An inventory model with temporary price discount when lead time links to order quantity

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This paper presents effects of a temporary price discount offered by supplier on a retailer’s replenishment policy, when lead time is linked to order quantity. A decision process for retailers is developed in deciding whether to adopt a regular or special order policy during a temporary sales period. Optimal special order quantity is determined by maximizing total cost saving between special and regular orders, and illustrated by several numerical examples along with sensitivity analysis of optimal solution.

Keywords: Inventory, Lead time, Temporary price discount

Introduction

Various inventory models have been proposed to gain deeper insight into relationship between price discounts and order policy. Naddor¹ proposed inventory models of response to a temporary price discount where special sale coincides with replenishment time. Barker & Vilcassim² studied situation where special sale coincides with retailer’s sales period. Ardalan³ developed optimal order policies with possible combinations of replenishment time and sales period. Tersine & Schwarzkopf⁴ relaxed assumption for length of special sale by permitting more than one replenishment cycle before sale expiration. Goyal⁵ discussed temporary price discount model for calculating average inventory of a regular policy. Aull-Hyde⁶ presented a temporary price discounts model for backlogs permissible. Various studies⁷-¹⁰ are available of response to a temporary sale price. However, in these studies, almost all the lead time (LT) was assumed to be constant and zero. In reality, LT is non-zero and variable due to time to fill order at source, shipping time, and time required to carry out paperwork. Several studies¹¹-¹⁵ are available on inventory problems with variable LT.

This study investigates effects of a temporary price discount offered by supplier on retailer’s replenishment policy with LT linked to order quantity.

Model Formulation

LT of a special order policy $L_s$ is always longer than that of regular order policy $L$, when retailer orders large quantities, $Q_s$. This may lead to an inventory shortage before special order quantity arrives, implying that a shortage cost is incurred. Therefore, retailer may place a large special order of quantity $Q_s$ (adopt $L_s$) or alternatively adopt $L$.

Temporary Price Discount Occurs at Retailer’s Reorder Time Point ($t_s = t_r$)

In this situation, retailer may place a large special order of quantity $Q_s$, or alternatively may place a regular order of quantity $Q^*$, at discounted price $(1-\delta)c$ at retailer’s replenishment time, followed by EOQ policy with original price $c$ for next replenishment (Fig. 1). Due to length of LT of $L_s$ being greater than length of LT for $L$, a shortage occur and backorder may be placed if retailer decides to order for $Q_s$. Therefore, total cost ($TCS_1$) consisting of costs for ordering, purchasing, shortage and holding is expressed as
\[ TCS_1 = A + (1 - \delta) c Q_s + \frac{bD}{2} \left( \frac{Q_s - Q^*}{P} \right)^2 + \frac{(1 - \delta) hc}{2D} \left[ \frac{(P - D)Q_s + DQ^*}{P} \right]^2 \] (1)

If retailer adopts a regular EOQ policy instead of placing a large special order quantity policy during the period \( Q_s / D \), then total cost \( TCN_1 \) is as

\[ TCN_1 = A + (1 - \delta) c Q^* + \frac{(1 - \delta) hc Q^*^2}{2D} + \frac{(Q_s - Q^*)}{D} \left( \frac{AD}{Q^*} + \frac{hcQ^*}{2D} + \frac{cD}{2} + \frac{hcQ^*}{2} \right) \] (2)

From Eqs (1) and (2), total cost saving between special and regular orders during time period \( Q_s / D \) can be obtained as

\[ g_1(Q_s) = TCN_1 - TCS_1 \]

\[ = \frac{Q_s - Q^*}{D} \left[ AD + \frac{\delta cD + \frac{hcQ^*}{2}}{2D} \right] + \frac{(1 - \delta) hc Q^*^2}{2D} - \frac{bD}{2} \left( \frac{Q_s - Q^*}{P} \right)^2 \]

\[ - \frac{(1 - \delta) hc}{2D} \left[ \frac{(P - D)Q_s + DQ^*}{P} \right]^2 \] (3)

Temporary Price Discount Does Not Occur At Retailer’s Reorder Time Point \( (t_s = t_r) \)

When supplier offers a temporary price discount at time point \( t_s \), if retailer decides to place a special order quantity \( Q_s \), then retailer receives items after time length \( L_s \). Consequently, inventory level \( q \) before special order quantity \( Q_s \) arrives is given by

\[ q = Q^* - \frac{DQ_s}{P} \] (Fig. 2).

