A finite element based backcalculation program for flexible pavements

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In recent years pavement maintenance planning has become a main task among highway agencies. In order to determine the structural capacity of a flexible pavement, non-destructive testing equipment is used. Many countries use the falling weight deflectometer (FWD). It is possible to obtain deflection data and backcalculate the pavements’ mechanical properties. However, the method used affects the backcalculated values. A user-friendly finite element code has been developed in order to backcalculate pavement properties. The problem in using any finite element program is to prepare mesh data. In order to facilitate the mesh preparation, a mesh generator have been developed. The constitutive equations for the bituminous mixture are visco-elastic, linear elastic, visco-elasto-plastic; for granular layers non-linear elastic (K-θ Model, Boyce Model, Pappin Model, Elhannani Model) and linear elastic, while for the subgrade non-linear elastic and linear elastic are used. Considerable deflection data are analyzed during the study using different models. The results indicate that the back-calculated properties depend upon the model which has been used. Therefore, using simple models can cause either over-design or under-design problems. The program is then tested against other back-calculation programs. The results indicate that the proposed program is better than other existing programs.

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In recent years, highway authorities have an increased pressure for the maintenance and rehabilitation planning of flexible pavements. Structural evaluation of the pavement is essential in order to assess the need for maintenance and rehabilitation. Efficient and economical methods are required for determining the structural capacity of existing flexible pavements. Since highway infrastructure deteriorates by the time, maintenance, rehabilitation and reconstruction of road networks have become important task. Nowadays, among highway agencies, a large proportion of highway funds is being used for maintenance, rehabilitation and reconstruction of pavements. The structural capacity of the existing pavement system needs to be evaluated as part of the rehabilitation design process. Considerable savings in maintenance and rehabilitation cost can be made by predicting accurately the remaining life of existing flexible pavement. Backcalculating the structural capacity of flexible pavements from non-destructive test data is one of the most efficient and economical method. Several non-destructive in-situ tests have been developed that provide information about the material properties of a flexible pavement. Among them, the Falling Weight Deflectometer (FWD) test has become one of the more popular because of the large amount of high-quality information that can be gathered in a quick, straightforward manner. The main purpose of carrying out this test is to find the stiffness characteristics of each layer. FWD test applies an impact load to the pavement to simulate the transient nature of traffic loading. Many countries use FWD as a non-destructive testing (NDT) equipment. Deflection data obtained from FWD is used to backcalculate the pavement mechanical properties such as elastic modulus, Poisson’s ratio. From an FWD data, interpretation of view, it is desirable to relate the coefficient of variation of surface deflections to the variations in layer moduli. However, the models used for backcalculation algorithm affect the backcalculated mechanical properties. Until now, many highway agencies throughout the world used simple methods such as linear elastic theory and equivalent layer thickness. Uzan developed the MODULUS program using linear elastic theory. Huang developed KENLAYER program with using the same method. The same method was used in developing many programs such as PADAL, ELSYM5, BISTRO, CHEVRON, MODCOMP, EFROMD, Ullidtz used the Equivalent Layer Thickness (ELT) Method and developed the ELMOD program. Zhou developed the BOUSDEF program in 1990 using the ELT method. Nowadays, the finite element method is extensively used paralleling with new technologies. The finite element method is especially attractive for the structural

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analysis of pavements when the non-linear behaviour of the granular and cohesive materials used in pavements is to be considered in mechanistic modelling. The finite element method—which is able to easily handle different geometries, materials properties, and boundary conditions—is widely used for solving complex problems. Barksdale developed the GAPS series and Wilson and Duncan developed the ILLI-PAVE program using the finite element method. This method was used in developing programs such as FENLAP, FEAD, and SAPIV. In this paper, a new backcalculation program, SDUFEM, is described.

The SDUFEM program uses the finite element method with non-linear material models. SDUFEM has also capability to simulate actual pavement loading conditions. A number of material models can be used to represent material characteristics such as elasticity, visco-elasticity, and plasticity. An iterative procedure is used to backcalculate the pavement layer moduli. A major effort in using any finite element program is to prepare the mesh data. In order to facilitate mesh preparation, a mesh generator routine was developed. The program also considers non-linearity of the wearing course layer, base layer, sub-base layer, and sub-grade materials. The program is user friendly. The theoretical basis for the software is first presented and SDUFEM is then used to analyze flexible pavement systems in the study.

