A comparative study of the effect of model lift coefficients on particle trajectory

Pankaj K Gupta\textsuperscript{a} & Krishnan V Pagalthivarthib\textsuperscript{*}

\textsuperscript{a}Department of Mechanical Engineering, MVGR College of Engineering, Vizianagaram, India
\textsuperscript{b}Department of Applied Mechanics, Indian Institute of Technology, New Delhi 110 016, India

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The objective of the present work is to (i) study the influence of different model lift coefficients, (ii) evaluate the importance of particle rotation, and (iii) compare the present computed results with two-way coupling results (in open literature) on the particle trajectory in dilute two-phase gas-solid horizontal channel flow. A hybrid Eulerian-Lagrangian method is employed where the Reynolds averaged Navier-Stokes (RANS) equations with $k-\varepsilon$ closure are used to model the fluid phase, and the particulate phase is treated via Lagrangian approach. Particle-wall collision is modeled using impulse-momentum equations. Five different models of lift coefficients (and/or lift forces) are selected from the open literature to evaluate the influence of lift force on the particle. The effect of shear lift is relatively very small as compared to lift due to particle rotation. Particle rotational velocities of the order of magnitude as high as $10^4$-$10^5$ rad/s are encountered due to particle-wall collision. Results of computed particle trajectories with simplified assumptions agree closely with (i) the simulated results of FLUENT\textsuperscript{®} (Version 6.1) with one-way and two-way coupling, and (ii) of other authors using two-way coupling. In terms of particle lift (and trajectory), the simulated results of FLUENT\textsuperscript{®} with one-way coupling are in agreement with the literature than two-way coupling.

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Dilute gas-solid two-phase flow studies find importance in many technological and industrial applications such as in fluidized beds, cyclone separators, vertical risers, pneumatic conveying of solids and so on. In pneumatic transport of solids, presence of solids has two major effects, viz. (i) additional pressure drop in the pipe/channel resulting from the particle-wall collision and due to the particle acceleration extracted from the fluid momentum, and (ii) erosion of the pipe/channel component. A strong inter-relation between these two factors and the economics of the overall operation makes it vital to understand the physics of confined gas-solid flows. Turbulent two-phase gas-solid flow in horizontal channel provides a classical example for developing robust numerical methods and to study the behaviour of particulate flow in such situations.

In many situations involving wall-bounded flows and mixing layers, the presence of strong shear layers can significantly influence the particle motion. Lee and Durst\textsuperscript{1} observed a particle free region near the wall in a fully-developed upward two-phase pipe flow. Later, Lee\textsuperscript{2} could demonstrate this phenomenon observed in the experiment\textsuperscript{1} in his numerical simulation of the particle motion and diffusion by including the Saffman\textsuperscript{3} lift force for large particles ($d_p=400$, 800 $\mu$m).

In wall-bounded gas-solid flows, particle experiences another phenomenon known as Magnus effect that arises due to the rotation of the particles induced by particle-wall and particle-particle collision. In pneumatic conveying, for example, very high particle rotational velocities are induced due to particle-wall collisions in confined flows. Matsumoto and Saito\textsuperscript{4,5} reported particle rotational velocities measured as high as 1800 revolutions per second induced due to particle-wall collision in a pipe flow. Such high particle rotational velocities are found\textsuperscript{6} to significantly influence the particle motion due to the Magnus effect.

Five different models, hereafter referred to as GROUP\textsuperscript{7-11}, have been chosen to study the effect of lift forces on particle trajectories. A hybrid Eulerian-Lagrangian approach is used in the present study, where the Reynolds averaged Navier-Stokes equations with the two-equation $k-\varepsilon$ model\textsuperscript{12} for turbulence are employed to obtain the carrier-phase flow field. Once the carrier-phase flow field is determined, the particulate phase is treated using

\*For correspondence (E-mail: kvp@am.iitd.ernet.in, amr02014@ccsun50.iitd.ernet.in)
Lagrangian method. Lagrangian method is found to be efficient in simulating dilute two-phase flows where the particle to fluid volumetric loading ratio is very small. Moreover, Lagrangian method is advantageous in understanding the physics of particle motion. Such studies could also lead to an insight into the applicable boundary condition for Eulerian-Eulerian models.

As a first approximation in this study, we assumed one-way coupling, i.e., the carrier-phase flow determines the particulate flow, but not vice versa. Particle-particle interaction is negligible since only dilute two-phase flow is considered. The dispersive effect of turbulent fluctuations on the particle motion is neglected since the density ratio $\rho_p/\rho \gg 1$, where $\rho_p$ and $\rho$ are the particle and fluid density, respectively. The rate of change of the angular momentum of a spherical particle interacting with a viscous fluid is handled using the relation by Dennis et al.\textsuperscript{13}

Particle-wall collision is handled using the model presented in Lun and Bent\textsuperscript{14}. All collisions are assumed instantaneous. During the impacts, the interstitial fluid effect is neglected. Rigid spherical particle has been considered. In general, roughness of the channel walls has statistical nature, both in terms of the height of the surface irregularities as well as in terms of their direction with respect to the impacting particle. Various models have been proposed\textsuperscript{4,5,7,15} to account for this randomness in simulating dilute two-phase flow through pipe/channel. As a first approximation, smooth walls are considered in this study, so that the direction of the normal to the wall is deterministic, not statistical.

