

## Development of delay models with quasi-Newton method resulting from TRANSYT traffic model

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This study presents delays at signalized intersection resulting from TRANSYT traffic model (TTM) for 3 and 4 phase individual junctions to develop new forms of delays to set up timing manual for practical users. Proposed total delay estimation (TDE) is solved with quasi-Newton method to obtain weighting parameters for delays resulting from TTM. With 3- and 4-legs signalized intersections, uniform and random plus over-saturation component of total delay may be estimated in a single model.

**Keywords:** Delay indexing, Optimization, Signalized intersection

### Introduction

Signalized junction delay may be analysed into uniform component that consists of signal timings, and random plus oversaturation component that includes vehicle queuing, random arrivals and over-saturation cases of traffic flows. Kimber & Daly<sup>1</sup> studied measurements of queue lengths and vehicle delays for testing predictions of time dependent queuing models. Akcelik<sup>2</sup> studied on Highway Capacity Manual<sup>3</sup> (HCM) delay formula and suggested a calibration process. Burrow<sup>4</sup> recommended additional factors for the formula improved by Akcelik<sup>2</sup>. Prevedouros & Koga<sup>5</sup> compared HCM<sup>3</sup> delay formula with that of HCM<sup>6</sup>. Powell<sup>7</sup> proposed some correction factors representing deceleration and acceleration delays of vehicles based on queuing to improve HCM<sup>8</sup> delay formula. Quiroga & Bullock<sup>9</sup> conducted a studied vehicle delays using Geographic Information and Global Positioning Systems. Qiao *et al*<sup>10</sup> developed a fuzzy logic model to simulate HCM delay formula. Dion *et al*<sup>11</sup> compared various analytic models with microscopic simulation models and addressed delays at signalized intersection controlled in fixed-time and operated in a range of conditions. Akcelik & Roupail<sup>12</sup> proposed a delay model for signalized intersections suitable for variable

demand conditions. Murat & Baskan<sup>13</sup> studied vehicle delays using artificial neural networks (ANN). HCM<sup>14</sup> calculates uniform and incremental delay on signalized intersections depending on degree of saturation and/or vehicle/capacity ratio.

TRANSYT traffic model<sup>15,16</sup> (TTM) is good at coordinated signalized intersections, but there is a problem of setting up signal timings based on Webster method. Traffic queues and delays at road junctions are reported<sup>17</sup>. Delay components are solved using coordinate transformation method<sup>18</sup>. Ceylan *et al*<sup>19</sup> developed approximate mathematical expressions for delay components at signalized intersections. Chiou<sup>20</sup> presented TRANSYT derivatives for network equilibrium flows. Farzaneh & Rakha<sup>21</sup> procedure overcame limitations of TRANSYT software with regard to modification of travel time factor. Fernandez *et al*<sup>22</sup> used TTM for cost-benefit analysis of infrastructure and traffic management projects in Chile. Wong *et al*<sup>23</sup> developed a general expression for sheared curves in different versions of TRANSYT. Wong<sup>24</sup> presented a base for phase-based optimization of signal timings for area traffic control.

This study proposed total delay estimation (TDE) models, which included uniform and oversaturation components for given timing period, and used Quasi-Newton method to solve weighting parameters of TDE models.