Nature of FWD Testing

In order to simulate the truck loading on the pavement, a circular plate is dropped onto the pavement from a set height. The height is adjusted according to the desired load level. Underneath the circular plate a rubber pad is mounted to prevent shock loading. Seven geophones (the number of geophones can change) are generally mounted on the trailer. When the vertical load is applied on the pavement, the geophones collect the data which is converted into the real deflections.

The Benkelman beam and dynaflact, which are mostly used in the developing countries only give information for underneath the circular plate. Whereas the FWD gives additional information for six other points away from the circular plate. The FWD is a trailer-mounted device which has been established worldwide as one of the most effective tools for measuring deflections for pavement evaluation purposes.

There are many types of FWDs which can apply the same loading. The frequencies of loading change between 0.025 and 0.030 s; the applied loads vary between 6.7-156 kN. The loads are generally applied in a sinusoidal form. The loading time of 0.030 s represents the wheel loading moving at a speed of 30 km/h and ±0.023 mm deviations can be seen from the FWD measurements. A crew can carry out 200-300 FWD measurements in a day.

Interpretation of FWD Measurements

A typical deflection bowl obtained from the FWD loading is shown in Fig. 1. The maximum deflection is obtained underneath the circular plate and points which are away from the circular load application point have smaller deflections.

When the FWD loading is applied, the load and the deflections due to the FWD loading are known. However, for the structural analysis, layer thicknesses must be known. Layer thickness can be determined from the highway plans (if available) or geo-radar. Therefore, there are only two unknowns in the problem, the elastic modulus and the Poisson’s ratio of each layer. Generally, the effect of Poisson’s ratio is negligible and can be ignored in the pavement analysis, therefore, a constant value of the Poisson’s ratios are generally assigned to each layer. For this purpose, computer programs are written which use linear elastic theory and the finite element method. In the linear elastic theory, all materials in the pavement are assumed to behave linearly, which is not a valid assumption for the pavement materials. Granular materials and soils especially behave in a non-linear
manner. Apart from this, there is a constraint in road width which is again not a valid assumption. Therefore, in recent years, the finite element method has been used to overcome the problems. Above finite element program is written by the first author. A problem in using finite element programs is the time taken to prepare the mesh for each problem. Further, an iteration approach is used to find the solution and this again increases the solution time.

Backcalculating the Layer Moduli of the Pavement

Non-destructive testing (NDT) enables the use of a mechanistic approach to pavement design and rehabilitation since the in-situ material properties can be backcalculated from the measured field data through appropriate analysis techniques. Backcalculating the layer moduli from pavement deflection bowls is a promising method of determining the performance of in-service pavements. The evaluation of material properties of existing in-service pavements is a fundamental problem in pavement engineering. Until now, highway agencies all over the world used traditional simple methods such as linear elastic theory and equivalent layer thickness. Using these simple methods, it is impossible to evaluate the material properties of in-service pavements in a realistic manner. But the finite element method is highly efficient in non-linear formulations for it can accommodate easily changes in material properties, allowing for variations in both the vertical and horizontal directions. The finite element method employs a process of discretisation whereby the structure is sub-divided into a number of elements, connected at the nodal points. Each element, can therefore, has an elastic stiffness consistent with its stress level. Nowadays, the finite element method is increasingly used with parallelising new technologies.

In this paper, the newly developed backcalculation program (SDUFEM) is described. SDUFEM is a user-friendly program in which the backcalculation layer moduli for flexible pavements based on deflections measured by a FWD is determined. SDUFEM uses the finite element method for calculating pavement deflections under a FWD loading. In order to facilitate the mesh preparation, a mesh generator was developed.

The deformed shape of a pavement when subjected to a vertically oriented surface loading is known as a deflection basin. The program attempts to match the measured deflection basin for a system with unknown layer moduli with the theoretical deflection basin for a system with known layer moduli. This process is called backcalculation of layer moduli.

Initially, the finite element mesh is generated using a mesh generator. The program then reads the input data sets which include: system mesh data, FWD load force and load radius, pavement layer thicknesses, Poisson’s ratio, chosen moduli of the layers, density of pavement materials, deflection data and the percent tolerance to stop calculations the deflection matching process. Deflections for the given FWD load and load radius are calculated using the finite element technique. The calculated deflections are compared to the measured deflections. If the calculated deflections do not match sufficiently to the measured deflections, moduli values are adjusted by using correction factors.

