Freeway traffic flow control by lumped parameter system approach

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Freeway traffic flows can be controlled by lumped parameter system approach owing to the space discretized system models. Such a control model incorporating a coordinated ramp metering mechanism is proposed in this study. The nonlinear state equations representing the traffic system dynamics are derived from the conservation law in difference equations form. The control model is obtained by a feedback linearization approach, so that the target density of the controller is chosen as the critical density of the traffic system. Simulation based test studies have been done in a VISSIM simulation environment in order to compare the data obtained by the shock wave modified feedback linearization method to the non-modified type, with respect to the uncontrolled system performance. Test results show that the main-link flow performance is increased sufficiently by the proposed control model.

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The steadily increasing traffic congestions on freeways worldwide have led to the use of several control mechanisms. Basically, these are formed by controlling the number of vehicles entering the freeway from an on-ramp (ramp metering), and/or by changing the free speed limit of the vehicles between the specified sections of the road (variable speed limiting). Ramp metering is the most common type of control mechanism. It has been recognized as an effective way for relieving freeway congestion which is typically the result of either a surge of demand during peak commuting hours or a temporary reduction of the road capacity.

The studies on ramp metering have been extended over the last 35 years. In this period a variety of algorithms based on either optimization techniques or automatic control theory have been proposed. Some of these are already being utilized for local ramp metering applications at many sites worldwide.

However, in recent years, many researchers have recognized that considering the effect of the coordinated controls over the traffic system level has many advantages compared to the local control. For the global results, non-linear control methods have become necessary. In this context, various linear and non-linear control methods available for lumped parameter systems can be adapted to the freeway traffic flow process by the use of space discretized models. In any case, it is important to analyse the control laws designed by using various methods, so that they can be compared for their effectiveness by experimental field studies or simulation based test studies.

Such a control model incorporating a coordinated ramp metering mechanism is proposed in this study. Each discrete space unit is made up according to the geometric conditions of the road, so that each may have one controlled on-ramp only. An approach adopted in this method is that, during the control period the state variables related to the macroscopic densities of the discrete space units are determined by evaluating the direction of the shock waves between the contiguous discrete space units.

Another approach in this study is that all the macroscopic flow parameters are determined by the accumulation of the occupancy times and vehicle counts gathered from the traffic detectors during discrete time periods. Thus, the dynamics of the freeway traffic flow process is reflected in an accurate way.

In this study, the basic considerations on freeway traffic system and the traffic control concept are introduced. The discrete system dynamics and the proposed control model are discussed. The simulation based test studies realized in VISSIM simulation environment, and their results are presented. The
performance data obtained by the shock wave modified feedback linearization method (proposed control method) and the non-modified feedback linearization method are evaluated with respect to the uncontrolled system performance data.

Basic Considerations on Freeway Traffic System

A freeway traffic system is considered as a distributed parameter system, having infinite number of flow parameters along the road. Traffic flow process on the road is characterized by three basic macroscopic parameters: flow, $q$, density, $\rho$, and velocity $v$. A freeway traffic system having $L$ length of the road with its distributed flow parameters, $q(x,t)$, $\rho(x,t)$, $v(x,t)$, is shown in Fig. 1.

Here flow is defined as the number of vehicles passing a specific point of the road in an hourly rate in a lane basis (veh/h/lane). Density is defined as the number of vehicles occupying per kilometre of the road in a lane basis (veh/km/lane). Velocity is defined as the average rate of motion in km/h. The road based flow, $Q$, is the total number of vehicles passing a specific point of the road in an hourly rate (veh/h). Thus, the road based flow which is also known as volume in some related literature is determined by multiplying the lane based flow, $q$, with the number of lanes, $\Delta$, at the point of the road under consideration:

$$Q(x,t) = q(x,t) \Delta(x)$$  \hspace{1cm} (1)

where $q$ and $Q$ represent the lane based flow and the road based flow, respectively.

