High field \( dc \) electrical conductivity in silicon

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An analytical expression for the conductivity of electrons in silicon in the presence of high \( dc \) electric field has been derived in the present paper by considering the asymmetric distribution function of electrons which is a function of electric field. This has resulted in dependence of the net number of electrons and their conductivity on electric field. The conductivity of electrons decreases drastically as electric field increases because of high \( dc \) electric field. The drift velocity of electrons experiences a transient effect and the time taken by electrons moving between successive collisions decreases as a result of quantum emission. The result of the present paper might be very important in characterising semiconductor devices where high field is primarily of concern.

Keywords: Electric conductivity, Silicon, Semiconductor devices, Hall coefficient

1 Introduction

Sharma\(^1\) had calculated the \( D/\mu \), i.e., diffusion mobility ratio in heavily doped \( n \)-type silicon using the modified density of states function. Tesfaye\(^2\) calculated Hall coefficient and coefficient of transverse magneto resistance in an intermediately doped \( n \)-type silicon with mixed scattering of electrons by both ionized impurities and acoustic phonons. However, these researchers\(^1,2\) considered only low fields (ohmic region).

In today’s technology, high field transport is of considerable interest because of the scaled down dimensions of sub-micron devices in the quasi free direction of carrier flow. In the presence of high electric field, the linearity of velocity-field characteristics breaks down that resulted in eventual saturation velocity of charge carriers under high electric field. Komirenko \textit{et al.}\(^3\) have investigated the problem of electron runaway at strong electric fields in polar semiconductors. Deug Yong Kim \textit{et al.}\(^4\) developed a non-linear transport theory of electrons in semiconductors in the presence of high electric field. Vijay\(^5\) presented analytic expressions for the Einstein ratio between diffusion coefficient \( (D) \) and mobility when a high electric field is applied to electrons in bulk silicon. The drift velocity of an electron is shown to be limited to the thermal velocity and diffusion coefficient remains unaffected in the absence of quantum emission. However, the presence of quantum emission in a high electric field lowers the saturation velocity and diffusion coefficients. The drift mobility of electrons measured by time-of-flight experiments in amorphous hydrogenated silicon showed dependence of mobility on the electric field.

However, not much theoretical work is reported on electrical conductivity in the presence of a high electric field. This paper presents analytic expressions for electrical conductivity in the presence of high field. The study of electrical conductivity in high \( dc \) field allows better design and commercialization of semiconductor devices for applications in the regime of high electric field. To derive analytic expressions for electrical conductivity in the presence of high field, the classical Boltzmann transport equation, steady state asymmetric distribution function for electrons, transient drift velocity of electrons, and non-degenerate electron approximation have been used.

2 Electrical Conductivity

In the presence of high electric field, the Fermi level and the band edge tilt parallel to each other and the Fermi energy changes by \( q\vec{E} \cdot \vec{l} \). Thus, the Fermi-Dirac distribution function \( f \) is a function of electric field and has the form:

\[
f = \left( 1 + \exp\left[ (\varepsilon - \mu + q\vec{E} \cdot \vec{l}) / k_B T \right] \right)^{-1}
\]

where \( q \) is the charge on electron, \( \mu \) the chemical potential, \( \vec{E} \) the applied electric field, \( \vec{l} \) the electron mean free path in vector form, \( T \) the absolute temperature and \( k_B \) is the Boltzmann constant. This
distribution function creates asymmetry in the number of electrons travelling parallel and antiparallel to the applied dc electric field, as shown in Fig. 1.

Assuming non-degenerate electrons, Boltzmann approximation and defining the probabilities of electrons travelling antiparallel and parallel to the applied electric field, the net number of electrons $dn_{net}$ contributing to electric current in a non-degenerate approximation is:

$$dn_{net} = dn \tanh\left(\frac{qE_l}{k_BT}\right)$$  \hspace{1cm} (2)$$

where $dn$ is the number of electrons per unit volume in the presence of low electric field. We see from this expression given in Eq. (2) that the number of charge carriers contributing to the electric current is a function of the applied field.

According to classical electrodynamics, a high electric field applied to semiconductors accelerates the electrons and the accelerated electrons emit quantum energy. This retards the motion of the electron. By taking the retarding force proportional to the velocity of the charge carriers, the equation of motion of conduction electrons indicates that after collision, the electrons drift velocity experiences a transient response given by:

$$v(t) = v_d \left(1 - \exp\left[-\frac{t}{\tau_o}\right]\right)$$  \hspace{1cm} (3)$$

where $v_d$ is the velocity of electrons in the low electric field, $\tau_o$ is called the ohmic relaxation time which is the mean time between collisions of a conduction electron and is related with the mean free path of electrons by $l = V_{Th} \tau$, where $V_{Th}$ is the thermal velocity of electrons.

In ohmic region (low electric field), the thermal velocity $V_T$ is much greater than the drift velocity and because of this fact the relaxation time $\tau$ is taken to be independent of the accelerating field. However, in the presence of high dc electric field, electrons can gain enough energy to emit quantum energy $\hbar \omega_o$. In this case the time $t$ in the expression of the drift velocity is constrained by $\tau_Q$ during which the energy gained by an electron in an electric field is enough to emit quantum energy $\hbar \omega_o$. The relaxation time $\tau$ in this high field $\tau$ is dependent on the accelerating field and will have the form:

$$\tau = \tau_o \left(1 - \exp\left(-\frac{\tau_Q}{\tau_o}\right)\right)$$  \hspace{1cm} (4)$$

where the time $\tau_Q$ at the onset of the emission of a quantum energy $\hbar \omega_o$ is given by:

$$\tau_Q = \frac{l_q}{v_{th}} = \frac{\hbar \omega_o}{(v_{th} qE)}$$  \hspace{1cm} (5)$$

with $l_q = \hbar \omega_o / qE$ being the inelastic scattering length for the onset of the quantum emission.