FLUENT\textsuperscript{®} (Version 6.1) Discrete Phase Modeling has also been used to evaluate and compare the particle trajectory using both one-way and two-way coupling. It should be noted that FLUENT\textsuperscript{®} (Version 6.1) offers only Saffman lift in its Discrete Phase Model [FLUENT\textsuperscript{®} 6.1 Documentation].

Mathematical Formulation

In the Eulerian-Lagrangian method for dilute two-phase flows, the carrier-phase flow field is determined first, followed by the Lagrangian method of particle tracking based on solving the particle momentum equation. Reynolds averaged Navier-Stokes equations governing carrier-phase flow field, turbulence transport equations, boundary conditions, and equation of particle motion considering the lift forces due to shear and rotation are described in this section, followed by a brief description on the Discrete Phase Modeling used in FLUENT\textsuperscript{®}.

Equations governing carrier-phase flow

The basic equations governing the two-dimensional, steady, incompressible flow in a horizontal channel (Fig. 1) can be written in non-dimensional form as

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0,$$  \hspace{1cm} (1)

and

$$\frac{\partial}{\partial x} \left( \frac{1}{\sigma_k} \left[ \frac{\partial}{\partial x} \left( \frac{1}{\sigma_k} \frac{\partial k}{\partial x} \right) \right] \right) = \frac{\partial}{\partial y} \left( \frac{1}{\sigma_k} \frac{\partial k}{\partial y} \right) + G - \varepsilon,$$ \hspace{1cm} (4)

and

$$\frac{\partial}{\partial x} \left( \frac{1}{\sigma_e} \frac{\partial e}{\partial x} \right) = \frac{\partial}{\partial y} \left[ \frac{1}{\sigma_e} \frac{\partial e}{\partial y} + C_{1e} \frac{\varepsilon}{k} \left( \frac{\partial k}{\partial x} + \frac{\partial k}{\partial y} \right) \right]$$ \hspace{1cm} and

$$\frac{\partial}{\partial x} \left( \frac{1}{\sigma_e} \frac{\partial e}{\partial x} \right) = \frac{\partial}{\partial y} \left[ \frac{1}{\sigma_e} \frac{\partial e}{\partial y} + C_{1e} \frac{\varepsilon}{k} G - C_{2e} \frac{\varepsilon}{k} \right],$$  \hspace{1cm} (5)
where $\sigma_k = 1$, $\sigma_\varepsilon = 1.3$, $C_{iu} = 1.44$, $C_{2\varepsilon} = 1.92$, ... (6)

and $G = \frac{\mu_r}{Re} \left[ 2 \left( \frac{\partial u}{\partial x} \right)^2 + 2 \left( \frac{\partial v}{\partial y} \right)^2 + \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 \right]$. ... (7)

**Boundary conditions**

(i) At the channel inlet:

$$U = U_0, V = 0, k = 1.5T_i^2, \varepsilon = C_{\mu} \frac{k^{3/2}}{l}, \quad ... (8)$$

where, $l = 0.07H$, $H$, is the characteristic length (taken as the width of the channel). $T_i$ is the turbulence intensity at the inlet (specified in the range of 1-5%).

(ii) At the exit, normal derivatives of all variables are zero.

(iii) From the flow field at the previous iteration, friction velocity is determined and wall boundary conditions are applied at wall nodes.

The eddy viscosity, $\mu_r$, is computed as

$$\mu_r = \rho C_{\mu} \frac{k^2}{\varepsilon}. \quad ... (9)$$

**Equation of particle motion**

Once the carrier-phase flow field is obtained, the motion of a particle is tracked via Lagrangian approach. First, we present a brief discussion on the lift forces due to velocity shear and particle rotation, respectively, followed by the mathematical formulation of the equation of particle motion.

**Slip-shear lift force**

A particle moving in a strong shear layer experiences a transverse lift force due to the non-uniform relative velocity over the particle and the resulting non-uniform pressure distribution. This lift force acts in the direction of higher slip velocity as shown in Fig. 1.

Using asymptotic expansion, Saffman\textsuperscript{3,17} derived the expression for shear lift force acting on a freely rotating particle moving with a constant velocity in a linear shear flow at very low particle Reynolds number as

$$F_{LS} = 6.46 \rho \left( \frac{d_p}{2} \right)^2 \left( u - u_p \right) \left| \frac{du}{dy} \right| \frac{d\omega_p}{dy} \sqrt{\frac{d\omega_p}{dy}}.$$

where $\rho$ and $\nu$ are the fluid density and kinematic viscosity, respectively; $d_p$ is the diameter of the sphere; $u$ and $u_p$ are the velocities of the fluid and the particle in $x$-direction; and $du/\partial y$ is the shear rate of mean flow. It was assumed in the derivation that

$$Re_p = \frac{|u - u_p| d_p}{\nu} << 1, \quad ... (11a)$$

$$Re_s = \frac{d^2_p |du/\partial y|}{\nu} << 1, \quad ... (11b)$$

and $Re_R = \frac{\omega_p d^2_p}{\nu} << 1$, \quad ... (11c)

where $Re_p$ is the particle Reynolds number, $Re_s$ is the shear Reynolds number, and $Re_R$ is the rotation Reynolds number, respectively, and $\omega_p$ is the rotational speed of the sphere. Eq. (10) can be used only when the above conditions [Eqs (11a)-(11c)] are satisfied. However, many practical situations involving particulate motion in turbulent flow with particle Reynolds number, $Re_p >> 1$, require modification of the shear lift force given by Eq. (10). Based on the calculations performed by Dandy and Dwyer\textsuperscript{18}, Mei\textsuperscript{19} proposed the following approximation for the shear lift force at finite $Re_p$ in the range $0.1 \leq Re_p \leq 100$ as

