with an inscribed regular *24,576-gon* and performed lengthy calculations involving hundreds of square roots carried out to nine decimal places.

Pi in India



Aryabhata (476–550 CE)

In India, we find its mention in the old Sanskrit text *Baudhayana Shulba Sutra* of the 6th century BCE that indicates this ratio as approximately equal to 3. Aryabhata (476–550 CE) was the first in the line of great mathematician-astronomers from the classical age of Indian mathematics and Indian astronomy. His most famous works are the *Aryabhatia* (499 CE, when he was 23 years old) and the *Arya-siddhanta*. Aryabhata worked on the approximation for Pi (π), and may have come to the conclusion that π is irrational. In the second part of the *Aryabhatiyam*, he writes:

> **"Add four to 100, multiply by eight, and then add 62,000. By this rule the circumference of a circle with a diameter of 20,000 can be approached."**

This implies that the ratio of the circumference to the diameter is ((4+100)×8+62000)/20000 = 3.1416, which is accurate to five significant figures. It is speculated that Aryabhata used the word *âsanna* (approaching), to mean that not only is this an approximation

Brahmagupta (598–668)

but that the value is incommensurable (or irrational). If this is correct, it is quite a sophisticated insight, because the irrationality of pi was proved in Europe only in 1761 by Lambert.

After *Aryabhatiya* was translated into Arabic (ca. 820 CE) this approximation was mentioned in Al Khwarizmi's book on algebra. Centuries later, in 825 CE Arab mathematician Mohammed Ibna Musa says that "This value has been given by the Hindus (Indians)".

Brahmagupta (598–668) was a great Indian mathematician and astronomer. Brahmagupta wrote important works on mathematics and astronomy. In particular, he wrote *Brahmasphutasiddhanta* (Correctly Established Doctrine of Brahma), in 628. The work was written in 25 chapters. In verse 40, he gives values of π, 12.40. The diameter and the square of the radius [each] multiplied by 3 are [respectively] the practical circumference and the area [of a circle]. The accurate [values] are the square-roots from the squares of those two multiplied by ten.

So, Brahmagupta uses 3 as a "practical" value of π, and square root of 10 as an "accurate" value of π.

Bhaskara (1114–1185), also known as Bhaskara II and Bhaskara Achârya ("Bhaskara the teacher") was an Indian mathematician and astronomer. He was born near Bijjada Bida (in present day Bijapur district in Karnataka). Bhaskara II wrote *Siddhanta Shiromani* at the age of 36 in 1150 AD. This colossal work is divided into four parts: *Lilawati*, *Beejaganit*, *Ganitadhyaya* and *Goladhyaya* and consists of about 1450 verses.

Each part of the book consists of a huge number of verses and can be considered as a separate book: Lilawati has 278, Beejaganit has 213, Ganitadhyaya has 451 and Goladhyaya has 501 verses. Bhaskara formulated simple ways of calculations from Arithmetic to Astronomy in this book. He wrote *Lilawati* in an excellent lucid and poetic language. It has been translated in various languages throughout the world. In *Lilawati*, he solved several problems on permutations and combinations and called the method as 'ankapaash'. He even gave an approximate value of pi as 22/7, which is 3.1416. He was even familiar with the concept of infinity and called it as 'khahar rashi', which means 'anant'.

Srinivasa Ramanujan (1887-1920) was one of India's greatest mathematical geniuses. He made substantial contributions to the analytical theory of numbers and worked on elliptic functions, continued fractions, and infinite series. He found several rapidly converging infinite series of π, which can compute 8 decimal places of π with each term in the series. Since the 1980s, his series have become the basis for the fastest algorithms currently used by Yasumasa Kanada and the Chudnovsky brothers to compute π.

Accurate Value

Since 4,000 years ago and up until this very day, people have been trying to get more and more accurate values for pi. Presently supercomputers are used to find its value with as many digits as possible. Recently a French computer scientist claimed to have set a new record in the calculation of the mathematical constant pi. Fabrice Bellard says he has calculated the value of pi to nearly 2.7 trillion digits, some 123 billion more than the previous record. Bellard used a desktop computer to perform the calculation, taking a total of 131 days to complete and also check the result. This version of pi takes over a terabyte of hard disk space to store. Previous records were established using supercomputers, but Bellard claims his method is 20 times more efficient.

The prior record of about 2.6 trillion digits, set in August 2009 by Daisuke Takahashi at the University of Tsukuba in Japan, took just 29 hours. However, that work employed a supercomputer 2,000 times faster and thousands of times more expensive than the desktop Bellard employed. These Herculean computations form part of a branch of mathematics known as arbitrary-precision arithmetic — simply put, knowing a given number to any amount of decimal places. It is hard to overstate just how long the currently determined pi is; reciting one number a second would take more than 49,000 years.

The calculation will stand as a monument of scientific trivia. The most fastidious engineer will never need more than the first few handful of those digits. Still, pi, the ratio of a circle's circumference to its diameter, is the longest known and most revered of all constants of nature, and lately there has been a surprising revival of interest in the act of computing it.