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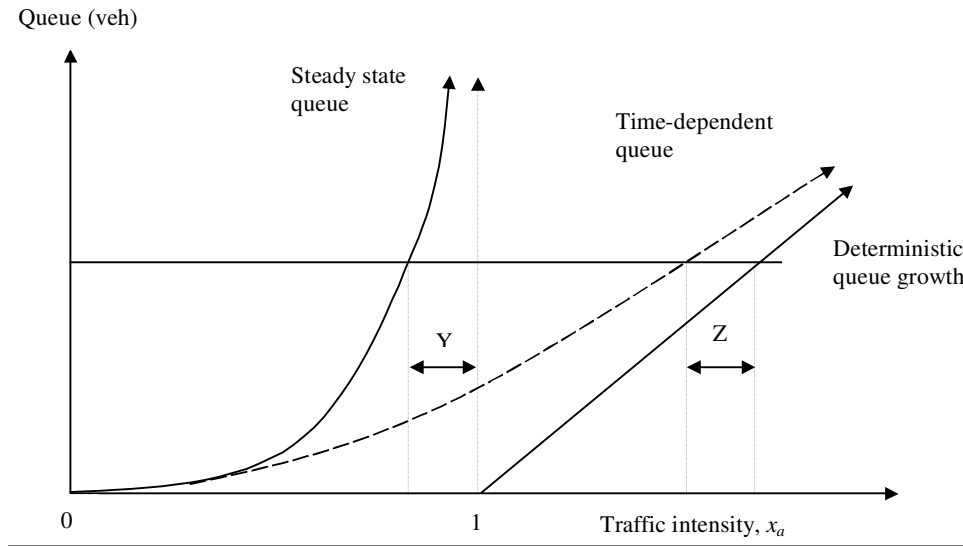


Fig. 1— Time-dependent delay formulation

**Experimental**

**Mathematical Formulations**

Steady-state queue formation (Fig. 1) takes place when saturation level is less than one and calculation of settings in this period is non-definitive. Deterministic queue growth happens when critical level of saturation degree is greater than one. Transformation is required between under-saturation and oversaturation to obtain delays. One of the well-known methods for transformation may be TTM<sup>15,16</sup> as

$$D_{aT}^{ro} = \frac{T}{4} [((q_a - \mu_a)^2 + \frac{4q_a}{T})^{0.5} + (q_a - \mu_a)] \quad \dots(1)$$

$$d_{aT}^{ro} = \frac{D_{aT}^{ro}}{q_a} \quad \dots(2)$$

where  $D_{aT}^{ro}$  and  $d_{aT}^{ro}$ , TRANSYT and average delay formula over time period  $T$  to a vehicle on link  $a$ ;  $q_a$ , link traffic volumes (veh/h);  $\mu_a$ , uniform departure rate;  $L$ , set of links. Delay components at signalized intersection may be analysed in following ways:

**Uniform Delay (UD) Component**

UD component with respect to each link  $a$  in  $L$  is calculated on the basis of whole time period  $T$ , as oversaturated links with  $x_a \geq 1$  and under-saturated

links with  $x_a < 1$ , where  $x_a$  is degree of saturation on link  $a$  in  $L$ . Uniform queues and delays are calculated on the basis of difference between cyclic cumulative departure graph and uniform departure rate  $\mu_a$  for each link  $a$  in  $L$  in time period  $T$  as

$$L_a^U = \frac{c\mu_a(1 - \Lambda_a)}{2} \quad \dots(3)$$

$$D_a^U = \frac{c\mu_a(1 - \Lambda_a)}{2} \quad \dots(4)$$

$$d_a^U = \frac{c(1 - \Lambda_a)}{2} \quad \dots(5)$$

where  $L_a^U$ , uniform queue ( $\forall a \in L$ );  $D_a^U$ , uniform delay ( $\forall a \in L$ );  $d_a^U$ , delay to a vehicle ( $\forall a \in L$ );  $\Lambda_a$ , proportion of green to cycle time ( $\forall a \in L$ );  $c$ , cycle time.

For under-saturated links, it is assumed that traffic queues develop at the start of effective red and clear at next effective green period. It is further assumed that  $t$  cycle time,  $c$ , is divided into an effective green and red period. Given a junction with under-saturated conditions without accumulation of queues, analytical expressions are given as

$$L_a^U = \frac{x_a \mu_a (1 - \Lambda_a)^2 c}{2(1 - \Lambda_a x_a)} \quad \dots(6)$$