Macroscopic flow properties

The basic flow model expressing the static relation among the macroscopic flow parameters, $q=f(\rho,v)$, is given as $^{12}$,

$$q(x,t) = \rho(x,t) v(x,t)$$  \hspace{1cm} (2)

In addition to Eq. (2), there are various other static models representing the relationship between the macroscopic flow parameters. One of them proposed by Greenshield $^{13}$ presents a linear relationship between the velocity and density parameters, $v=f(\rho)$, given as follows,

$$v=v_f \left(1-\rho / \rho_{jam}\right)$$ \hspace{1cm} (3)

Here $v_f$ and $\rho_{jam}$ are the constants corresponding to the free speed limit and jam density of the road, respectively. They can be determined depending on the road conditions and vehicle compositions related to the considered traffic system. By substituting Eq. (3) into Eq. (2), it can easily be seen that the static relation between the flow-density and flow-velocity pairs, $q=f(\rho)$ and $q=f(v)$, are formulated with the following parabolic functions,

$$q = v_f \left(\rho-\rho^2/\rho_{jam}\right)$$ \hspace{1cm} (4)

$$q = \rho_{jam} \left(v-v^2/v_f\right)$$ \hspace{1cm} (5)

According to Eq. (4), the maximum flow corresponding to $\partial q(\rho,v)/\partial \rho=0$ at any considered point of the road has its optimum density, which is named as the critical density, $\rho_c$. Similarly, due to Eq. (5), the maximum flow corresponding to $\partial q(\rho,v)/\partial v=0$ has its optimum velocity, $v_o$, at any considered point of the road. Thus, according to the flow models given with Eqs (2) and (3), the critical density and optimum velocity related to the traffic system can be determined as:

$$\rho_c = \rho_{jam}/2$$

$$v_o = v_f / 2$$ \hspace{1cm} (6)

Depending on the above formulations, the basic static relations between the macroscopic flow parameters, $v=f(\rho)$, $q=f(\rho)$, $q=f(v)$, can be represented by the graphical forms given in Fig. 2.

Fig. 1—Freeway traffic system with distributed flow parameters.

Fig. 2—Basic static relations between the macroscopic flow parameters.
Road capacity usage

The maximum flow, $q_{max}$, which can also be named as the lane based capacity, corresponding to the optimum velocity and/or critical density of the road can be determined depending on the critical density, $\rho_c$, and the optimum velocity, $v_o$, values given with Eq. (6) as follows,

$$q_{max} = \rho_c \cdot v_o = \frac{\rho_{jam} \cdot v_f}{4} \quad \ldots \quad (7)$$

By substituting Eq. (7) into Eq. (1), the road capacity at the considered point of the road, $Q_{max}$, is determined as,

$$Q_{max} = q_{max} \cdot \Delta \quad \ldots \quad (8)$$

where $\Delta$ is the number of lanes.

Referring to the traffic flow characteristics given in Fig. 2, it is clear that when the density exceeds the critical density, both the flow and velocity begin to decrease, so that both become zero at the jam density, $\rho_{jam}$. Because of that reason the road capacity usage becomes very important, when the density exceeds the critical density, $\rho(x,t) > \rho_c$. Therefore, any control approach must take this fact into consideration when designing a control process.

Traffic Control Concept

Taking the above considerations into account, the flow-density relationship, $q=f(\rho)$, can be referred to as the fundamental diagram of the traffic flow process, for designing a control method. Thus, the aim of the freeway traffic control process can be stated as to ensure that, $$(\rho(x,t), \rho_c, \rho_{jam}) / 2, \, \forall x \in L, \, t \in R^+ \quad \ldots \quad (9)$$

where $R^+$ represents the set of positive real numbers. Due to the statement in Eq. (9) the control aim can be achieved by having the critical density, $\rho_c$, related to the considered traffic system as the target density of the controller.$^{10}$

Ramp metering approach

The ramp metering approach supports the control aim given with Eq. (9), by activating the traffic signal which indicates whether the vehicles can go into the freeway or not. The freeway traffic control mechanism maintained by ramp metering approach is shown in Fig. 3.

In Fig. 3, $Q_{in}=q_{in} \Delta_{in}$ and $Q_{out}=q_{out} \Delta_{out}$ represent the inflow (upstream) volume and outflow (downstream) volume, respectively. $d$ is the demand, and $r$ is the controlled ramp volume. $r_{max}$ and $r_{min}$ are the maximum and the minimum ramp volumes, respectively. $c$, $G$, and $R_d$ indicates the cycle time, the green phase duration, and the red phase duration of the traffic signal, respectively, so that $R_d = c \cdot G^{9}$.