With regard to total cost saving, two possible cases are found based on value of \( q \): (1) \( q \geq 0 \) and (2) \( q < 0 \) (Fig. 2).

**Case 1:** \( q \geq 0 \)

When \( q \geq 0 \), it means that inventory level either + or 0 before \( Q_s \) arrives. Total cost of special order during time period \( (Q_s + q) / D \) is given as

\[ TCS_{21} = A + (1 - \delta) c Q_s + \frac{hc}{2D} \left( Q^* - D t_s - \frac{DQ_s}{P} \right) + \frac{(1 - \delta) hc Q_s}{2D} \left[ Q_s + 2 \left( Q^* - D t_s - \frac{DQ_s}{P} \right) \right] \] (4)
Total cost of time period \((Q_s + q)/D\) under EOQ policy is represented by

\[
TCN_{21} = \frac{hc}{2D} \left( Q^* - D t_s - \frac{DQ_s}{P} \right)^2 + \frac{Q_s}{D} \left[ \frac{AD}{Q^*} + cD + \frac{hCQ^*}{2} \right] - A
\]

\[\ldots(5)\]

Therefore, total cost saving during time period \((Q_s + q)/D\) can be obtained as

\[
g_{21}(Q_s) = TCN_{21} - TCN_{21} - TCS_{21}
\]

\[\frac{Q}{D} \left[ \frac{AD}{Q^*} + \delta cD + \frac{hCQ^*}{2} \right] \left[ (1-\delta)hCQ \left[ Q_s + 2 \left( Q^* - D t_s - \frac{DQ_s}{P} \right) \right] - A \right]
\]

\[\ldots(6)\]

Case 2: \(q < 0\)

When \(q < 0\), it means that shortage occurs before special order quantity \(Q_s\) arrives. Total cost of adopting a special order policy is given as
\[ TCS_{22} = A + (1 - \delta) c Q_s + \frac{b}{2D} \left( \frac{DQ_s}{P} - Q^* + D t_s \right)^2 \]
\[ + \frac{(1 - \delta) cD}{2D} \left( Q_s + Q^* - D t_s - \frac{DQ_s}{P} \right)^2 \]  

Total cost of time period \( Q_s / D \) under EOQ policy is given as
\[ TCN_{22} = \frac{Q_s}{D} \left[ \frac{AD}{Q^*} + cD + \frac{hcQ^*}{2} \right] \]  

Therefore, total cost saving during time period \( Q_s / D \) can be obtained as
\[ g_{22}(Q_s) = TCN_{22} - TCS_{22} \]
\[ = \frac{Q_s}{D} \left[ \frac{AD}{Q^*} + \delta cD + \frac{hcQ^*}{2} \right] - \frac{b}{2D} \left( \frac{DQ_s}{P} - Q^* + D t_s \right)^2 \]
\[ - \frac{(1 - \delta) cD}{2D} \left( Q_s + Q^* - D t_s - \frac{DQ_s}{P} \right)^2 \]  

\textbf{Theoretical Results}

It is worth placing a special order when total cost saving is positive. Otherwise, retailer should adopt regular order policy.

\textbf{Temporary Price Discount Occurs at Retailer's Reorder Time Point (} \( t_s = t_r \) \textbf{)}

Taking first and second order derivatives of \( g_1(Q_s) \) in Eq. (3) with respect to \( Q_s \), one gets
\[ \frac{dg_1(Q_s)}{dQ_s} = \frac{1}{D} \left[ \frac{AD}{Q^*} + \delta cD + \frac{hcQ^*}{2} \right] - \frac{b}{P} \left( \frac{Q_s - Q^*}{P} \right) \]
\[ - \frac{(1 - \delta) cD}{D} \left( \frac{P - D}{P} \right) \left\{ \left( \frac{P - D}{P} \right) Q_s + \frac{DQ_s^*}{P} \right\} \]
\[ \frac{d^2 g_1(Q_s)}{dQ_s^2} = -\frac{bD}{P^2} - \frac{(1 - \delta) cD}{D} \left( \frac{P - D}{P} \right)^2 \]  