$$C_{LS} = \frac{F_{LS}}{F_{LS, Saff}} = \frac{C_{LS, Saff}}{F_{LS, Saff}} = \left( 1 - 0.3314 \beta^{1/2} \right) \exp \left( \frac{-Re_p}{10} \right) + 0.3314 \beta^{3/2}, \quad Re_p \leq 40,$$

$$= 0.0524 \left( \beta Re_p \right)^{1/2}, \quad Re_p > 40,$$

where, $\beta = 0.5 \frac{Re_s}{Re_p}$, and $F_{LS, Saff}$ (or $C_{LS, Saff}$) indicates the corresponding result obtained by Saffman\textsuperscript{3}.

**Fig. 1—Schematic of a sphere in a shear flow**
Slip-rotation lift force

The rotation of the particle in wall-bounded flows is induced primarily due to particle-wall and/or particle-particle collision. This rotation results in a deformation of the flow field around the particle, associated with a shift of the stagnation points, and a transverse lift force as shown in Fig. 2.

An analytical expression for the force acting on a spherical particle due to Magnus effect was derived for small particle Reynolds number as

\[ F_{LR} = \pi \left( \frac{d_p^3}{8} \right) \rho \left( \omega \times (\vec{u} - \vec{u}_p) \right), \]  

... (13)

where, \( \omega \) is the relative rotation given by:

\[ \omega = \frac{1}{2} \nabla \times \vec{u} - \omega_p. \]  

... (14)

Crowe et al. extended the rotation lift force for higher particle Reynolds number as

\[ F_{LR} = \frac{\rho \pi}{2} d_p^2 C_{LR} \left( \vec{u} - \vec{u}_p \right) \frac{\omega \times (\vec{u} - \vec{u}_p)}{|\omega|}. \]  

... (15)

For higher particle Reynolds number, the lift coefficient, \( C_{LR} \), in Eq. (15) is modified using the following correlation based on the data available in literature and experiments for \( \text{Re}_p < 140 \):

\[ C_{LR} = 0.45 + \left( \frac{\text{Re}_k}{\text{Re}_p} - 0.45 \right) \exp \left( -0.05684 \cdot \text{Re}_k^{0.4} \cdot \text{Re}_p^{0.3} \right) \]

\[ \text{for } \text{Re}_p < 140 \]  

... (16)

In the present study, the modified expressions of shear lift coefficient, \( C_{LS} \), and rotation coefficient, \( C_{LR} \), for higher particle Reynolds number have been used, as and where considered by the GROUP7-11.

The equation of motion of the particle can be written as

\[ \rho_p \frac{d\vec{u}_p}{dt} = -\nabla P + \frac{C_D \rho}{2} \left( \vec{u} - \vec{u}_p \right) \left( \vec{u} - \vec{u}_p \right) \]

\[ + \frac{C_L \rho}{2} \left( \vec{u} - \vec{u}_p \right) \left( \vec{u} - \vec{u}_p \right) \]

\[ + \nabla \left( \rho \omega \right) \left( \frac{d\vec{u}}{dt} - \frac{d\vec{u}_p}{dt} \right). \]  

... (17)

where \( \rho_p \) is the particle density; \( \forall \) is the particle volume; \( t \) is the time; \( \vec{u}_p \) and \( \vec{u} \) are, respectively, the particle and fluid mean velocities; \( p \) is the fluid pressure; \( C_D \) is the drag coefficient; \( C_L \) is the lift coefficient, \( A \) is the projected area, \( \pi d_p^2 / 4 \), of the particle of diameter \( d_p \); \( \rho \) is the fluid density; and \( C_v \) is the virtual mass coefficient. For spherical particles, \( C_v = 0.5 \) is taken.

The first term on the right-hand side of Eq. (17) is the force on the particle due to fluid pressure. The second term describes the viscous force due to relative motion of the particle in the fluid. The third term describes the combined effects of gravity and buoyancy. The fourth term describes the combined effects of lift forces due to shear and rotation. This term would be replaced by expression of lift force due to particle rotation and/or shear as considered by the GROUP7-11. The fifth term describes the inertia of the added mass (entrained with the relative motion). This last term is negligible when \( \rho \ll \rho_p \) (as in gas-solid systems).

The scalar and non-dimensional form of Eq. (17) can be expressed as

\[ \frac{du_p}{dt} = -\frac{1}{S + C_v} \frac{\partial P}{\partial x} + \frac{3C_D}{4d_p (S + C_v)} (u - u_p) \]

\[ + \frac{3C_L}{4d_p (S + C_v)} (v - v_p) \]

\[ + \frac{C_v}{S + C_v} \left( \frac{\partial u_p}{\partial x} + v \frac{\partial u}{\partial y} \right). \]

... (18)
and

\[
\frac{dv_p}{dt} = - \frac{1}{S + C_v} \frac{\partial p}{\partial y} + \frac{3C_p}{4d_p(S + C_v)} \left[ \vec{u} - \vec{u}_p \right] (y - v_p) + \frac{3C_l}{4d_p(S + C_v)} (-u + u_p) + \frac{1 - S}{S + C_v} \vec{g} + \frac{C_v}{S + C_v} \left( u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right).
\]

... (19)