All ramp metering algorithms calculate suitable ramp volumes, $r$. In the traffic signal cycle realization process, $r$ is converted to the green phase duration (green time), $G$, as,

$$G = \left( \frac{r(t)}{r_{max}} \right) \cdot c \quad \ldots \quad (10)$$

where $c$ is the fixed cycle time. The green phase duration $G$ is constrained by a $G_{min}>0$ and $G_{max} \leq c$, corresponding to the minimum and maximum ramp volumes, respectively. The green time starts at the beginning of each discrete time period (cycle time), and continues up to the end of green phase duration. Remaining duration up to the end of the cycle time traffic signal is converted to the red phase, which indicates that the ramp flow is stopped.

Optimal control problem

Theoretically, ramp metering is effective if the inflow volume, $Q_{in}$, plus the ramp volume, $r$, is less than the outflow volume, $Q_{out}$,

$$[Q_{in}(t) + r(t)] \leq Q_{out}(t) \quad \ldots \quad (11)$$

Therefore, a feedback solution is needed for metering problem, in which the ramp flow, $r$, can be adjusted between the maximum and minimum ramp volumes depending on the inflow and/or outflow variations. Then the control problem can be stated as an optimal control problem, to find the optimal $r(t)$ which minimizes,
\[ J(r) = \int_{t_0}^{t_f} \left( (r - r_{\text{max}})^2 + w \int_0^t (\rho(x,t) - \rho_c)^2 \, dx \right) \, dt \]  
\[ \cdots (12) \]

where, \( t_f \) is the final time, and \( w \) is the relative weighing factor \(^{10} \).

A feedback solution can be maintained either by deciding the structure of the control, such as a proportional integral derivative (PID) control with constant gains, and solve numerically for the optimal values of the gains, or by setting up the control objective for a standard feedback control problem, such as steady state asymptotic stability as \(^{10} \),

\[ \lim_{\tau \to \infty} \int_0^L (\rho(x,t) - \rho_c)^2 \, dx \to 0 \]  
\[ \cdots (13) \]

**Performance index**

The objective of the freeway traffic control process is to optimize a performance index that consists of the state variables \((\rho, q)\) and control variables \((r)\) under the certain constraints imposed on the control. Performance index can be stated to minimize the travel times, delays, number of stops, or some other parameters such as fuel consumption and environmental pollution.

As given in Eq. (12), any performance index formulation may include the variables related to both the main-link flows and on-ramp flows. So, it is clear that the best performance results can be achieved by both maximizing the volume of the on-ramp flow, \( r \), and optimizing the density of the main-link flow, \( \rho \).

On the other hand, the on-ramp queues may be extended as a result of the ramp metering process, particularly during the peak commuting hours or a temporary reduction of the road capacity. So, the control process increasing the main-link flow performance may give rise to some decrease on the on-ramp flow performance. In this situation both ramp metering and variable speed limiting mechanisms must be taken into consideration in a coordinated control process to increase the performance of whole system.

However, only the ramp metering mechanism is used for maintaining the control process in this study. Also, only for the main-link flows the performance index is taken into consideration.

**Discrete System Dynamics**

To attempt a controller design, first the macroscopic model representing the dynamics of the traffic flow process must be decided on. For this aim, it is assumed that the traffic flow process is governed by the macroscopic model proposed by Lighthill & Whitham\(^{14} \) and Richards\(^{15} \), which is based on the principle of fluid dynamics:

\[ \frac{\partial}{\partial x} q(x,t) + \frac{\partial}{\partial t} \rho(x,t) = 0 \]  
\[ \cdots (14) \]

Eq. (14) is a steady-state representation of the freeway traffic flow dynamics, which is known as the hyperbolic conservation law in partial differential equations (PDE) form. Due to this representation, the freeway traffic system is considered as a distributed parameter system, having infinite number of flow parameters along the road.

**Discretization need**

However, practically it is not possible to measure the infinite number of flow parameters at infinite number of discrete points. In reality, sensor measurements are made at discrete points with 500-1000 m intervals\(^{9,10} \), depending on the geometric structure of the way. This means that, the space base of the freeway traffic system needs to be discretized.

Depending on the above requirements, space discretization is performed by dividing the considered length of the road into the discrete space units represented by the space index \( i \), as shown in Fig. 4.

On the other hand, for real time control applications macroscopic flow parameters must be determined within the very short time intervals\(^{9,10} \). Because of this, measurements are made within the short time intervals by gathering the microscopic flow parameters from the traffic detectors. Thus, the macroscopic flow parameters related to any considered point of the road can be represented in discrete time form: \( q(k), \rho(k), v(k) \). In this representation \( k \) is the discrete time index indicating the time interval between \((k-1)T\) and \(kT\), in which \( T \) is the duration of the discrete time period.