Given that \( \frac{d^2 g_1(Q_s)}{dQ_s^2} < 0 \), then \( g_1(Q_s) \) is a concave function of \( Q_s \). Therefore, a unique value exists of \( Q_s \) (say \( Q_{s1} \)), which maximizes \( g_1(Q_s) \). Solving Eq. \( \frac{dg_1(Q_s)}{dQ_s} = 0 \) gives
\[ Q_{s1} = \frac{P(ADQ^* + \delta cD + hcQ^*/2) + bDQ^* - (1 - \delta)hc(P - D)DQ^*}{bD^2 + (1 - \delta)hc(P - D)^2} \]  

Substituting Eq. (12) into Eq. (3), corresponding maximum total cost saving can be obtained as
\[ g_1(Q_{s1}) = \frac{DQ^*}{2DQ^*[bD^2 + (1 - \delta)hc(P - D)^2]^2} \left[ \frac{1}{D} \left( \frac{AD}{Q^*} + \delta cD + \frac{hcQ^*}{2} \right) \right. \]
\[ - \frac{(1 - \delta) hc}{D} \left( \frac{P - D}{P} \right) Q_{s1}^* \right]^2 > 0 \]

Thus, when temporary price discount occurs at retailer’s reorder point, optimal special quantity \( Q_s^* = Q_{s1} \).

\textbf{Temporary Price Discount Does Not Occur at Retailer’s Reorder Time Point (} \( t_s \neq t_r \) \textbf{)}

\textbf{Case 1:} \( q \geq 0 \) \( / Q_s \leq P/Q^* - D t_s \) \( / D \) \( l \)

Taking first order derivative of \( g_{21}(Q_s) \) in Eq. (6) with respect to \( Q_s \) and letting result equal zero gives
\[ \frac{dg_{21}(Q_s)}{dQ_s} = \frac{1}{D} \left[ \frac{AD}{Q^*} + \delta cD + \frac{hcQ^*}{2} \right] - \frac{(1 - \delta) hcQ_s}{2D} \left( \frac{P - 2D}{P} \right) \]
\[ - \frac{(1 - \delta) hc}{2D} \left[ Q_s + 2 \left( \frac{Q^* - D t_s - DQ_s^*}{P} \right) \right] = 0 \]  

By solving Eq. (13), a unique solution of \( Q_s \) (say \( Q_{s21} \)) is obtained as
\[ Q_{s21} = \frac{P(AD/Q^* + \delta cD + hcQ^*/2) - (1 - \delta)hcQ^* - t_s D}{(1 - \delta)hc(P - 2D)} \]
and second order derivative of \( g_{21}(Q_s) \) in Eq. (6) with respect to \( Q_s \) is
Given that \( \frac{d^2 g_{22}(Q_s)}{dQ_s^2} < 0 \), \( g_{22}(Q_s) \) is a concave function of \( Q_s \), a unique value of \( Q_s \) (say \( Q_{s22} \)) which maximizes \( g_{22}(Q_s) \) can be found. Solving Eq. \( \frac{dg_{22}(Q_s)}{dQ_s} = 0 \) gives

\[
Q_{s22} = \frac{DP^2}{bD^2 + (1-\delta)hc(P-D)^2} \left\{ \frac{1}{D} \left( \frac{AD}{Q_s} + \delta cD + \frac{hcQ_s^*}{2} \right) + \left(\frac{Q_s^* - D t_s}{b} - \frac{(1-\delta)hc(P-D)}{DP}\right) \right\}^{\frac{1}{2}} - A \]

Substituting Eq. (19) into Eq. (9), corresponding maximum total cost saving for Case 2 is

\[
g_{22}(Q_{s22}) = \left[ \frac{b+(1-\delta)hc(P-D)^2}{2D^2} \right] Q_{s2}^2 - \left(\frac{b+(1-\delta)hc}{2D} \right) \left(\frac{Q_s^* - D t_s}{b} - \frac{(1-\delta)hc(P-D)}{DP}\right)^{\frac{1}{2}} - A \]