All quantities in Eqs (18) and (19) are in non-dimensional form. Velocity is non-dimensionalized with respect to the inlet uniform flow velocity, \( U_o \); pressure with respect to \( \rho U_o^2 \); time with respect to \( L/U_o \); and all lengths with respect to the channel length \( L \). The non-dimensional quantity, \( \ddot{g} \), is defined as

\[
\ddot{g} = gL/U_o^2.
\]

... (20)

The drag coefficient, \( C_D \), is calculated from the empirical correlation,

\[
C_D = \begin{cases} 
0.44 & \text{when } Re_p > 1000, \\
(24/Re_p)(1 + 0.14Re_p^{0.7}) & \text{when } Re_p \leq 1000.
\end{cases}
\]

... (21)

Table 1 lists the lift coefficients used by the GROUP\textsuperscript{7-11} in their models. Table 2 lists the various expressions for slip-shear lift force, \( F_{LS} \); slip-rotation lift force, \( F_{LR} \); and the rate of change of particle angular momentum\textsuperscript{13} due to its interaction with surrounding fluid.

**Discrete phase modeling using FLUENT\textsuperscript{*}**

The Lagrangian discrete phase formulation used by FLUENT\textsuperscript{*} (Version 6.1) assumes that the second phase is sufficiently dilute so that particle-particle interactions and the effects of the particle volume fraction on the gas phase are negligible. In practice, these issues imply that the discrete phase must be present at a fairly low volume fraction, usually less than 10-12%.

FLUENT\textsuperscript{*} offers both one-way and two-way coupling between the two-phases. In one-way coupling (Uncoupled approach), the discrete phase patterns are predicted based on a fixed continuous phase (or carrier-phase) flow field. Whereas, the continuous phase flow pattern is impacted by the discrete phase (and vice versa) in two-way coupling, and calculations of the continuous phase and discrete phase equations are alternated until a converged coupled solution is achieved.

The trajectory is initiated by defining in the discrete phase model, the initial position, velocity, size, and temperature (if applicable) of the individual particles along with inputs defining the physical properties of the discrete phase. However, FLUENT\textsuperscript{*} does not offer the choice of specifying initial particle rotation in its discrete phase model.

The trajectory calculations are based on the force balance on the particle using the continuous phase conditions as the particle moves through the flow field. This force balance equates the particle inertia with the forces acting on the particle. FLUENT\textsuperscript{*} considers various forces [FLUENT\textsuperscript{*} 6.1 Documentation] acting on the particle along with drag and gravity forces, such as the virtual mass force, forces (Coriolis and centrifugal) in rotating reference frames (if applicable), thermophoretic force, Brownian force and Saffman’s lift force. The lift force due to particle rotation is not included in the FLUENT\textsuperscript{*} Lagrangian discrete phase model. Moreover, the Saffman’s lift force in FLUENT\textsuperscript{*} is intended for small particle Reynolds numbers. A restriction is imposed that the particle Reynolds number, \( Re_p \), must be smaller than the square root of the particle Reynolds number based on the shear field. Since this restriction is valid for submicron particles [FLUENT\textsuperscript{*} 6.1 Documentation], FLUENT\textsuperscript{*} recommends to use this option only for submicron particles.

In summary, the general procedure for setting up and solving a discrete phase problem [FLUENT\textsuperscript{*} 6.1 Documentation] is as follows: (i) Solve the continuous phase flow; (ii) Set the discrete phase injections, i.e., particle initial position, velocity, size and so on; (iii) Solve the coupled (two-way coupling) flow, if desired; and (iv) Track the discrete phase injections and plot the results.

Step (iii) is necessary only when two-way coupling is considered.