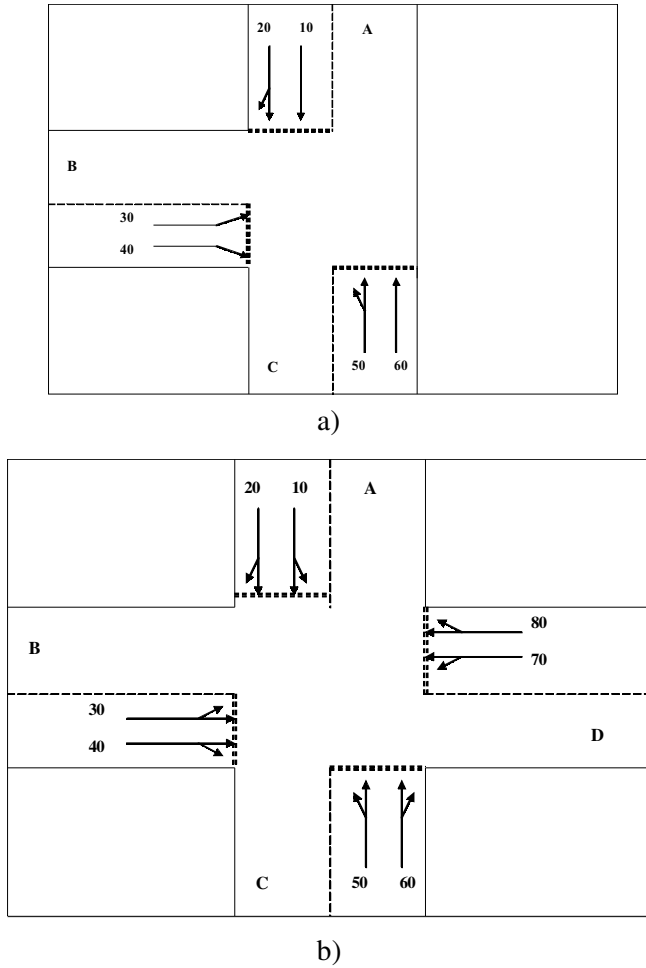


Fig. 2— Idealized intersection with: a) 3 approaching legs and 6 traffic streams; b) 4 approaching legs and 8 traffic streams

$$D_a^U = L_a^U \quad \dots(7)$$

$$d_a^U = \frac{D_a^U}{q_a} \quad \dots(8)$$

#### Random plus Oversaturation (RO) Delay Component

There is a need to obtain time-dependent delay calculation, based on steady state and deterministic approaches. As degree of saturation  $x_a$  approaches 1, steady-state queue length tends to infinity. Due to complexity of mathematical expressions used in queuing analysis, Kimber & Hollis<sup>17</sup> proposed coordinate transformation method. TRANSYT<sup>15</sup> uses time dependent delay formula in a simulation period. This study used TRANSYT delay formula [Eqs (1) & (2)] for RO of delay.

