Parametric study of induced draft counter flow rectangular cooling tower based on exergy analysis

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Received 20 May 2009; revised 19 January 2010; accepted 27 January 2010

This study presents development of a simple and efficient mathematical model, in which energy, draft and pressure equations are solved simultaneously. Model was validated with experimental data collected from an induced draft counter flow rectangular cooling tower of a refrigeration plant (capacity, 220 tonnes) in a dairy industry at Ambala (India). Model predictions and experimental data match satisfactorily. In a parametric study, wet bulb temperature of inlet air plays a significant role on air and water outlet temperatures, evaporation loss, fan power, thermal efficiency, exergy destruction and second law efficiency.

Keywords: Draft equation, Exergy analysis, Induced draft cooling tower, Parametric study

Introduction

A cooling tower reduces temperature of circulating water so that water may be reused in condensers and other heat exchange equipment. Walker et al.¹ proposed basic theory of cooling tower operation. Merkel² developed theory for thermal evaluation of cooling towers. Jaber & Webb³ developed equations necessary to apply e-NTU method directly to counter flow or cross flow cooling towers. Kloppers & Kroger⁴,⁵ investigated critical differences in heat and mass transfer analyses and performance evaluation of Merkel, e-NTU and Poppe methods for a certain fill material at different operating and ambient conditions. Kloppers & Kroger⁶,⁷ studied effect of Lewis factor on performance prediction of natural draft and mechanical draft wet cooling towers and proposed a new form of empirical equation that correlates fill loss coefficient data more effectively when compared to other forms of empirical equation commonly found. Naphon⁸ investigated heat transfer characteristics of cooling tower. Kachhwaha et al.⁹ carried out heat and mass transfer analysis of a counter flow wet cooling tower. Sutherland¹⁰ compared accurate analysis of mechanical draft counter flow cooling tower, including water loss by evaporation, with approximate common method based on enthalpy driving force (Merkel method) for wide range of inlet water and air conditions. Fisenko et al.¹¹ developed a new mathematical model of a mechanical draft cooling tower performance. Soylemez¹² carried out a thermo-hydraulic optimized performance analysis yielding simple algebraic formulation for estimating optimum performance point of counter current mechanical draft wet cooling towers using e-NTU method. Qureshi & Zubair¹³ modeled three zones of cooling tower (spray, packing and rain zones). Again, Qureshi and Zubair¹⁴ presented thermodynamic analysis of counter flow wet cooling towers and evaporative heat exchangers using both first and second laws of thermodynamics. Muangnoi et al.¹⁵ carried out an exergy analysis to indicate exergy and exergy destruction of water and air flowing through cooling tower. Muangnoi et al.¹⁶ studied effects of inlet relative humidity and inlet temperature on the performance of counter flow wet cooling tower based on exergy analysis. Limited information is available regarding simultaneous solution of heat and mass transfer formulations coupled with draft equation for fan power evaluation¹⁸.

This study presents numerical procedure for simultaneous solution of heat and mass transfer formulation incorporated with draft equation. Parametric study was carried out for wide range of inlet air conditions for induced draft counter flow rectangular cooling tower (IDRCT).

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Mathematical Model Formulation

Mathematical model for IDRCT (Fig. 1) consists of following three main equations:

Energy Equation
Amount of heat transferred to air stream from circulating water is expressed as
\[ q = m_w \cdot c_{pwm} \cdot (t_{wi} - t_{wo}) = m_a (i_{mas5} - i_{ma1}) \]  \hspace{1cm} (1)

where, \( i_{mas5} \), enthalpy of saturated air-vapour at 5; \( i_{ma1} \), enthalpy of air-vapour at inlet of IDRCT.

Draft Equation
For IDRCT (Fig. 1) while ignoring pressure differences due to gravity field, draft equation obtained by matching fan performance curve and flow characteristics is expressed as
\[ (K_{ilfi} + K_{rzfi} + K_{fsfi} + K_{wdfi} + K_{defi} + K_{ctfi} + K_{spfi}) \times \left( \frac{m_{av15}}{A_{tr}} \right)^2 / \left( 2 \rho_{av15} \right) = 0 \]  \hspace{1cm} (2)

Various loss coefficients in Eq. (2) are calculated using empirical equations\(^\text{17}\).