Timeline of efforts to estimate the approximate value of Pi

Person/People/Computer	Year	Value
Babylonians	~2000 B.C.	3 1/8
Egyptians	~2000 B.C.	(16/9)^2= 3.1605
Chinese	~1200 B.C.	3
Old Testament	~550 B.C.	3
Archimedes	~300 B.C.	proves 3 10/71<Pi<3 1/7uses 211875/67441=3.14163
Ptolemy	~200 A.D.	377/120=3.14166...
Chung Huing	~300 A.D.	sqrt(10)=3.16...
Wang Fau	263 A.D.	157/50=3.14
Tsu Chung-Chi	~500 A.D.	proves 3.1415926<Pi<3.1415929
Aryabhatta	~500	3.1416
Brahmagupta	~600	sqrt(10)
Fibonacci	1220	3.141818
Ludolph van Ceulen	1596	Calculates Pi to 35 decimal places
Machin	1706	100 decimal places
Lambert	1766	Proves Pi is irrational
Richter	1855	500 decimal places
Lindeman	1882	Proves Pi is transcendental
Ferguson	1947	808 decimal places
Pegasus Computer	1957	7,840 decimal places
IBM 7090	1961	100,000 decimal places
CDC 6600	1967	500,000 decimal places
CDC 7600	1973	1,001,250 decimal places
HITAC M-280H	1983	16,777,206 decimal places
William Gosper, Symbolics 3670	October 1985	17,526,200 decimal places
David H. Bailey, CRAY-2	January 1986	29,360,111 decimal places

Sometimes Pi Minute is also celebrated; this occurs on March 14 at 1:59 p.m. If Pi is truncated to seven decimal places, it becomes 3.1415926, making March 14 at 1:59:26 p.m., Pi Second (or sometimes March 14, 1592 at 6:53:58 a.m.). An upcoming Pi Day in 2015 will have the date 3-14-15 and a Pi Second of 9:26:53.589... a.m.

Pi is a mathematical constant whose value is the ratio of any circle's circumference to its diameter; this is the same value as the ratio of a circle's area to the square of its radius.

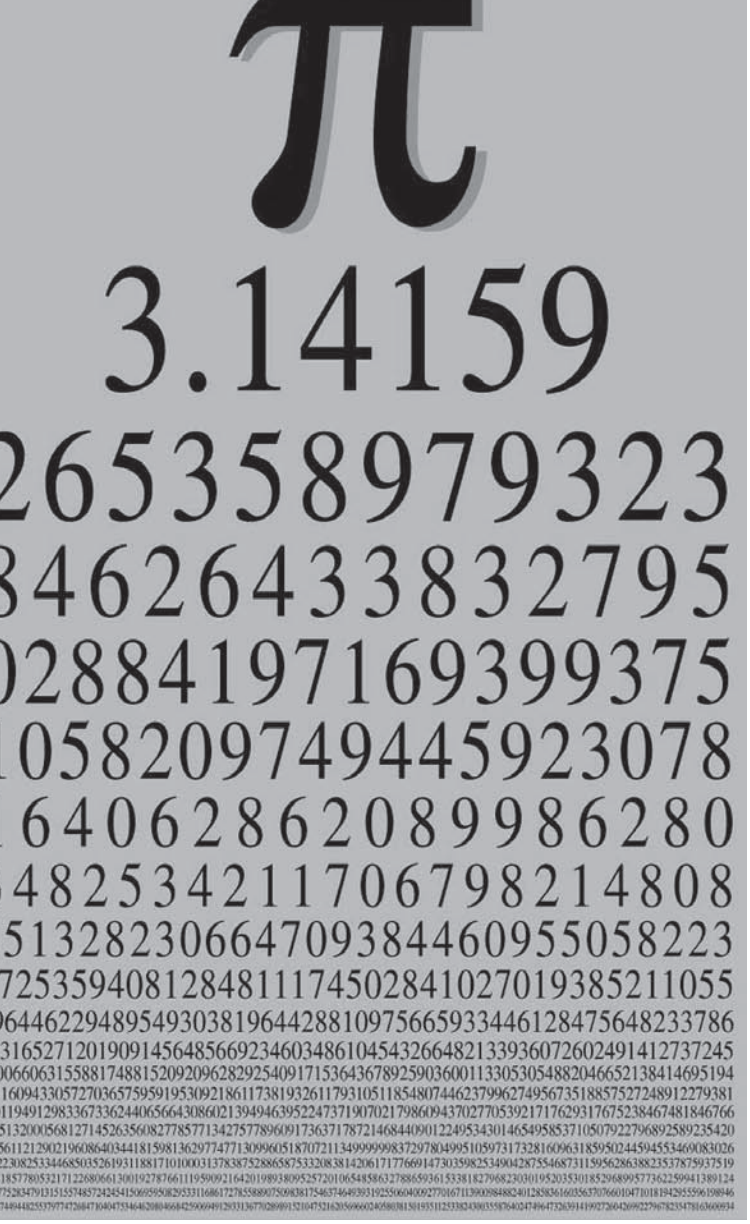
The calculation has become part of the fast-growing science of complexity theory, which is devoted to analyzing the difficulty of various problems and the efficiency of different problem-solving methods, or algorithms.