For the backcalculation analysis, an initial elastic modulus is set-up for each layer. The FWD loading is applied vertically onto the mesh and the resulting deflections are compared with the error functions shown below. If the error is not within the acceptable range, the elastic modulus is changed until the error function is satisfied. Following the satisfying the error function, the elastic modulus for each layer is assumed to be known. Hence, a forward analysis is carried out with the obtained values, in order to find the tensile strain underneath the bituminous mixture and the vertical strain on the sub-grade. The results obtained from the forward analysis are entered into the fatigue and plastic deformation graphics. From these two graphics, the remaining life of the pavement is determined. Finally, a decision is made whether an overlay is necessary. For an objective decision, the following relative error square summation (RSS), definition is employed:

\[
RSS = \sum_{s} \left( \frac{d_i^m - d_i^h}{d_i^m} \right)^2 = \sum_{s} \left(1 - \frac{d_i^h}{d_i^m} \right)^2 \quad \ldots (1)
\]

where \(d_i^h\) = calculated deflections in \(i\)-th geophone; \(d_i^m\) = measured deflections in \(i\)-th geophone, and \(s\) = sensor number from \(i\) to \(s\). Many backcalculation programs have limitations on the maximum allowable number of layers in their analyses but SDUFEM has no such limitation.

Development of the SDUFEM Program

There are many programs developed for the structural analysis of flexible pavements, however it
is difficult to alter many of these programs for any special problems, since these programs are complex or access to the sources of programs is not possible. For this reason, the development of the SDUFEM program has carried out. SDUFEM is written in FORTRAN.

The program requires input of the layer thickness of each layer, the vertical loads, the initial assumed elasticity moduli of the layers, and the loaded radius. For displacements and loads, the vertical downwards and the horizontal right directions are positive. Inputs of program data can be checked on screen, before running.

In the program, the finite element mesh is automatically formed using a mesh generator. Each layer consists of varying specific characteristic values and is introduced as a block. Meanwhile, the number of elements is defined and blocks are formed by user. The number of blocks is equal to that of the number of layers. The coefficients of each block describe the number of element in each block. Material type number is also entered at this stage. The co-ordinates of all elements are automatically determined by the program. The mesh generation process is therefore extremely simple. It is possible to indicate each block and to show the mesh with its nodal numbers and element numbers and to enlarge the mesh.

Granular materials used for flexible pavements behave in a non-linear elastic manner while bituminous materials behave in visco-elastic manner. In the program, K-θ, Boyce, Elhannani and Uzan models for granular materials and four different models for visco-elastic materials can be used. The flow diagram of the program is shown in Fig. 2.

### Finite Element Method for Pavement Analysis used in SDUFEM

Since the finite element method has advantageous over the layered analysis in modelling the non-linear behaviour of materials, it could potentially provide a framework for a backcalculation procedure. FWD loading on the pavement represents an axisymmetric problem. Though the pavement layers are horizontally layered, the stiffness characteristics of these layers vary in lateral as well as in the vertical direction because of the stress-dependent nature of the unbound materials. The finite element method is best suited for such circumstances. According to the symmetry, both geometrical and mechanical, around a vertical axis which passes through the loading centre, it is advantageous to use cylindrical coordinates, \( z \), \( r \) and \( \theta \) for analysis. The radial symmetry makes the problem independent of the coordinate \( \theta \) Thus, the strains and stresses at any point are only functions of its vertical and radial coordinates, \( z \) and \( r \) respectively. The displacements can also be described in terms of the components in the vertical and radial directions, respectively \( v \) and \( u \). The components of stress and strain are schematically shown in Fig. 3. The corresponding vectors have the following form:

\[
\{ \sigma \} = \begin{bmatrix} \sigma_{zz} \\ \sigma_{rr} \\ \sigma_{\theta\theta} \\ \tau_{zr} \end{bmatrix} \quad \{ \varepsilon \} = \begin{bmatrix} \varepsilon_{zz} \\ \varepsilon_{rr} \\ \varepsilon_{\theta\theta} \\ \gamma_{zr} \end{bmatrix}
\]

According to the hypothesis of small displacements, strains are expressed in terms of the first derivations of the displacements. Using the differential operator \([\mathbf{S}]\),

\[
\{ \varepsilon \} = [\mathbf{S}]{\{ \varphi \}} \quad \ldots(3)
\]

can be written.
where \[ [S] = \begin{bmatrix} \frac{\partial}{\partial z} & 0 \\ 0 & \frac{\partial}{\partial r} \\ 0 & \frac{1}{r} \end{bmatrix} \]

and \( \{r\} = \begin{bmatrix} v \\ u \end{bmatrix} \)