Pressure Equation
Pressure of air upstream of fan (\( p_{a5} \)) is expressed as
\[ p_{a5} = p_{a1} \left[ 1 - (0.009754(H_3 + L_{tr}/2)/t_{tr})^{1.5} + 0.622(w1)/(w1 + 0.622) \right] \times (K_{ilfi} + K_{rzfi} + K_{fsfi} + K_{wdfi} + K_{defi} + K_{ctfi}) \times \left( \frac{m_{av15}}{A_{tr}} \right)^2 / \left( 2 \rho_{av15} \right) \]  \hspace{1cm} (3)

Air-vapour leaving cooling tower is assumed to be saturated. Same condition of air at inlet of fan was assumed as condition at outlet of fill, so that properties of air-vapour at section 5 and 6 (Fig. 1) are taken same.

Formulation for Exergy Analysis
Exergy analysis consists of using first and second law of thermodynamics together, for analyzing performance in reversible limit, and estimating departure from this limit\(^\text{14}\). Exergy represents true potential of a system to perform an optimal work with respect to a dead state or surrounding. Greater the difference between energy source and its surroundings, greater is the capacity to extract work from system. In a counter flow cooling tower, water and air are the only two kinds of working fluids revealed in operation.

Exergy of Water
Exergy of water is given as\(^\text{15}\).
\[ X_w = m_w \left[ (h_{fw} - h_{fwo}) - t_r (s_{fw} - s_{fwo}) - R_v \cdot t_r \cdot \ln (\theta) \right] \]

where \( \theta = p_{w,w} / (0.622 + w) \)

\[ \text{Exergy of Air-Vapour} \]

Exergy of air-vapour is sum of exergy of dry air and exergy of vapour. Specific exergy of dry air is given as

\[ \Psi_a = \left[ x_a (c_{pa}/M_a) \left\{ t - t_r - t_r \cdot \ln (t/t_r) \right\} + (R/M_a) \cdot t_r \cdot (p/p_r) + (R/M_a) \cdot t_r \cdot x_a \cdot \ln (x_a/x_{a'}) \right] \]

With above equations, exergy of air-vapour mixture becomes

\[ X_{av} = m_a [\Psi_a + \Psi_v] \]

\[ \text{Exergy Destruction (X_d)} \]

Exergy destruction is given by

\[ X_d = (X_{wi} + X_{awi} + X_{wimakeup}) - (X_{wo} + X_{avo}) \]

\[ \text{Second Law Efficiency (}\eta_{II}\text{)} \]

Second law efficiency is given by

\[ \eta_{II} = 1 - \left[ X_d / (X_{wi} + X_{awi} + X_{wimakeup}) \right] \]

\[ \text{Thermal Efficiency (}\eta_{th}\text{)} \]

Thermal efficiency of a cooling tower or efficiency of evaporative cooling is given by

\[ \eta_{th} = \frac{t_{wi} - t_{wo}}{t_{wi} - t_{wb1}} \]

For heat and mass transfer analysis, Merkel formulation has been used. In its four-point form, approximate formula is given as

\[ Me_M = \frac{h_d a_f L_d}{m_w} = \frac{h_d a_f L_d}{G_w} = \frac{C_p w}{t_{wi} (i_{marw} - i_{mu})} \]

\[ = \left( \frac{t_{wi} - t_{wo}}{4} \right) \left[ \frac{C_{pw1} + C_{pw2} + C_{pw3} + C_{pw4}}{\Delta i_{1}} + \frac{1}{\Delta i_{2}} + \frac{1}{\Delta i_{3}} + \frac{1}{\Delta i_{4}} \right] \]

\[ = \frac{C_{pwm} (t_{wi} - t_{wo})}{4} \left[ \frac{1}{\Delta i_{1}} + \frac{1}{\Delta i_{2}} + \frac{1}{\Delta i_{3}} + \frac{1}{\Delta i_{4}} \right] \]

\[ \text{...(11)} \]