Similarly, to obtain optimal special order quantity \( Q_{s22}^* \) for Case 2, let \( \Delta_3 = Q_{s22} - P(Q_s^* - D t_s)/D \) and \( \Delta_4 = g_{22}(Q_{s22}) \), then

**Theorem 2**

Optimal special order quantity for Case 2 as follows:

\[
Q_{s22}^* = \begin{cases} 
Q_{s22}, & \text{if } \Delta_3 > 0 \text{ and } \Delta_4 > 0, \\
0, & \text{otherwise}.
\end{cases}
\]

**Algorithm**

**Step 1**

If \( t_s = t_f \), then optimal special order quantity is \( Q_{s1}^* = Q_{s1} \). Go to Step 6.
Table 1—Optimal solutions of Example 1 under different values of $P$

<table>
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<tr>
<th>$P$</th>
<th>$t_r$</th>
<th>$t_s$</th>
<th>$Q_s^*$</th>
<th>$TCN^*$</th>
<th>$TCS^*$</th>
<th>$g^*$</th>
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<tr>
<td>1500</td>
<td>0.1054</td>
<td>0.1054</td>
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<td>8248.44</td>
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</table>

$TCN^*$ and $TCS^*$ denote minimum total cost for regular and special order policy, respectively; $g^*$ is corresponding maximum total cost saving; — indicates retailer will not order until next replenishment time and adopts regular EOQ policy.

**Step 2**

Determine values of $Q_{21}$ and $Q_{22}$ from Eq. (14) and Eq. (19), respectively.

**Step 3**

Calculate values of $\Delta_1 = Q_{s1}^* - P\left(Q - D\gamma_s\right)/D$, and $\Delta_2 = g_21(Q_{s21}^*)$ in Eq. (16). If $P > 2D$, $\Delta_1 \leq 0$ and $\Delta_2 > 0$, then set $Q_{s21}^* = Q_{s21}$ and $g_21(Q_{s21}^*) = \Delta_2$; otherwise, set $Q_{s21}^* = 0$ and $g_21(Q_{s21}^*) = 0$.

**Step 4**

Calculate values of $\Delta_3 = Q_{s22}^* - P(Q^* - D\gamma_s)/D$, and $\Delta_4 = g_22(Q_{s22}^*)$ in Eq. (20). If $\Delta_3 > 0$ and $\Delta_4 > 0$, then set $Q_{s22}^* = Q_{s22}$ and $g_22(Q_{s22}^*) = \Delta_4$; otherwise, set $Q_{s22}^* = 0$ and $g_22(Q_{s22}^*) = 0$.

**Step 5**

Find $\max \{g_21(Q_{s21}^*), g_22(Q_{s22}^*)\}$. If $g_21(Q_{s21}^*) \geq g_22(Q_{s22}^*)$, then $Q_s^* = Q_{s21}^*$; otherwise, $Q_s^* = Q_{s22}^*$.

**Step 6**

Stop.

Once optimal solution is obtained, corresponding minimum total costs can be found for regular and special order, and subsequently determining maximum cost saving.

**Numerical Examples**

To illustrate solution process for two situations, following examples are presented:

**Example 1**

An inventory system with following data is considered: $c = \$10/\text{unit}$, $D = 1000\text{ units/year}$, $A = \$150/\text{order}$, $b = \$5/\text{unit/year}$, $\delta = 0.05$ and $h = 0.3$. From these data, it can be found that $Q^* = 316.23$. Sets of values of $P$ and $t_s$ are assumed as $P \in \{1500, 2000, 2500, 3000, 3500\}$ and $t_s \in \{t_r, 0.01\}$, where $t_r = (Q^*/D) - L = (Q^*/D) - (Q^*/P)$. From algorithm, optimal order policies depending on different values of $P$, $t_r$ and $t_s$ can be obtained (Table 1).

From Table 1, it is observed that retailer will determine optimal special order quantity by trading off benefits of price discount against additional holding cost and shortage cost, if necessary. For example, for
$t_s = 0.01$ and $P = 3000$ or 3500 in Table 1, optimal value of $Q^* = 0$, which implies that retailer should ignore temporary price discount offered by supplier and will not order until next replenishment time and then adopt regular EOQ policy. For other cases, optimal policy of retailer is to order a special quantity. In addition, when supplier’s production rate increases, special order quantity increases at the beginning and decreases later. However, total cost saving decreases.