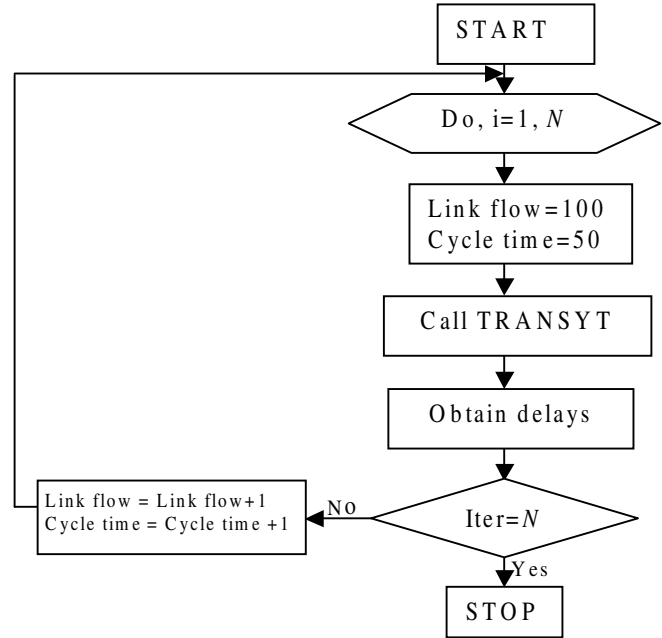
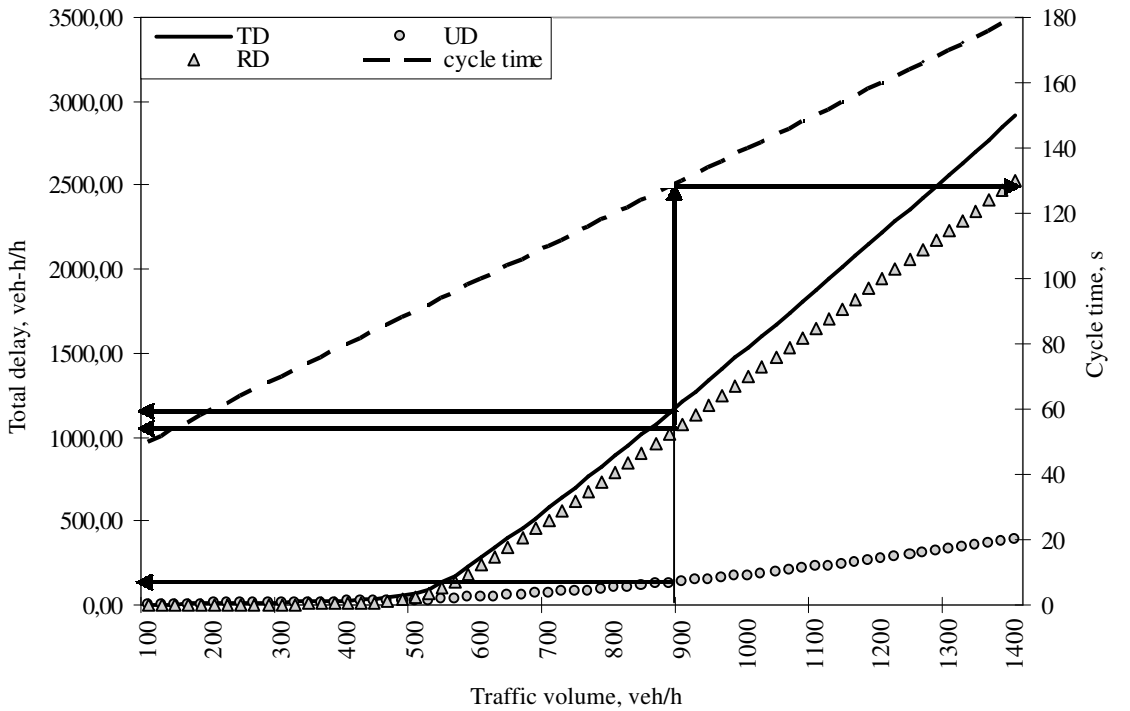