Enthalpy differentials (\( \Delta i \)) are dependent on following intermediate temperatures:

\[ t_{w(1)} = t_{wo} + 0.1 (t_{wi} - t_{wo}) \]

\[ t_{w(2)} = t_{wo} + 0.4 (t_{wi} - t_{wo}) \]

\[ t_{w(3)} = t_{wo} + 0.6 (t_{wi} - t_{wo}) \]

\[ t_{w(4)} = t_{wo} + 0.9 (t_{wi} - t_{wo}) \]

Subscripts 1, 2, 3, 4 used in Eq. (11) refer to intervals in Chebyshev’s integral. \[ \text{Transfer characteristic in spray zone (Me}_{sp}\text{) of cooling tower is given as} \]

\[ Me_{sp} = 0.2 L_{sp} \cdot (G_w/G_w)^{0.5} \]

\[ \text{Transfer characteristic in fill zone (Me}_{fi}\text{) of cooling tower is given as} \]

\[ Me_{fi} = a_d L_{fi} G_{bd} G_{cd} \]

Values of coefficients for existing film type fill (from manufacturer) are as follows: \( a_d = 0.2692; b_d = -0.094; c_{di} = 0.6023 \) for fill.

Loss coefficient of fill due to frictional and drag effects (\( K_{dlm} \)) at mean conditions of cooling tower is given by \( K_{dlm} = a_{dl} L_{dl} G_{bd} G_{cd} \). Values of coefficients for existing film type fill (from manufacturer) are as follows: \( a_{di} = 1.9277; b_{di} = 1.2752; c_{di} = -1.0356 \) for fill.

Transfer characteristic in rain zone (\( Me_{rz} \)) of an induced draft rectangular cooling tower is given as
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Calculate heat load, \( q \) (Eq. 1)

Solve energy equation (Eq. 2) with guess values of \( m_{av15} \) and \( p_{a5} \) for \( t_{a5} \)

Determine water outlet temperature (\( t_{wo} \)) for \( MeC = Me_{ip} + Me_{o} + Me_{e} \)

Read initial conditions

Solve draft equation (Eq. 3) with calculated value of \( t_{a5} \) and guess values of \( p_{a5} \) to get new value of \( m_{av15} \)

Solve pressure equation with calculated value of \( t_{a5} \) and \( m_{av15} \) to get new value of \( t_{a5} \)

\[ \text{tol}^* = 0.00001 \]

Convergence criterion satisfied

Calculate output parameters (inlet air/water mass flow rate ratio, fan power, evaporation loss, exergy destruction, thermal and 2nd law efficiency).

End

Fig. 2 — Flowchart of solution procedure
Eq. (14) is applicable with following restrictions:

\[ 0.927 - 1.289 \text{ kg/m}^3 \]

\( \text{Me}_m; \text{ and } \)

\( \text{Eq. (15)} \)

\( (\text{Me}_k) \) is given as

\[ \text{Me}_r = \text{Me}_{sp} + \text{Me}_{fi} + \text{Me}_{rz} \] ...

\( \text{Evaporation Loss} \)

Amount of water lost due to evaporation is given as

\[ m_{w\text{ (evap)}} = (m_{av15} - m_{av1}) \]

\( \text{Solution Procedure} \)

Using initial conditions and model formulation, water outlet temperature is iteratively calculated by equating Merkel Numbers obtained from Chebyshev method and total transfer characteristics of cooling tower for spray, fill and rain zones. Energy equation [Eq (1)], draft equation [Eq (2)] and pressure equation [Eq (3)] are highly non-linear and therefore, an iterative procedure has been developed to solve these equations. Air-vapour outlet temperature \( (t_{av5}) \), average mass flow rate of air-vapour \( (m_{av15}) \) through cooling tower and pressure \( (p_{av5}) \) at section 5 in IDRCT are unknown parameters in these equations. Initially, guess values of \( m_{av15} \) and \( p_{av5} \) were supplied to Eq (1). \( m_{av15} \) was chosen nearly equal to mass flow rate of inlet water \( (m_i) \). Value of \( p_{av5} \) was chosen slightly less than atmospheric pressure \( (p_{at}) \) at ground level at section 1 in IDRCT. By using these guess values; energy equation was solved for air-vapour outlet temperature \( (t_{av}) \). Further, calculated value of \( t_{av5} \) and initial guess value of \( p_{av5} \) were supplied to Eq (2) to obtain new value of \( m_{av15} \). Now, calculated values of \( m_{av15} \) and \( t_{av5} \) were supplied to Eq (3) to determine calculated value of \( p_{av5} \). At this juncture, calculated values of \( m_{av15} \) and \( p_{av5} \) were compared with initial guess values and modified accordingly. Using modified values, procedure stated above was repeated till achievement of convergence within prescribed tolerance limit.