**Numerical Method**

Galerkin finite element method is used to formulate the governing carrier-phase flow and turbulent transport equations. Particle equation of motion is solved numerically using the fourth-order Runge-Kutta method.
<table>
<thead>
<tr>
<th>Case</th>
<th>Models by</th>
<th>$C_{L_5}$ (Shear lift coefficient)</th>
<th>$C_{L_R}$ (Rotation lift coefficient)</th>
<th>$C_R$ (Rotation coefficient)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Tsuji et al</td>
<td>$C_{L_5} = \begin{cases} 0.5\sigma \quad (\sigma \leq 1.0) \ 0.5 \quad (\sigma &gt; 1.0) \end{cases}$</td>
<td>$C_{L_R} = \frac{6.45}{Re_x^{0.5}} + \frac{32.1}{Re_x}$</td>
<td>$C_R = \frac{6.45}{Re_x^{0.5}} + \frac{32.1}{Re_x}$</td>
</tr>
<tr>
<td></td>
<td>Not considered</td>
<td>$C_{L_5} = (0.4 \pm 0.1) \frac{d_{p}}{u_{*}} \frac{\phi}{(u - u_{p})}$</td>
<td>$C_{L_R} = \frac{d_{p}}{u_{*}} \frac{\phi}{(u - u_{p})}$ for $Re_p \leq 0.1$</td>
<td>$C_R = \frac{6.45}{Re_x^{0.5}} + \frac{32.1}{Re_x}$</td>
</tr>
<tr>
<td>2.</td>
<td>Frank et al</td>
<td>$C_{L_5} = (1 - 0.3314Re_{p}^{0.5}) \exp(-1.0Re_{p}) + 0.3314Re_{p}^{0.5}$ for $(Re_p \leq 0.1)$</td>
<td>$C_{L_R} = \frac{d_{p}}{u_{*}} \frac{\phi}{(u - u_{p})}$ for $Re_p &gt; 0.1$</td>
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<tr>
<td></td>
<td>Not considered</td>
<td>$C_{L_5} = 0.0524(Re_{p}Re_{p})^{0.5}$ for $(Re_p &gt; 0.1)$</td>
<td>$C_{L_R} = \frac{d_{p}}{u_{*}} \frac{\phi}{(u - u_{p})}$ for $0.1 &lt; Re_p &lt; 1000$</td>
<td>$C_R = \frac{6.45}{Re_x^{0.5}} + \frac{32.1}{Re_x}$ for $20 \leq Re_p \leq 1000$</td>
</tr>
<tr>
<td></td>
<td>Laun &amp; Liu</td>
<td>$Re_p = \frac{d_{p}}{d_{p}/(2v_0)}$</td>
<td>$C_{L_5} = (0.178 + 0.822Re_{p}^{0.52})$ for $1 &lt; Re_p &lt; 1000$</td>
<td>$C_{L_5} = \frac{d_{p}}{u_{*}} \frac{\phi}{(u - u_{p})}$ for $20 \leq Re_p \leq 1000$</td>
</tr>
<tr>
<td>3.</td>
<td>Sommerfeld</td>
<td>$C_{L_5} = \begin{cases} 0.45 \frac{Re_{p}}{Re_{p} - 45} \exp(-0.05684Re_{p}^{0.6}Re_{p}^{0.3}) \quad for \quad Re_p &lt; 140 \frac{Re_{p}}{Re_{p} - 45} \end{cases}$ for $Re_p &lt; 140$</td>
<td>$C_{L_R} = \frac{d_{p}}{u_{*}} \frac{\phi}{(u - u_{p})}$ for $20 \leq Re_p \leq 1000$</td>
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<tr>
<td>4.</td>
<td>Pan et al.</td>
<td>$Re_p = \frac{d_{p}}{d_{p}/(2v_0)}$</td>
<td>$C_{L_5} = \begin{cases} 0.45 \frac{Re_{p}}{Re_{p} - 45} \exp(-0.05684Re_{p}^{0.6}Re_{p}^{0.3}) \quad for \quad Re_p &lt; 140 \frac{Re_{p}}{Re_{p} - 45} \end{cases}$ for $Re_p &lt; 140$</td>
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<tr>
<td>5.</td>
<td></td>
<td>$Re_p = \frac{d_{p}}{d_{p}/(2v_0)}$</td>
<td>$C_{L_5} = \begin{cases} 0.45 \frac{Re_{p}}{Re_{p} - 45} \exp(-0.05684Re_{p}^{0.6}Re_{p}^{0.3}) \quad for \quad Re_p &lt; 140 \frac{Re_{p}}{Re_{p} - 45} \end{cases}$ for $Re_p &lt; 140$</td>
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</tr>
</tbody>
</table>

where, $\bar{V} = \bar{u} - \bar{u}_p$, and $\Omega = \frac{1}{2} \nabla \times \bar{u} - \bar{\omega}_p$
Table 2—Lift forces due to shear and particle rotation considered by the GROUP

<table>
<thead>
<tr>
<th>Case</th>
<th>Models by</th>
<th>Shear Lift</th>
<th>Rotation Lift</th>
<th>Torque Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Tsuj et al.\textsuperscript{7}</td>
<td>Not considered</td>
<td>( F_{\text{LS}} = \frac{3}{4} \frac{\rho}{\rho_p} (C_{\text{LS}}/d_p)(v - v_p) \left( \sqrt{(u - u_p)^2 + v_p^2} \right) )</td>
<td>( I \frac{d\omega_p}{dt} = -\frac{\rho}{2} \left( \frac{d_x}{2} \right)^3 C_x \omega_p^3 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>( F'<em>{\text{LS}} = -\frac{3}{4} \frac{\rho}{\rho_p} (C</em>{\text{LS}}/d_p)(u - u_p)^2 )</td>
<td></td>
</tr>
<tr>
<td>2.</td>
<td>Frank et al.\textsuperscript{8}</td>
<td>Not considered</td>
<td>[ \begin{bmatrix} F'<em>{\text{LS}} \ F</em>{\text{LS}} \end{bmatrix} \right] = K_m \text{Re}<em>p C</em>{\text{LS}} \begin{bmatrix} v - v_p \ -(u - u_p) \end{bmatrix} ]</td>
<td>( I \frac{d\omega_p}{dt} = -\frac{\rho}{2} \left( \frac{d_x}{2} \right)^3 C_x \omega_p^3 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>where, ( K_m = \frac{3}{4} \frac{\nu_p}{\rho_p d_p^2} )</td>
<td></td>
</tr>
<tr>
<td>3.</td>
<td>Lun &amp; Liu\textsuperscript{9}</td>
<td>( F_{\text{LS}} = 1.615 (\rho u)^{1/2} d_j C_{\text{LS}} \frac{du}{dy} (u - u_p) e_y / \sqrt{\frac{du}{dy}} )</td>
<td>( F_{\text{LS}} = \frac{1}{2} \rho \nu_p \frac{\pi d_j^2}{4} C_{\text{LS}} \Omega \times \bar{v} )</td>
<td>( I \frac{d\omega_p}{dt} = -\frac{\rho d_j^3}{64} C_x</td>
</tr>
<tr>
<td>4.</td>
<td>Sommerfeld\textsuperscript{10}</td>
<td>( F_{\text{LS}} = \frac{\rho \pi}{2} d_j C_{\text{LS}} \left((\bar{u} - \bar{u}_p) \times \bar{\omega} \right) )</td>
<td>( F_{\text{LS}} = \rho \pi d_j^2 C_{\text{LS}} \left</td>
<td>\bar{u} - \bar{u}_p \right</td>
</tr>
<tr>
<td>5.</td>
<td>Pan et al.\textsuperscript{11}</td>
<td>( F_{\text{LS}} = -1.62 d_j^2 \rho u \sqrt{\left</td>
<td>\frac{\partial u_x}{\partial y} \right</td>
<td>} )</td>
</tr>
</tbody>
</table>