The electrical conductivity due to the net electrons moving with a drift velocity $v(t)$ given by Eq. (3) in the presence of high electric field, has the form:

$$\sigma(E) = \tanh\left[\Omega\left(1 - \exp\left[-\frac{\xi}{\Omega}\right]\right)\left(1 - \exp\left[-\frac{\xi}{\Omega}\right]\right)\right] \times \exp[\Omega\left(1 - \exp\left[-\frac{\xi}{\Omega}\right]\right)]$$

$$\times \left\{\frac{8\pi (2m^*_e)^{3/2}}{3\hbar^3 k_BT} \int_0^{\xi} \epsilon^{3/2} \tau_o \frac{\partial f_o}{\partial \epsilon} d\epsilon\right\}$$  \hspace{1cm} (6)$$

where, $\Omega \equiv \frac{E}{E_{\infty}}$ is the normalized applied field with $E_{\infty} = \frac{k_BT}{ql_o}$ being the critical electric field that defines the onset of non-ohmic behaviour for the drifting electrons with $l_o$ being the mean free path in low electric field, $\xi = \frac{h \omega_o}{k_BT}$ the normalized quantum of energy emitted by accelerated electrons, and $f_o$ is the Maxwell Boltzmann distribution function for non-degenerate electrons in low electric field.

The last expression in Eq. (6), in the parenthesis, is just the expression of electrical conductivity in low electric field (ohmic) region. Hence, the normalized
electrical conductivity, the ratio of the electrical conductivity in high electric field to the ohmic electrical conductivity, is:

\[
\frac{\sigma(E)}{\sigma(E_o)} = \frac{\tanh[\Omega(1-\exp[-\xi/\Omega])][1-\exp[-\xi/\Omega]]^2 \exp[\Omega(1-\exp[-\xi/\Omega])]}
\]

with \( \Omega \) and \( \xi \) being defined under Eq. (6).

3 Results and Discussion

The ratio of the electrical conductivity in high electric field to the ohmic electrical conductivity is plotted against the parameter \( \Omega \), the normalized applied field for different values of \( \xi \), the normalized quantum of energy emitted by the charge carriers, electrons. The variation of normalized electrical conductivity, \( \sigma(E)/\sigma(E_o) \), with dimensionless applied electric field \( \Omega \) is shown in Fig. 1 for various values of normalized quantum of energy \( \xi = \frac{h\omega}{k_B T} \) emitted by electrons due to high electric field.

It is noted from Fig. 2 that, \( \frac{\sigma(E)}{\sigma(E_o)} \) decreases with increasing electric field for each values of the normalized quantum energy emitted by conduction electrons or it increases with decreasing temperature for a fixed value of quanta of energy \( h\omega_o \). This can be explained as follows. In low electric field region, the electrical conductivity is not a function of electric field. However, in the presence of high \( dc \) electric field, the electrical conductivity is a function of an electric field. These combined function looks like as \( \sigma \) is inversely proportional to the electric field (Fig. 1). The normalized electrical conductivity decreases with increasing temperature for fixed quantum of energy \( h\omega_o \). This is because of the fact that as temperature increases the collision frequency due to high thermal vibration reduces the relaxation time and then electric current of the charge carriers. Hence, the decrease in conductivity in the presence of high temperature is attributed by acoustic phonon scattering.

In Fig. 1, the values of \( \xi \) and \( \frac{\sigma(E)}{\sigma(E_o)} \) can be explained in terms of temperature if we plug the reported experimental value of the quantum of energy \( h\omega_o = 63 \) meV (at \( T = 300^\circ K \)) and assume that the dependence of quantum of energy emitted by accelerated electrons on temperature to be negligible. Under these conditions, the values of normalized quantum energy emitted by conduction electrons, \( \xi = 1.5, 2, 2.5 \) and \( 3 \) approximately correspond to temperatures \( T = 500, 377, 300 \) and \( 250 \) K, respectively. The higher the temperature, the lesser the amount of normalized electrical conductivity and lower the amount of the applied field at which the maximum electrical conductivity occurs. However, it is important to note that in the region of low electric field, increasing temperature significantly increases the electric current but its effect on electric current is overwhelmed by the presence of high electric field. In line with this, from the time-of-flight and conductivity experiments the field effect has been observed to be particularly strong at low temperatures. Antoniadis and Schiff used transient photoconductivity in conjunction with time of flight experiments and deduced an exponential field dependence of the mobility where the field effect is increasing due to the lowering of temperatures.

4 Conclusions

The analytical expression of electrical conductivity of non-degenerate electrons in silicon has been derived by considering the asymmetric distribution function of electrons which is a function of electric field. This has resulted in an electric field dependence of the net number of electrons and their conductivity.
This analytic expression was used to plot normalized electrical conductivity versus normalized applied electric field using the software Mathematica. The conductivity of electrons decreases drastically as electric field increases. This is actually because of high dc electric field, the drift velocity of electrons experiences a transient effect and the time that electrons move between successive collisions decreased as result of quantum emission. The result of present paper is very important in characterising the design of semiconductor devices for applications where high electric field is primarily of concern.

References