If the materials assume homogeneous, isotropic and elastic, their stresses and strains can be related as follows:\(^{14}\):

\[
\{\sigma\} = [D]\{\varepsilon\} \quad \ldots(4)
\]

where \( [D] \) is matrix of elastic constants. The elasticity matrix \( [D] \) will not necessarily be the same for the all system, since Young’s modulus \( E \) and Poisson’s ratio \( \nu \) may vary, both vertically and horizontally. In SDUFEM, non-linear material models were used. Therefore, Young’s modulus \( E \) and Poisson’s ratio \( \nu \) values in elasticity matrix \( [D] \) are computed by using non-linear material models.

Mesh generation

In finite element technique, the preparation of input data is very often slow, tedious and prone to error. However, using an automatic mesh generation routine can save time.

As can be seen in Fig. 4, pavement analysis usually involves a relatively simple geometry of domain.

However, the mesh generator is very convenient for processing a large amount of data. In SDUFEM, there is a procedure that automatically generates the finite element mesh. The boundary conditions of each pavement layer, which has different material properties, are described as blocks and then element numbers of each block are input. Once each block is considered as an element, the coordinates of eight nodal points of the block are input. That is, a layer is firstly modelled using eight-node rectangular element, with four nodes at the vertices and four nodes at the mid-distances of the sides of block. The number of layers is input as the number of blocks. Then, the numbers of elements in radial and vertical directions are defined. Also, element frequency constants are input. So, the finite element mesh is generated. In addition, the number of the materials to be used in the pavement system can also be described at this stage.

All coordinates of elements in the finite element mesh are automatically appointed due to the input data in SDUFEM.

It is possible to have computer image of the blocks in different colours, which have different material properties and geometry and the finite element mesh generated. However, finite element mesh can also be seen along with its elements and nodal point numbers and the mesh can be enlarged.

Pavement system is modelled using eight-node rectangular elements, with four nodes at the vertices and four nodes at the mid distance of the sides (see Fig. 5). This type of elements has been extensively used in finite element applications for it gives a satisfactory level of accuracy despite its simplicity.

In order to obtain the displacement of a point from nodal displacement, it is essential to determine the shape functions for eight-node elements. The shape
functions are generally polynomial and must have the number of terms as same as the number of nodal displacements for each element. In SDUFEM, surface pressures are applied vertically, so the nodal forces exist only in the $z$ direction.

**Boundary conditions, material models adopted and program output**

The boundary conditions adopted in SDUFEM are shown in Fig. 6. User can input restrained nodal points and restraint type. In SDUFEM, all nodes along the bottom of the mesh are fixed. Elements are free for vertical displacement and radial displacement are not allowed in SDUFEM.

Bituminous layer of flexible pavements shows the visco-elastic behaviour. A visco-elastic material possesses both the elastic property of a solid and the viscous behaviour of liquid. Due to the viscous component, the behaviour of visco-elastic materials is time dependent and the longer the time the more the flow. As bituminous layer is a visco-elastic material whose behaviour depends on the time of loading, it is natural to apply the theory of visco-elasticity to the analysis of layered systems. The general procedure is based on the elastic-visco-elastic correspondence principle.

Figure 7 shows various mechanical models for characterising visco-elastic materials. The models are formed of two basic elements: a spring and a dashpot. An elastic material is characterised by a spring, as indicated in Fig. 7a, and obeys Hooke’s Law, whereby stress is proportional to strain:

$$\sigma = E \varepsilon$$  \(\ldots (5)\)

in which $\sigma$ is stress, $\varepsilon$ is strain, and $E$ is the elastic modulus. A viscous material is characterised by a dashpot, as indicated in Fig. 7b, and obeys Newton’s Law, whereby stress is proportional to the time rate of strain:

$$\sigma = \lambda \frac{\partial \varepsilon}{\partial t}$$  \(\ldots (6)\)

in which $\lambda$ is viscosity and $t$ is time. Under a constant stress, Eq. (6) can be easily integrated and becomes

$$\varepsilon = \frac{\sigma}{\lambda}$$  \(\ldots (7)\)