In Fig. 4, \( q_i(k), \rho_i(k) \) and \( v_i(k) \) represent the macroscopic flow, density and velocity parameters, respectively, related to the discrete space unit represented by the space index \( i \). All of the discrete space
parameters are also represented in discrete time form in Fig. 4.

Discrete space unit

Each discrete space unit is created in a way that it can include only one controlled on-ramp and the number of lanes cannot change along the discrete space unit length. So, each discrete space unit (road segment) indicated in Fig. 4 can be represented with a generalized form of the geometric structure as shown in Fig. 5.

In Fig. 5, \( L_i \) is the length of the discrete space unit \( i \), \( \Delta_i \) is the number of lanes along its length, and \( \rho_i(k) \) is the macroscopic density of the discrete space unit. \( q_{i-1}(k) \) and \( q_i(k) \) are the inflow and outflow of the discrete space unit; \( Q_{i-1}(k) \) and \( Q_i(k) \) are corresponding volumes, respectively. The inflow of any discrete space unit is the outflow of previous discrete space unit in the same time. \( r_i(k) \) is the ramp volume, \( d_i(k) \) is the demand volume of the on-ramp, and \( e_i(k) \) is the exit flow volume.

Measuring the macroscopic flow parameters

Each discrete space unit represented by the space index \( i \) (\( 1 \leq i \leq n \)) has its own traffic detector, \( D_{ti} \), placed near the on-ramp entrance of the road, as shown in Fig. 6.

At the end of each discrete time period, the average degree of saturations, \( DS_i(k) \), related to the outflow of the discrete space units are determined, as:

\[
DS_i(k) = \frac{\sum_{j=1}^{N_i(k)} o_j j}{T}, \quad j \leq N_i(k) \quad \text{... (15)}
\]

\( 1 \leq i \leq n, \quad 0 \leq DS_i(k) \leq 1 \)

where, \( N_i(k) \) is the number of vehicles passing from traffic detector through the discrete time period. \( j \) represents the vehicle number \([0 \leq j \leq N_i(k)]\), and \( o_j \) its occupancy time, which is a measure of the time, depending on the length and speed of the vehicle occupying the traffic detector.

Then, the measured macroscopic densities related to the outflow position of the discrete space units, \( \rho_i^{\text{out}}(k) \), are determined by multiplying the average degree of saturations given with Eq. (15) by the jam density of the road, \( \rho_{\text{jam}} \).

\[
\rho_i^{\text{out}}(k) = \rho_{\text{jam}} DS_i(k), \quad 1 \leq i \leq n \quad \text{... (16)}
\]

The lane based macroscopic flow, \( q_i(k) \), is calculated due to the following equation at the end of each discrete time period:

\[
q_i(k) = 3600 \frac{N_i(k)}{T}, \quad 1 \leq i \leq n \quad \text{... (17)}
\]

With the Eqs (15)-(17), the macroscopic flow parameters are calculated by using the cumulative microscopic flow parameters gathered from the traffic detectors. As result, the macroscopic parameters obtained by this way can reflect the dynamics of the traffic flow process in an accurate way.

Control Model

Actually, the time and space discretized form of the freeway traffic system can be considered as a lumped parameter system for each discrete space unit. Then, the non-linear state equations can be derived from the conservation law stated with Eq. (14) either in ordinary differential equations (ODE) form or in difference equations form for optimal control of the traffic flow process. For the aim of this study, the dynamics of traffic system is derived difference equations form in the following.

Discrete system dynamics

The discrete system dynamics representing the freeway traffic system having \( n \) number of discrete space units can be derived form the conservation law given with Eq. (14) in difference equation form as:

![Fig. 6—Measuring the macroscopic flow parameters related to the discrete space units.](image-url)
\[ \rho_i(k+1) = \rho_i(k) + \frac{T}{3600} L_i \Delta_i \times [Q_{i+1}(k) - Q_i(k) + r_i(k) - e_i(k)] \quad \ldots \quad (18) \]

\[ 1 \leq i \leq n \]

where all the volumes represented by \( Q_i(k), Q_{i+1}(k), r_i(k) \) and \( e_i(k) \) are in veh/h, and the macroscopic density of the discrete space unit represented by \( \rho_i(k) \) is in veh/km/lane. The segment length \( L_i \) is in kilometers, and the discrete time period \( T \) is in seconds.