Modern algorithms for computing pi rely on sophisticated formulas, some of which have only recently been proved correct. The formulas use simple arithmetical operations – mostly multiplications and square roots – and produce closer and closer approximations that converge on pi with remarkable speed. Although the individual operations are simple, they quickly lead to very big numbers. One algorithm would reach 500-million-digit accuracy in just 14 steps, if anybody could actually compute it. In the latest record-setting calculations, supercomputers must manipulate gigantic numbers in their memories, multiplying 100 million digits at a time.

Memorizing Pi

Even long before computers have calculated π, memorizing a record number of digits became an obsession for some people. There are many ways to memorize π, including the use of "piems", which are poems that represent π in a way such that the length of each word (in letters) represents a digit. Here is an example of a piem, originally devised by Sir James Jeans: *How I need* (or: *want*) *a drink, alcoholic in nature* (or: *of course*), *after the heavy lectures* (or: *chapters*) *involving quantum mechanics*. Notice how the first word has 3 letters, the second word has 1, the third has 4, the fourth has 1, the fifth has 5, and so on.

Piems are related to the entire field of humorous yet serious study that involves the use of mnemonic techniques to remember the digits of π, known as piphilology. In other languages there are similar methods of memorization. Pi is known for turning up in all sorts

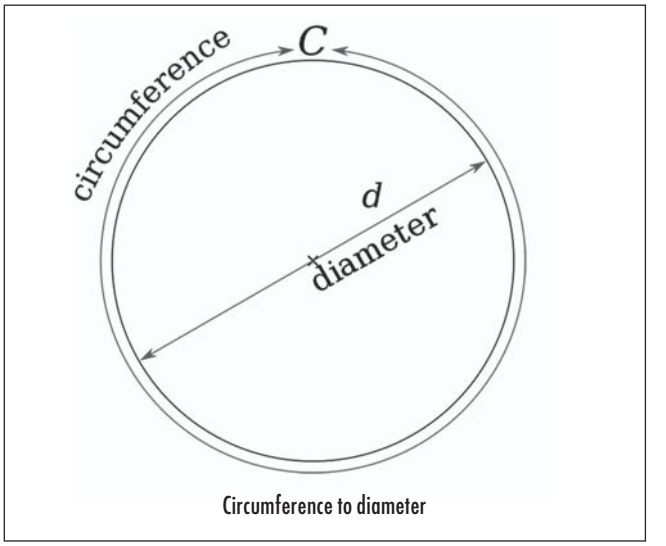


of scientific equations, including those describing the DNA double helix, a rainbow, ripples spreading from where a raindrop fell into water, waves, navigation and more.

In 2006, Akira Haraguchi, a retired Japanese engineer, claimed to have recited 100,000 decimal places. This, however, has yet to be verified by Guinness World Records. The Guinness-recognized record for remembered digits of π is 67,890 digits, held by Lu Chao, a 24-year-old graduate student from China. It took him 24 hours and 4 minutes to recite to the 67,890th decimal place of π without an error. Akira Haraguchi, a 59-year-old Japanese mental health counselor, managed to recite the number's first 83,431 decimal places, almost doubling the previous record held by his country man.

Celebrating Pi

Many countries in the world celebrate Pi Day and Pi Approximation Day to commemorate the significance of the value of Pi. The objective to celebrate the day is not only about exploring the beauty of this number, but also to make continuous rigorous efforts to understand the complexity for finer and finest approximation of the same.



Circumference to diameter

Pi Day is observed on March 14 (in the mm/dd date notation: 3/14); since 3, 1 and 4 are the first three digits of π. Pi Approximation Day is observed on July 22, because of Archimedes' popular approximation of π being 22/7. However, this may be considered misleading, as all cited dates are "approximation days" (as π is an irrational number) and 22/7 is actually a closer approximation of π than 3.14. Typically, March 14 is more popular for countries using the month/day format (22/7 being an impossible date in this format), and the 22nd of July is more popular for countries using the day/month format (since 3/14 and 31/4 are impossible dates in this format).

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The first Pi Day celebration was held at the San Francisco Exploratorium in 1988, with staff and public marching around one of its circular spaces, then consuming fruit pies. The museum has since added pizza pies to its Pi Day menu. The founder of Pi Day was Larry Shaw, a now-retired physicist at the Exploratorium who still helps out with the celebrations and in organizing many related activities for students and general public. The Massachusetts Institute of Technology often mails its acceptance letters to be delivered to prospective students on Pi Day.

In India, however, efforts to celebrate Pi Day are rarely seen.

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