Note: linear relative velocity, \( \bar{v}_p = \bar{u} - \bar{u}_p \); fluid rotation \( \bar{\omega} = \nabla \times \bar{u} \); relative rotation, \( \hat{\Omega} = \frac{1}{2} \nabla \times \bar{u} - \bar{\omega} \); \( u_{x_0} \) = relative velocity in x-direction
Numerical formulation of carrier-phase flow

The governing equations (1-5) are cast into weak algebraic form using Galerkin finite element method. Velocity components, \( U \) and \( V \), and turbulent quantities, \( k \) and \( \varepsilon \), are interpolated bilinearly using \( \Omega_1\Omega_0 \) elements. Pressure, \( P \), within an element is taken as constant. \( N_i \) is chosen as the weight function for both the momentum equations and the turbulent transport equations; whereas, for the continuity equation, the weight function is simply unity. Details of complete finite element modeling of governing equations and turbulent transport equations are presented in Pagalthivarthi and Gupta \(^{24}\) and Gupta and Pagalthivarthi \(^{16}\), respectively. Combined quasi-Newton’s method is employed to solve for the carrier-phase flow field, whereas, Picard type iteration is used to solve for the turbulent quantities. RAM-based frontal solution technique is used to solve the linear system of equations resulting from both Newton’s algorithm and Picard’s algorithm.

Numerical procedure for particle motion

Eqs (18) and (19) represent initial value problem, and are numerically integrated using the fourth order Runge-Kutta method\(^{26}\). The initial position, velocity and the angular speed of the particle are specified at the channel inlet. The initial particle velocity is taken as equal to 50% of the fully developed carrier fluid velocity.

Particle-wall collision is modeled using the sticking-sliding collision model\(^{14}\). Figure 3 shows the schematic of the particle-wall collision process.

For a particle colliding with a flat wall, two types of collision can be distinguished, namely, collision without sliding, and collision with sliding, estimated by using impulsive equations\(^6\).

(a) For a non-sliding collision,

\[
\frac{u_{p1} - d_p}{2} \omega_{p1} < \frac{7}{2} \mu_0 (1 + e) v_{p1}, \quad \ldots \quad (22)
\]

then, the particle translational and rotational velocities after the collision are obtained as

\[
u_{p2} = \frac{1}{7} \left( 5u_{p1} + d_p \omega_{p1} \right), \quad \ldots \quad (23a)
\]

\[
u_{p2} = -ev_{p1}, \quad \ldots \quad (23b)
\]

and \( \omega_{p2} = 2 \frac{u_{p2}}{d_p} \). \quad \ldots \quad (23c)

(b) For a sliding collision,

\[
u_{p2} = u_{p1} - \mu_d (1 + e)v_{p1} \varepsilon_0, \quad \ldots \quad (24a)
\]

\[
u_{p2} = -ev_{p1}, \quad \ldots \quad (24b)
\]

and \( \omega_{p2} = \omega_{p1} + 5 \mu_d (1 + e) \frac{v_{p1}}{d_p} \varepsilon_0, \quad \ldots \quad (24c)
\]

where,

\[
\varepsilon_0 = \text{sign} \left( u_{p1} - \frac{d_p}{2} \omega_{p1} \right), \quad \ldots \quad (25)
\]

In the Eqs (22)-(25), \( e \) is the coefficient of restitution; \( \mu_0 \) and \( \mu_d \) are the coefficients of static and dynamic friction, respectively; \( \varepsilon_0 \) indicates the direction of the motion of the particle surface relative to the wall; and subscripts 1 and 2 indicate quantities just before and after the collision, respectively.

Results and Discussion

Computational economy of \( \Omega_1\Omega_0 \) elements, mesh refinement studies and comparison of carrier-phase flow results with existing experimental results in the open literature are discussed elsewhere\(^{24}\). Extensive mesh refinement studies\(^{24}\) carried over 100×16, 150×24 and 200×32 elements establish computational economy of \( \Omega_1\Omega_0 \) elements and partial validation of the code. For the sake of completeness, some results of carrier-phase flow are shown in Fig. 4. Non-dimensional contours of turbulent kinetic energy, \( k \), turbulent dissipation rate, \( \varepsilon \), and ratio of turbulent viscosity to laminar viscosity are presented in Figs 4a-4c, respectively. Contours in Figs 4(a)-4(c) are symmetrical about the centerline axis. As anticipated, both kinetic energy, \( k \), and dissipation rate, \( \varepsilon \) increase drastically near the walls. In Fig. 4d, velocity vectors of carrier-phase flow are shown. Note that at the walls, the non-zero velocity vector is due to the velocity being computed using the wall functions.
In order to validate the code, results obtained by the present study are compared with previously reported simulations in a horizontal channel. The parameters shown in Table 3 have been used throughout this study. In the figures of this section, all quantities are non-dimensionalized, unless otherwise stated. For the purpose of clarity, particle(s) released at \( y = H/2 \), and/or \( y = H/4 \) and \( y = 3H/4 \) are considered.