A Maxwell model is a combination of spring and dashpot in series, as indicated in Fig. 7c. Under a constant stress, the total strain is the sum of the strains of both spring and dashpot, or from Eqs (5) and (7)

$$\varepsilon = \frac{\sigma}{E_0} + \frac{\sigma t}{\lambda_0} = \frac{\sigma}{E_0} \left(1 + \frac{t}{T_0}\right)$$  \(\ldots (8)\)

in which $T_0 = \lambda_0 / E_0$ = relaxation time. A subscript 0 is used to indicate a Maxwell model.
Visco-elastic models used in SDUFEM

Four different visco-elastic models were assumed for the bituminous layer in SDUFEM. First, a Maxwell model was used for modelling bituminous layer (Fig. 8). The mechanical models may be converted to differential operators. Let $\frac{\partial}{\partial t} = D = \text{differential operators}$. Second, one Maxwell model and one Kelvin model, third, one Maxwell model and two Kelvin models, and fourth, one Maxwell model and three Kelvin models were used for bituminous layers in SDUFEM.

For a generalised model with one Maxwell and $n$ Kelvin models in series, by the reciprocal principle

$$\frac{\sigma}{\varepsilon} = \frac{1}{E_0 T_0 D_1 + \sum_{i=1}^{n} \frac{1}{E_i (T_i D_i + 1)}} \quad \ldots(9)$$

Granular material models used in SDUFEM

K-θ, Boyce, Elhannani and Uzan models were adopted for the granular layers, and the K-θ model was used for subgrade in SDUFEM. Granular material exhibit non-linear, stress dependent behaviour. The K-θ model has been widely used by pavement engineers to characterize the non-linear behaviour of granular material in which the resilient modulus is expressed in terms of the bulk stress$^{15}$. The model is written in the form$^{16}$:

$$E = A(3p_{\text{max}})^b \quad \text{or} \quad E = A p_a \left( \frac{3p_{\text{max}}}{p_a} \right)^b \quad \ldots(10)$$

where $A$, $B$ are material, $p_{\text{max}}$ is maximum mean normal stress and $p_a$ is atmospheric pressure. The model has been found extremely useful and simple and is still in use for pavement evaluation$^{17}$.

Another model used in SDUFEM is the Uzan model. The form of the model is shown below$^{15}$:

$$E = A(3p_{\text{max}})^b q^c$$

or

$$E = A p_a \left( \frac{3p_{\text{max}}}{p_a} \right)^b \left( \frac{q}{p_a} \right)^c \quad \ldots(11)$$

where $q$ is the deviatoric stress and $C$ is a material constant. The Boyce model was developed using the theorem of reciprocity. The form of the model can be expressed as follows:

$$\varepsilon_v = p_a \left[ \frac{1}{A} - \frac{(1 - B) q^2}{6C p^2} \right] \quad \ldots(12)$$

$$\varepsilon_s = \frac{p_a q}{3C p} \quad \ldots(13)$$

where $\varepsilon_v$, $\varepsilon_s$, are the volumetric and shear strain respectively; $p$, $q$ are the mean and deviatoric stresses respectively; $A$, $B$, $C$ are the material constants. And,

$$p = \frac{\sigma + \sigma_1 + \sigma_3}{3}, \quad q = \sigma_1 - \sigma_3$$

and $\varepsilon_v = \frac{q}{3G}$ equalities are known. Where for $K$ and $G$, $K = \frac{E}{3(1 - 2v)}$, and $G = \frac{E}{2(1 + v)}$ equalities are valid. The Elhannani model was also used in SDUFEM. Elhannani modified the Boyce model and the model then takes the form$^{18}$:

$$\varepsilon_v = p_a \left[ \frac{1}{A} - \frac{(1 - B) q^2}{6C p^2} - \frac{B q}{D p} \right] \quad \ldots(14)$$

$$\varepsilon_s = p_a \left[ \frac{1}{3C p} - \frac{1}{D} \right] \quad \ldots(15)$$

where $p_a$ is atmospheric pressure; $B$ and $D$ are the unitless coefficients. $\varepsilon_v = \frac{p}{K}$, and $\varepsilon_s = \frac{q}{3G}$.
The presentation of results has been provided with a graphical interface. It is possible to plot the nodal displacements, which are printed out in such a way that the differences between the original and deformed meshes are indicated. The graphic output of stress and strain contours has also been developed.