**Non-linear optimal control model**

The non-linear optimal control model is derived from discrete system dynamics given with Eq. (18) by using the feedback linearization approach, so that the target density of the controller is chosen as the critical density of the considered traffic system, \( \rho_i(k+1) = \rho_c \).

Hence, the ramp flows related to the next discrete time period can be obtained as:

\[ r_i(k+1) = \kappa \left[ \rho_c - \rho_i(k) \right] + [Q_i(k) - (1-s) Q_{i+1}(k)] \quad \ldots \quad (19) \]

\[ \kappa = \frac{3600 L_i \Delta_i}{T}, \quad 1 \leq i \leq n \]

where, \( s \) represents a constant value for calculating the exit flow volumes. It is determined according to the field experiments, so that,

\[ e_i(k) = s Q_{i+1}(k), \quad 0 < s < 1 \quad \ldots \quad (20) \]

As indicated above, in the control model given with Eq. (19) \( \rho_i(k) \) represent the macroscopic densities related to the discrete space units. To decide the structure of the control model (non-modified control model), it can be assigned as outflow density of the corresponding discrete space unit,

\[ \rho_i(k) = \rho_{i\text{ out}}(k), \quad 1 \leq i \leq n \quad \ldots \quad (21) \]

However, as a result of the proposed control method, it is determined by making a simple decision analysis based on the shock wave phenomena between the contiguous discrete space units.

The velocity of the shock waves, \( v_{c,i}(k) \), between the contiguous discrete space units are calculated at the end of each discrete time period, depending on the measured macroscopic flows and densities related to the previous and next discrete space units \(^8,^{15} \), as follows,

\[ v_{c,i}(k) = \frac{q_{i+1}(k) - q_i(k)}{\rho_{i\text{ out}}(k) - \rho_{i\text{ out}}(k)}, \quad \text{if} \quad \rho_{i\text{ out}}(k) \neq \rho_{i\text{ out}}(k) \quad \ldots \quad (22) \]

The densities related to the discrete space units in Eq. (19) are assigned either the next discrete space unit output density, \( \rho_{i\text{ out}}(k) \), or the previous discrete space unit output density, \( \rho_{i-1\text{ out}}(k) \), depending on the existing shock wave directions, as stated in the following,

\[ \rho_i(k) = \begin{cases} \rho_{i\text{ out}}(k), \text{ if } v_{c,i}(k) > 0 \\ \rho_i(k), \text{ if } v_{c,i}(k) < 0 \text{ or } \rho_{i-1\text{ out}}(k) = \rho_{i\text{ out}}(k) \end{cases} \quad \ldots \quad (23) \]

**Proposed control model**

Then, the block diagram of the proposed non-linear optimal control model (namely lumped parameter system approach) is given in Fig. 7.

Here, the outflow volumes related to the next and previous discrete space units are determined by multiplying the corresponding flows with their number of lanes, \( \Delta_i \) and \( \Delta_{i-1} \), as defined in Eq. (1),

\[ Q_i(k) = q_i(k) \Delta_i, \quad Q_{i-1}(k) = q_{i-1}(k) \Delta_{i-1}, \quad 1 \leq i \leq n \quad \ldots \quad (24) \]

At the end of each discrete time period, the green phase durations related to the next discrete time period, \( G_i(k+1) \), are determined by substituting the ramp volumes, \( r_i(k+1) \), calculated with Eq. (19) into the Eq. (10), as,

\[ \begin{cases} [r_i(k+1)/r_{\text{max},i}] \Delta_i, \text{ if } G_{\min,i} \leq G_i(k+1) \leq G_{\max,i} \\ G_{\max,i}, \text{ if } G_i(k+1) > G_{\max,i} \\ G_{\min,i}, \text{ if } G_i(k+1) < G_{\min,i} \end{cases} \quad \ldots \quad (25) \]

where \( r_{\text{max},i} \) represents the constraints related to the maximum volume of the ramp flows related to the discrete space units, \( G_{\max,i} \) and \( G_{\min,i} \) are the constraints corresponding to the maximum and minimum ramp volumes, respectively.