Fan power calculations were performed using standard equations. Evaporation loss was calculated using Eq. (16). Exergy destruction \( (X_e) \), second law efficiency \( (\eta_{th}) \) and thermal efficiency \( (\eta_{th}) \) of cooling tower were calculated using Eqs (8), (9) and (10) respectively (Fig. 2).

**Results and Discussion**

**Initial Conditions**

Initial conditions supplied to the model were collected from an IDRCT of a refrigeration plant (capacity, 220 tonnes) installed in a dairy industry at Ambala (India). Air/water conditions were as follows: atmospheric pressure at ground level 1, \( p_{at} \), 101325 Pa; inlet air dry bulb temperature, \( t_{wi} \), 304.15 K; inlet air wet bulb temperature, \( t_{wb1} \), 296.85 K; water inlet temperature, \( t_w \), 307.65 K; inlet water mass flow rate, \( m_w \), 20.8 kg/s; and mean droplet diameter in rain zone, \( d_r \), 0.0040 m. Cooling tower parameters were as follows: tower height, \( H_f \), 3.429 m; fan height, \( H_f \), 3.048 m; tower inlet height, \( H_i \), 0.6096 m; tower inlet width, \( W_i \), 1.8288 m; tower breadth or length, \( B_i \), 3.048 m; plenum chamber height, \( H_f \), 0.2286 m; fan diameter, \( d_f \), 1.2 m; and fan rotational speed, \( N_f \), 710 r/min. Various Loss coefficients were as follows: loss coefficient for inlet louvers, \( K_{il} \), 3.5; loss coefficient for fill support, \( K_{fs} \), 0.5; loss coefficient for fan supports, \( K_{fr} \), 1.536 m; reference rotational speed, \( N_f \), 750 r/min; and reference air density, \( \rho_r \), 1.2 kg/m$^3$. Ambient conditions for exergy analysis were: pressure, \( p \), 101325 Pa; \( t_{wb1} \), 298.15 K; \( t_{wb1} \), 295.45 K; and relative humidity, 80%.

**Model Validation**

Initial conditions were supplied to the model and predicted values were compared with measured data (Table 1). Model predictions and experimental data match satisfactorily. Predicted water outlet temperature was within 7.30% of cooling range. Predicted average mass flow rate of air-vapour was within 3.12% of measured value. Predicted fan power was around 58% of installed capacity. Model also predicts following
parameters: air outlet temperature at section 5, $t_{a5}$, 304.33 K; pressure of air at section 5 upstream of fan, $P_{a5}$, 101249.45 Pa; transfer characteristic in spray zone, $M_{esp}$, 0.0341; transfer characteristic in fill zone, $M_{efi}$, 0.5810; transfer characteristic in rain zone, $M_{erz}$, 0.0488; total transfer characteristic (Merkel number) of cooling tower, $M_T$, 0.6639; Merkel number by Chebyshev’s formula, $M_C$, 0.6624; thermal efficiency of cooling tower, $\eta_{th}$, 36.76 %; water lost due to evaporation, $m_{wevap}$, 0.132 kg/s; air/water mass flow rate ratio at inlet $(m_{av1}/m_w)$, 0.4737; evaporation loss of water, $\text{evap loss}$, 0.634 %; exergy destruction, $X_d$, 9.885 kW; and second law efficiency, $\eta_{II}$, 93.96 %.