In Fig. 5a, the computed particle trajectories of the present study are compared with the simulated results of Case 1 of Table 2. Lift due to shear was not considered by Tsuji et al.\(^7\) The lift coefficient, \( C_{LR} \), given in case 1 of Table 1 was used in computing particle trajectories. The qualitative trends, particularly the peaks reached by the particles after colliding with channel walls agree closely even without two-way coupling being used in the present study. The slight discrepancies in the trajectories (refer Fig. 5a) could be attributed (Tsuji Y, personal communication) due to the reasons that Tsuji et al.\(^7\) used mixing length model for the eddy viscosity while the \( k - \varepsilon \) model is used in the present study for predicting carrier-phase flow field; and Tsuji et al.\(^7\) used the drag coefficient after Morsi and Alexander\(^25\) which is different from the one used in the present study. These small differences caused by reasons aforementioned are expanded in the result (Fig. 5).

Based on parameters in Table 3, the present computed results are also compared with that of FLUENT® using one-way coupling [see Fig. 5(b)]. FLUENT® offers only Saffman lift force. In the present computation (see Figure 5), lift force due to shear (Saffman lift) is neglected for the purpose of comparison. The two results are fairly comparable even without the Saffman lift force being considered in the present computation. However, a significant

![Figure 4](image-url)

**Table 3**—Parameters used in the present study

<table>
<thead>
<tr>
<th>( U_0 ) (m/s)</th>
<th>( L ) (m)</th>
<th>( H ) (m)</th>
<th>( \rho_p ) (kg/m(^3))</th>
<th>( \rho ) (kg/m(^3))</th>
<th>( \varepsilon )</th>
<th>( \mu_0 )</th>
<th>( \mu_d )</th>
</tr>
</thead>
<tbody>
<tr>
<td>15.0</td>
<td>5</td>
<td>0.025</td>
<td>1000</td>
<td>1.31</td>
<td>0.8</td>
<td>0.4</td>
<td>0.4</td>
</tr>
</tbody>
</table>
influence on the particle trajectory is observed by including the lift force due to particle rotation [see Figure 5(b)].

In Fig. 6a, the successive peaks attained by the particle(s) are shown for the present study and case 1 of Table 2. The peaks match very closely except the initial peak(s) reached by the particle(s). This quantitative difference is due to unavailability of the particle initial rotation specified at the channel inlet (Tsuji, Y, personal communication). Similarly, Fig. 6b shows the position of particle impacting the channel base. The impacting positions of the particle released at \( y=H/4 \) do not match closely.

Figure 7 shows the comparison of simulated trajectories with FLUENT® 6.1 using one-way and two-way coupling. The trajectories are simulated with lift force due to shear only and no rotation. In the absence of specifying initial particle rotation, and quantitatively, in terms of particle lift (and trajectory), one-way coupling results seem to be in better agreement than the results of two-way coupling and agree closely for the identical parameters of Table 3. For practical purposes, two-way coupling makes no significant qualitative difference in the results. Moreover, two-way coupling requires more computation effort than one-way coupling.

The particle trajectories with and without lift forces are compared in Fig. 8. The effect of including lift forces is distinctly noticeable. The major contribution to the lift force comes from particle rotation. Particle-wall collision induces high particle rotational velocities that significantly contribute to slip-rotation lift force whereas slip-shear lift force is relatively very small.

Figure 9 shows the comparison of computed particle trajectories using the cases (1-5) of Tables 1 and 2. Figures 9a-9e are qualitatively similar. The influence of including different lift coefficients in computing the particle trajectories is significant. However, the computed particle trajectories using
Cases 3 and 4 of Tables 1 and 2 considerably vary from the other three cases. This could be reasoned as due to (i) considering shear lift coefficient, $C_{LS}$, and (ii) accounting for particle Reynolds number, $Re_p >> 1$.

In Fig. 10, for a particle released at $y=H/2$, variation of the particle rotational velocity along the channel length is compared for the GROUP7-11 [Cases (1-5) of Tables 1 and 2] considered in this study. The magnitude of the particle angular velocity goes on increasing along the flow due to repeated collisions. The sharp cusps (indicating an increase in the magnitude of particle rotation) correspond to the point of particle-wall collisions which confirms the results4,5 from open literature. Figure 10 also shows the variation of particle longitudinal velocity along the flow. At the channel inlet, $u_p=0.5$, 50% of the corresponding fluid velocity. Thereafter, the particle $u_p$ goes on increasing along the length of the channel since the fluid, which has greater velocity, drags the particle.

Figure 11 shows the variation of $x$-component ($R_{fx}$) and $y$-component ($R_{fy}$) of the slip-rotation lift force ($F_{LR}$) for all the five cases of Table 2. From Table 2 it can be observed that the $x$-component or ($R_{fx}$) of slip-rotation lift force is proportional to the $y$-component of the relative velocity, $v−v_p$. The $y$-component of the
particle velocity, \( v_p \) changes sign at the points of impact causing discontinuity at those points. Therefore, we observe a qualitatively similar trend in \( R_{fx} \) for all the five cases. \( R_{fx} \) attains zero value when the particle no longer bounces as \( v_p \) becomes zero. However, interesting results are observed in the variation of \( R_{fy} \) along the channel length. For cases 1, 2 and 5 of Table 2, we observe qualitatively similar trend in \( R_{fy} \). Since \( R_{fy} \) is directly proportional to \( \omega_p \); and since \( \omega_p \) increases along the flow direction (refer Fig. 10), it can be seen that \( R_{fy} \) follows similar trend as \( \omega_p \). For the case 4 of Table 2, a continuous decrease in the magnitude of \( R_{fy} \) is noticed as the particle traverses along the channel length.