Example 1

In this example, a backcalculation process was applied on a three-layer pavement as shown in Fig. 9. Deflections were measured at seven points and the values used for backcalculating the layer’s elastic moduli as shown in Table 1. The top layer of asphaltic concrete has a thickness of 10 cm, the thickness of the unbound base layer is 30 cm. Poisson’s ratio values for the bituminous layer, and base layer and subgrade are equal to 0.35, 0.30 and 0.40, respectively. The representative contact pressure of 700 kPa is applied on the pavement surface. The pavement section of Fig. 9 was analysed with the LEAD, FEAD and SDUFEM programs. LEAD uses linear elastic theory, while FEAD and SDUFEM use the finite element method. Comparison with the results of other programs for Example 1 is given in Table 2. Backcalculated moduli values are close to other program results. SDUFEM gives acceptable backcalculated moduli values. Comparison of moduli values backcalculated from different programs is presented in Fig. 10. Resulting surface deflection values from forward calculations by using backcalculated moduli values are shown in Table 3. The smallest sum of least squares was obtained from SDUFEM. Visco-elastic material model for bituminous layer, Uzan model for base layer and K-θ model for sub-grade were used for Example 1.

Example 2

In this example, a backcalculation process was applied on a two-layer pavement as shown in Fig. 11.
Deflections were measured at five points and these values were used in backcalculating the layer’s elastic moduli (Table 4). Asphalt concrete layer has a thickness of 23 cm, the thickness of the unbound base layer is 460 cm. Poisson’s ratio values for the bituminous layer, base layer and sub-grade are equal to 0.35, 0.40 and 0.40, respectively. The representative contact pressure of 757 kPa is applied on the pavement surface. This pavement section was analysed using eight programs. Comparison with the results of other programs is given in Table 5. This examination of the new finite element program very closely represented the other backcalculation programs can be made by comparing the backcalculated results. Close agreement with the other programs is observed in Fig. 12. It can be seen the figure that backcalculated moduli values from SDUFEM matched the other program results. Resulting surface deflection values from forward calculations by using backcalculated moduli values are shown in Table 6. The smallest sum of least squares was obtained from SDUFEM. Visco-elastic material model for bituminous layer, Elhannani model for base layer and K-0 model for sub-grade were used for the example.

| Table 4—Deflections for example 2 |
|-----------------|-----------------|-----------------|-----------------|
| d1              | d2              | d3              | d4              | d5              |
| Deflections (cm)| 0.0584          | 0.0425          | 0.0325          | 0.0249          | 0.0116          |
| Sensor spacings(cm) | 20.32          | 25.4            | 30.48           | 76.2            |

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<tr>
<th>Table 5—Backcalculated elastic moduli values for example 2</th>
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<tr>
<td>Program</td>
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<tr>
<td>BISDEF</td>
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<td>BOUSDEF</td>
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<td>CHEVDEF</td>
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<td>ELSDEF</td>
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<td>MODCOMP</td>
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<td>BISAR</td>
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<td>BKGREEN</td>
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<td>SDUFEM</td>
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| Table 6—Computed deflections for different programs and sum of square errors |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Program         | MODCOMP         | BISDEF          | BOUSDEF         | CHEVDEF         | ELSDEF          | BISAR           | BKGREEN         | SDUFEM          |
| Elastmoduli (MPa) | d1              | d2              | d3              | d4              | d5              |                 |                 |                 |
| Bituminous Layer | 56.1            | 55.2            | 58.4            | 55.78           | 65.16           | 55.16           | 55.29           | 52.71           |
| Granular Base   | 46.69           | 46.41           | 48.45           | 46.31           | 46.45           | 46.46           | 45.54           | 39.80           |
| Subgrade        | 34.61           | 34.13           | 35.15           | 34.30           | 34.11           | 34.78           | 33.74           | 27.45           |
| Sum of square errors | 0.1934          | 0.1184          | 0.1582          | 0.0859          | 0.109           | 0.1311          | 0.1813          | 0.0846          |

Fig. 12—Comparison of moduli for example 2
Conclusions
In this study, the development of a finite element model to simulate the flexible pavement systems and to analyze flexible pavements is presented. SDUFEM has the capability to simulate actual traffic loads and can include linear and non-linear material models. Four different visco-elastic models and four different non-linear elastic models were introduced to model various materials and sub-grade in the program. The program is user-friendly. For benchmarking two examples were given. SDUFEM predictions were compared with actual pavement conditions by using literature examples. Computations indicated that the SDUFEM program gives the least sum of square errors. The finite element computer program developed in this study provides a useful numerical model for simulating the flexible pavement systems. The examples presented here have demonstrated the flexible pavement layer properties can be backcalculated easily and efficiently. The principal advantage of this new software is its computational efficiency and mesh generation easiness.

References