Table 1—Output results

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Predicted value</th>
<th>Measured value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water outlet temperature, $t_{wo}$</td>
<td>303.68 K</td>
<td>303.95 K</td>
</tr>
<tr>
<td>Water cooling range, $\Delta t$</td>
<td>3.97 K</td>
<td>3.70 K</td>
</tr>
<tr>
<td>Average mass flow rate of air-vapour, $m_{av15}$</td>
<td>9.914 kg/s</td>
<td>9.614 kg/s</td>
</tr>
<tr>
<td>Fan power, $P_f$</td>
<td>1.038 kW</td>
<td>1.790 kW</td>
</tr>
</tbody>
</table>

Table 2—Effect of variation in wet bulb temperature of inlet air

<table>
<thead>
<tr>
<th>Run</th>
<th>$t_{a1}$ K</th>
<th>$t_{wb1}$ K</th>
<th>$t_{wo}$ K</th>
<th>$m_{av15}$ kg/s</th>
<th>$t_{a5}$ K</th>
<th>$m_{av1}/m_w$</th>
<th>evap loss %</th>
<th>$P_f$ kW</th>
<th>$\eta_{th}$ %</th>
<th>$X_d$ kW</th>
<th>$\eta_{II}$ %</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>304.15</td>
<td>293.00</td>
<td>302.62</td>
<td>9.943</td>
<td>303.41</td>
<td>0.4738</td>
<td>0.835</td>
<td>1.041</td>
<td>34.33</td>
<td>20.878</td>
<td>87.28</td>
</tr>
<tr>
<td>2</td>
<td>304.15</td>
<td>295.00</td>
<td>303.15</td>
<td>9.928</td>
<td>303.87</td>
<td>0.4736</td>
<td>0.740</td>
<td>1.039</td>
<td>35.57</td>
<td>15.352</td>
<td>90.63</td>
</tr>
<tr>
<td>3</td>
<td>304.15</td>
<td>296.85</td>
<td>303.68</td>
<td>9.914</td>
<td>304.33</td>
<td>0.4734</td>
<td>0.644</td>
<td>1.038</td>
<td>36.76</td>
<td>9.885</td>
<td>93.96</td>
</tr>
<tr>
<td>4</td>
<td>304.15</td>
<td>299.00</td>
<td>304.33</td>
<td>9.894</td>
<td>304.92</td>
<td>0.4731</td>
<td>0.527</td>
<td>1.036</td>
<td>38.38</td>
<td>2.828</td>
<td>98.27</td>
</tr>
<tr>
<td>5</td>
<td>304.15</td>
<td>300.00</td>
<td>304.71</td>
<td>9.890</td>
<td>305.12</td>
<td>0.4732</td>
<td>0.461</td>
<td>1.035</td>
<td>38.43</td>
<td>0.054</td>
<td>99.97</td>
</tr>
</tbody>
</table>

Effect of variation in wet bulb temperature of inlet air on various performance parameters was studied (Table 2) with reference to base case (Run 3).

a) **Air Outlet Temperature v/s Wet Bulb Temperature of Inlet Air**

It was observed that in Run 1 and 2, air outlet temperature is less than air inlet dry bulb temperature whereas from Run 3 to Run 5, air outlet temperature increases with increase in wet bulb temperature of inlet air (Fig. 3a). Thus, in Run 1 and 2, both air and water are cooled. This is possible in very hot and extreme dry conditions, because of latent heat transfer from water to air ($W_{sw} > W$) and sensible heat transfer from air to water ($t_a > t_w$). Net enthalpy transfer is from water to air since $i_{masw} > i_{ma}$.

b) **Evaporation Loss v/s Wet Bulb Temperature of Inlet Air**

It was observed that evaporation loss decreases with increase in wet bulb temperature of inlet air (Fig. 3b). Due to increase in air outlet temperature, sensible heat component increases and latent heat component decreases resulting in reduction in evaporation loss.

c) **Thermal Efficiency v/s Wet Bulb Temperature of Inlet Air**

It was observed that thermal efficiency of cooling tower increases continuously with increase in wet bulb temperature of inlet air (Fig. 3c).

d) **Exergy Destruction v/s Wet Bulb Temperature of Inlet Air**

It was observed that exergy destruction in cooling tower decreases continuously with increase in wet bulb temperature of inlet air (Fig. 3d). For a fixed inlet water temperature, exergy of outlet water increases resulting in reduction in exergy destruction. As air outlet temperature increases with increase in wet bulb temperature of inlet air, exergy of outlet air stream increases but exergy destruction of air decreases. As evaporation loss decreases with increase in wet bulb temperature of inlet air, therefore, required quantity of make-up water reduces. Hence, exergy destruction decreases with increase in wet bulb temperature of inlet air.
It was observed that second law efficiency increases continuously with increase in wet bulb temperature of inlet air (Fig. 3e) due to decrease in exergy destruction.