![Fig. 7—Proposed non-linear optimal control model.](image)
In line with the proposed control method, the cycle times of traffic signals are chosen equal to the discrete time period, \( c = T \), for all the controlled on-ramps. All calculated green times start at the beginning of the following discrete time period and continue up to the end of calculated green phase duration. Up to the end of the cycle time, the traffic signals are converted to the red phase, which indicates that the ramp flow is stopping.

**Simulation Based Test Studies**

Simulation based test studies were realized in VISSIM simulation environment to compare the performance data obtained by the shock wave modified feedback linearization method (proposed control method) and non-modified type of it with respect to the uncontrolled system performance. Three different tests were completed under the same traffic conditions: Test 1, for the uncontrolled case; Test 2, for the modified control case; Test 3, for the non-modified control case.

**Simulation package**

The simulation package VISSIM consists of two different programs: traffic simulator and signal state generator. The traffic simulator is a microscopic traffic simulation program based on Weidmann’s statistical model including the car following and lane changes logic. The model has been calibrated through the multiple field measurements at the Technical University of Karlsruhe, Germany. Periodical field measurements and their resulting updates of model parameters ensure that changes in driver behaviour and vehicle improvements are accounted for. The signal state generator is the traffic signal control software polling the traffic detectors which process information from the traffic simulator on a second by second basis. It then determines the actual signal status for the following second and gives this information back to the traffic simulator.

The result of the simulation on-line is the animation of traffic operations and off-line the generation of output files gathering statistical data such as travel time, delay and number of stops. The simulation process in VISSIM has been outlined in Fig. 8.

Traffic control program can be transferred to the VISSIM test environment by using an additional Vehicle Activated Program (VAP) module. Any control model can be written as a text file by using the functions and commands of VAP.
The proposed control model and the non-modified type of it were programmed in this way. The flow chart of the control programs written for the tests in this study is shown in Fig. 9.

Assignments
The traffic detectors, signal groups and other hardware elements of the freeway traffic system consisting of 1.8 km length of the main-link segment with its on-ramps and off-ramps were configured in simulation environment for tests, as illustrated by Fig. 10.

The configured traffic system was divided into 3 discrete space units, so that each has one controlled on-ramp and one off-ramp only. The lengths of the main-link segments $L_1$, $L_2$, $L_3$ were chosen as 0.44 km, 0.565 km and 0.595 km, respectively.

Other assignments about the configured freeway traffic system have been created as follows: (i) number of lanes for all main-link segments: $\Delta=3$, (ii) number of lanes for all the ramps: $\Delta=2$, (iii) exit flow volume calculation coefficients for the off-ramp exit flows: $s_1=0.20$, $s_2=0.15$, $s_3=0.17$, respectively, (iv) maximum speed distributions: $v_f=80-100$ km/h for cars, $v_f=50-80$ km/h for heavy good vehicles (HGV), (v) vehicle length distributions: between 8-18 m, (vi) HGV ratio: 0.20, (vii) Initial green times for the signal groups: 70, 75, 80 s, respectively.

The constant parameters were assigned as follows: (i) traffic signal cycle time: $c=100$ s, (ii) $G_{min}=5$ s and $G_{max}=90$ s for all the traffic signals, (iii) $r_{max}=2000$ veh/h for all the ramp flows, (iv) $\rho_c=40$ and $\rho_{jam}=80$ veh/km/lane for all the main-link segments.

A close-up view related to one of the on-ramp approach in the simulation screen seen in Fig. 10 is shown in Fig. 11.

Performance Evaluations
The performance index has been created due to the total travel times along the main-link segment, as defined with the following equation,

$$PI = T_s = T \sum_{k=1}^{K} \left[ \sum_{i=1}^{N} \rho_j(k) \cdot L_i \cdot \Delta_i \right]$$

However, in addition to travel times; total delay, the total number of stops, and capacity usage for chosen main-link segment were also evaluated by using VISSIM data gathered through the simulations. All of these performance data were accepted from the simulator as average values in 300 s intervals.

The traffic scenario reflecting the dynamics of the environmental changes has been chosen as shown in Table 1, for all the 3 h (10800 s) of simulations.

For performance evaluations, 1793 m of the main-link segment length including all 3 main-link segments are marked in simulation screen.