Example 2

In this example, a sensitivity analysis is conducted demonstrating robustness of model, wherein effects of changes in system parameters ($c$, $\delta$, $D$, $A$, $h$, and $b$) on optimal special order quantity and optimal total cost saving are examined. Data used are same as those in Example 1. For convenience, considering the case when $P = 2000$ and $t_s = 0.01$. Sensitivity analysis is conducted by changing each of the parameters by $-50\%$, $-25\%$, $+25\%$ and $+50\%$ and simultaneously altering one parameter at a time while all other parameters remain unchanged (Table 2).

Based on results in Table 2, following managerial insights are gained: 1) When purchase cost $c$ increases, special order quantity $Q^*$ decreases. However, total cost for regular order policy $TCN^*$, total cost for special order policy $TCS^*$, and total cost saving $g^*$ increases. Thus as purchase cost increases, it is more beneficial for retailer to adopt special order policy. Nevertheless, special order quantity will decrease; 2) As $Q^*$, $TCN^*$, $TCS^*$ and $g^*$ increase, $\delta$ increases. This implies that greater the price discount, larger the special order quantity, total cost of regular order, total cost of special order and total cost saving for retailer. Results are consistent with basic common theory. Moreover, $\delta$ is sensitive to total cost saving. From supplier’s viewpoints, offering retailer a high temporary discount rate will lead to increased sales volume; 3) $Q^*$, $TCN^*$, $TCS^*$ and $g^*$ increase when $D$ and $A$ increase. It implies that when either market demand rate or ordering cost or both increases, retailer will order a larger quantity. Moreover, when market demand rate $D$ is sufficiently low (value of $D$ reduces to 50% in Table 2) it is not worthwhile for retailer to place a special order; and 4) Accordingly, when either $h$ or $b$ or both increases, $Q^*$, $TCN^*$, $TCS^*$, and $g^*$ decrease. It implies that when either holding costs or shortage costs or both are higher, retailer will order a lower quantity and total cost saving will decrease. Additionally, holding cost is more sensitive to optimal solutions than shortage cost.

Conclusions

This study presented effects of a temporary price discount offered by supplier on a retailer’s replenishment policy. An algorithm was established to determine optimal order polices. Numerical examples were presented to illustrate solution procedure.

Acknowledgements

Authors greatly appreciate referees for their valuable and helpful suggestions.

Notation and Assumptions

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Table 2—Effect of changes in various parameters of Example 2

Notation

- $P$: production rate of the supplier.
- $D$: market demand rate.
- $c$: unit purchasing price.
- $\delta$: price discounted fraction from the supplier ($0 < \delta < 1$).
- $A$: ordering cost per regular or special order.
- $h$: holding cost rate.
$b$, shortage cost per unit per unit time.
$q$, inventory level before the special order quantity arrives.
$L$, the length of lead time of the regular order quantity.
$L_s$, the length of lead time of the special order quantity.
$r$, the reorder point of regular order policy (in terms of inventory level).
$t_r$, the reorder time at reorder point of regular order policy.
$t_s$, the time at which the supplier offers a temporary price discount.
$Q$, order quantity under regular order policy.
$Q^*$, optimal order quantity under regular order policy.
$Q_s$, order quantity at discount price.
$Q^*_s$, optimal order quantity at discount price.

Assumptions
(1) Deterioration of item is not considered in the model.
(2) Supplier offers retailer a temporary price discount at discount rate $\delta$. Temporary discount occurs at only one instant in time.
(3) Inventory is continuously reviewed and it is replenished whenever inventory level falls to $r$ under regular order policy.
(4) Prior to and after temporary price discount, retailer adopts an economic order quantity (EOQ) policy and optimal order quantity is $Q^* = \sqrt{\frac{2AD}{hc}}$.
(5) Length of lead time depends on order quantity.
(6) Shortages are allowed and items can be backordered under special order policy.
(7) None of the price discount is passed on to customers.

References