Figure 12 shows the variation of slip-shear lift force, \( F_{LS} \), along the length of the channel for cases 3 and 4 of Table 2, respectively, for a particle released at \( y=H/2 \). The direction of the shear gradient and sign of the slip velocity determines the direction of the slip-shear force. In both the cases 3 and 4, the magnitude of shear force is very small, and thus it has a negligible effect on the particle trajectories. Though the slip-shear lift coefficient, \( C_{LS} \), is similar in both cases 3 and 4, considerable difference is observed in the variation of \( F_{LS} \) due to different expression used by cases 3 and 4 for \( F_{LS} \).

In Fig. 13, slip-rotation lift coefficient, \( C_{LR} \), is plotted as a function of particle Reynolds number, \( Re_p \), with the Reynolds number of rotation, \( Re_\sigma \), as a parameter for the cases 1, 3, 4 and 5 of Table 1. Figures 13a-13c are qualitatively similar. Figure 13d corresponds to case 1 of Table 1 where, 

\[
\sigma = \left( \frac{d_p}{2} \right) \frac{\omega_p}{(u-u_p)}
\]

can be expressed as \( \sigma = 0.5 \).
In the present computations, the range of $\text{Re}_p$ (for Case 1 of Table 1) was 0 – 0.5 as shown in Fig. 13d. For $\text{Re}_p > 1$, all models for rotation lift predict $C_{LR} < 0.5$, although the trends are somewhat different. In the present computations of channel flow, $\text{Re}_p > 1$ for practically all cases. It is observed that $C_{LR}$ is higher for higher rotation Reynolds number. This shows that with increasing angular velocity of the particle, the rotation lift force increases, and this is confirmed in the results (refer Figs 11a-11c).

In general, $C_{LR}$ is found to be significantly influenced by the particle rotation. Similarly, Fig. 14 shows the shear lift coefficient, $C_{LS}$, as a function of particle Reynolds number with the shear Reynolds number, $\text{Re}_s$, as a parameter (Cases 3 and 4 of Table 2).

$\text{Re}_p/\text{Re}_p$. For the operating range of particle Reynolds number $\text{Re}_p$ in the present computations, the range of $C_{LR}$ (for Case 1 of Table 1) was 0 – 0.5 as shown in Fig. 13d. For $\text{Re}_p > 1$, all models for rotation lift predict $C_{LR} < 0.5$, although the trends are somewhat different. In the present computations of channel flow, $\text{Re}_p > 1$ for practically all cases. It is observed that $C_{LR}$ is higher for higher rotation Reynolds number. This shows that with increasing angular velocity of the particle, the rotation lift force increases, and this is confirmed in the results (refer Figs 11a-11c).

In general, $C_{LR}$ is found to be significantly influenced by the particle rotation. Similarly, Fig. 14 shows the shear lift coefficient, $C_{LS}$, as a function of particle Reynolds number, $\text{Re}_p$, with shear Reynolds number $\text{Re}_s$ as a parameter. In the GROUP7-11 considered here, only cases 3 and 4 included the force due to velocity shear. It may be observed that the effect of shear force is significant for $\text{Re}_p \approx 1$.

Conclusions

Particle trajectory computations have been carried out using one-way and two-way coupling between particles and carrier-phase. Carrier-phase computations, validated in our previous studies, are performed using standard $k$-$\epsilon$ turbulence model and Galerkin finite element method. Five different models are considered to study the influence of lift forces on the trajectory of particle(s) entrained in a wall-bounded confined flow. The present code is validated for its computed particle trajectories with simulation of particle trajectories reported earlier. The following conclusions are drawn from the present study:

(i) Particle trajectories in wall-bounded confined flow are influenced considerably by using different lift coefficients. Results also indicate that the particle trajectories are influenced by using different expressions for the lift forces ($F_{LR}, F_{LS}$) though the corresponding lift coefficients ($C_{LR}, C_{LS}$) remain identical.

(ii) Lift force due to particle rotation is found to significantly affect the particle trajectory, whereas lift force due to shear has negligible effect on the particle trajectory. Very high particle rotational velocities of the order $10^4$-$10^5$ rad/s were induced due to the particle-wall collisions. These results are confirmed by the experimental studies reported in the open literature.

(iii) Computed particle trajectories with one-way coupling agree closely with the simulations reported by previous authors using two-way coupling. The results of particle trajectories with identical parameters using FLUENT® with one-way and two-way coupling also agree closely, and qualitatively, the difference is not very significant for practical purposes. One-way coupling on the other hand requires less computational effort.

(iv) The quantitative difference between the computed results and results reported by previous authors are attributed due to different specified particle rotation rate at the channel inlet, different model for predicting the carrier-phase flow field, and different drag coefficients used.

References

16. Gupta P K & Pagalthivarthi K V, Comparison of zero-equation and k-$\epsilon$ models in rotating channel flow prediction.