<table>
<thead>
<tr>
<th>Simulation time (s)</th>
<th>Main-link input volume $G_o$ (veh./h)</th>
<th>Ramp 1 demand volume $d_1$ (veh./h)</th>
<th>Ramp 2 demand volume $d_2$ (veh./h)</th>
<th>Ramp 3 demand volume $d_3$ (veh./h)</th>
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</thead>
<tbody>
<tr>
<td>0-3600</td>
<td>6000</td>
<td>500</td>
<td>600</td>
<td>700</td>
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<tr>
<td>3601-7200</td>
<td>6000</td>
<td>1200</td>
<td>1300</td>
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<td>7201-10800</td>
<td>7000</td>
<td>700</td>
<td>500</td>
<td>500</td>
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</tbody>
</table>

Fig. 10—1.8 km length of freeway traffic system configured in VISSIM.

Fig. 11—The on-ramp close-up view of Fig. 10.
Results and Discussion

Test results

The test results on average travel times along the 1793 m of the main-link segment are shown in Fig. 12 for Test 1, Test 2, and Test 3, respectively.

According to the results obtained, average travel times realized through the 3 h of simulations are 172.34, 142.37, 149.04 s/veh for Test 1, Test 2 and Test 3, respectively. According to these results, main-link average travel time is decreased by 17.4% within the proposed model, while uncontrolled condition it is decreased by 13.5% within the non-modified model.

The average delays realized through the 3 h of simulations are 78.8, 64.2, 71.2 s/veh for Test 1, Test 2 and Test 3, respectively. These results indicate that the average delay is decreased by 18.5% within the proposed model, while uncontrolled condition it is decreased by 9.6% within the non-modified model.

The test results on average number of stops along the 1793 m of the main-link segments are shown in Fig. 13 for Test 1, Test 2, and Test 3, respectively.

According to the results obtained, the average number of stops realized through the 3 h of simulations are 1.93, 0.73, 0.83 stops/veh for Test 1, Test 2 and Test 3, respectively. According to these results, average number of stops is decreased by 62.2% within the proposed model, while uncontrolled condition it is decreased by 57% within the non-modified model.

The test results about the capacity usage related to the 1793 m of the main-link segment are shown in Fig. 14 for Test 1, Test 2, and Test 3, respectively.

Fig. 12—Test results for the average travel times along the main-link segment.

Fig. 13—Test results on the average number of stops along the main-link segment.
According to the results obtained, the total number of vehicles passed completely the 1793 m of the main-link segment through the 10800 s of simulations are 6809, 7409, 7497 vehicles for Test 1, Test 2 and Test 3, respectively. These results indicate that the capacity usage of the main-link segment is increased by 8.8% within the proposed model, while unconstrained condition it is increased by 10.1% within the non-modified model.

The total travel times for Test 1, Test 2 and Test 3 which are defined as the performance index with Eq. (26) can be calculated by using the obtained results (from the 3 h of simulations) about the average travel times, and the total number of the vehicles which passed completely the 1793 m of the main-link segment as follows:

- Test 1: Average travel time = 172.34 s, total number of the vehicles passed = 6809, calculated total travel time = 172.34 × 6809 = 325.96 h
- Test 2: Average travel time = 142.37 s, total number of the vehicles passed = 7409, calculated total travel time = 142.37 × 7409 = 293 h
- Test 3: Average travel time = 149.04 s, total number of the vehicles passed = 7497, calculated total travel time = 149.04 × 7497 = 310.38 h.

According to these results, the total travel time related to 1793 m of the main-link segment is decreased by 10.1% within the proposed model, while unconstrained condition it is decreased by 4.8% within the non-modified model.

Conclusions

A time and space discretized non-linear optimal control model, based on the shock wave modified feedback linearization method for freeway traffic control namely lumped parameter system approach, is proposed in this study. As indicated by simulation based test results, the effectiveness of the main-link traffic flow process is increased in satisfied manner by the proposed control model, referring to the average travel times, average delays, average number of stops, and total travel times. Besides this, the main-link capacity usage is increased by nearly the same percent within modified and non-modified feedback linearization methods while under unconstrained conditions.

On the other hand, the on-ramp queues may be extended as a result of the ramp metering process, particularly during the peak commuting hours or a temporary reduction of the road capacity. In this situation, the control process increasing the main-link flow performance may give rise to some decreases regarding the on-ramp flow performance. However, the effects of the shock wave modified feedback linearization method and non-modified feedback linearization method on the ramp queues fall outside the limitation of this paper, and the performance index has been taken into consideration for only the main-link flows. Therefore, new studies should be planned in this context, including coordinated control models with the ramp metering and variable speed limiting approaches, to increase the overall performance of system.
References