**Conclusions**

For a rectangular cooling tower load, the model successfully predicts air and water outlet temperatures, fan power requirements, and make-up water requirement, besides inlet air/water mass flow rate ratio,
thermal efficiency, exergy destruction and second law efficiency. Increase in wet bulb temperature of inlet air causes increase in air and water outlet temperatures, thermal efficiency and second law efficiency and decrease in evaporation loss and exergy destruction. Verification of simulation model was conducted with experimental data of an IDRCT of a refrigeration plant of (capacity, 220 tonnes) installed in a dairy industry at Ambala (India).

Nomenclature

- \( a \): area, \( \text{m}^2/\text{m}^3 \)
- \( B \): breadth of cooling tower, m
- \( b_1, b_4 \): dimensional coefficients
- \( c_p \): specific heat at constant pressure, J/kg K
- \( D \): diffusion coefficient
- \( d \): differential element, droplet
- \( G \): mass velocity, kg/m
- \( g \): gravity, m/s
- \( H \): height, m
- \( h \): heat transfer coefficient, W/m² K; enthalpy, J/kg
- \( h_d \): mass transfer coefficient, kg/m² s
- \( i \): enthalpy, J/kg
- \( i_m \): enthalpy of dry air at wet bulb temperature, J/kg
- \( i_{ma} \): enthalpy of saturated air at the local bulk water temperature, J/kg
- \( k \): loss coefficient
- \( k \): thermal conductivity, W/m K
- \( L \): length, m
- \( m \): mass flow rate, kg/s
- \( M \): molecular mass, kg/mole
- \( N \): rotational speed, r/min
- \( NTU \): number of transfer units
- \( P \): power, W or kW
- \( p \): pressure, N/m² or Pa
- \( q \): heat transfer rate, J/s or W
- \( r \): rounding, m
- \( s \): entropy, J/kg K
- \( t \): temperature, K
- \( v \): velocity, m/s
- \( W \): width of cooling tower, m
- \( w \): humidity ratio, kg water vapour/kg dry air
- \( X \): exergy, W or kW
- \( x \): mole fraction
- \( \Delta \): differential
- \( \mu \): dynamic viscosity, kg/ms
- \( \eta \): efficiency
- \( \theta \): relative humidity
- \( \Psi \): specific exergy, J/kg
- \( Me \): Merkel number, \((b_1 \cdot a_p \cdot L_p) / G_w)\)
- \( Sc \): Schmidt number
- \( \tilde{\sigma} \): Surface tension, N/m

Subscripts

- \( a \): air, atmospheric
- \( av \): mixture of dry air and water vapour
- \( C \): Chebyshev method
- \( c \): casing of fan
- \( d \): destruction, diameter (m)
- \( de \): drift eliminator
- \( dif \): diffuser
- \( e \): effective
- \( evap \): evaporated
- \( F \): fan
- \( F/\text{dif} \): fan/diffuser
- \( fd \): frictional and drag
- \( fi \): fill
- \( fr \): frontol
- \( fs \): fill support
- \( II \): second law
- \( i \): inlet
- \( il \): inlet louvers
- \( M \): Merkel method
- \( m \): mean
- \( o \): outlet
- \( pl \): plenum chamber
- \( r \): reference
- \( rz \): rain zone
- \( s \): saturation, static
- \( sp \): spray
- \( T \): total
- \( th \): thermal
- \( up \): upstream
- \( v \): vapour
- \( w \): water
- \( wb \): wet bulb
- \( wd \): water distribution system
- \( wv \): water vapour

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