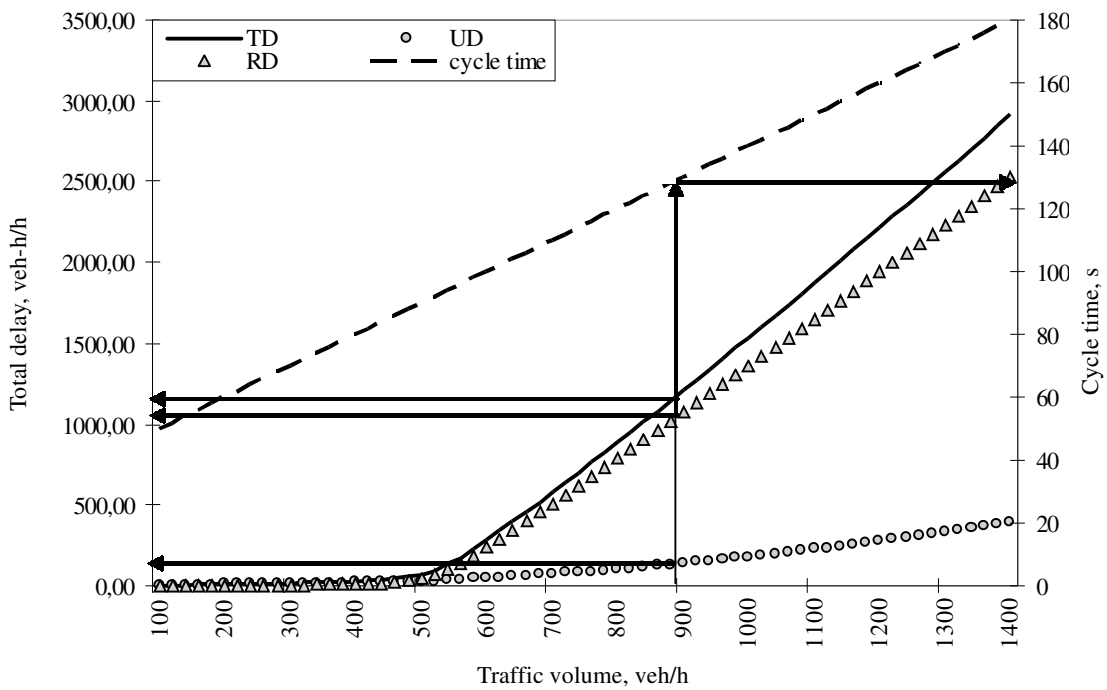
Fig. 3— Obtaining generated theoretical delays via TRANSYT model

#### Delay Modelling using TRANSYT Output

Development of TDE models at signalized intersections is applied on 3- and 4-leg intersections, where TRANSYT was used to predict delays to further develop analytical TDE expressions for estimating junction delays. Each approaching link consists of two lanes, through, left and right turning movements. Saturation flows equally distributed to each lane are taken as 1800 veh/h for each traffic stream. Obtaining delays were calculated with three stages for 3-leg intersection and four stages for 4-leg intersection. TDE models were built on TRANSYT outputs for both intersections. Traffic volumes for each approaching links varied (100-1200 veh/h). Cycle time was changed (50-180 s) in a linear fashion for each change on traffic volumes for each approaching links for both types of junctions. Delays were noted for every increase on traffic volumes for every 1 h. Two example signalized intersections consisted of 3 legs with 6 approaching links (Fig. 2a) and 4 legs with 8 approaching links (Fig. 2b). Flowchart (Fig. 3) of obtaining delay components using TRANSYT for 1 h indicates that program starts with changing signal cycle and traffic volumes for each approaching links. And then, by calling TRANSYT,  $UD$  and  $RO$  delays are estimated. This program run until the desired number of iterations is achieved. During the delay calculation, green timings to stages are distributed



a)



b)

Fig. 4—Indicators and timing parameter of an example signalized junction for: a) 3-leg intersection; b) 4-leg intersection

according to the “*equisaturation*” facility by TRANSYT traffic model. Maximum iteration number is set as 180. A FORTRAN 95 code is written for obtaining theoretical delays at the example signalized intersections by internally calling TRANSYT traffic model using the expressions (Eqs.2-8). Signal timings were noted for each run of the TRANSYT model.

**TDE Models**

Developed TDE models based on cycle time and traffic volumes, were having UD and RO components. General structure of TDE model is given as

$$TD_{model} = w_1 \cdot q^2 + w_2 \cdot q + w_3 \cdot c^2 + w_4 \cdot c + w_5 \cdot q \cdot c \dots(9)$$

Table 1—Comparison of total junction delay with HCM, Akcelik and TDE models

q veh/h	c(sn)	g (s) (green time)	Three stage Total delay, veh-h/h			g (s) (green time)	Four stage Total delay, veh-h/h		
			HCM2000	Akcelik	TDE		HCM2000	Akcelik	TDE
100	50	11.67	15.87	29.20	3.37	7.50	19.71	49.38	6.82
200	60	15.00	19.78	38.78	8.17	10.00	25.43	84.00	16.05
300	70	18.33	24.63	55.16	15.50	12.50	41.00	188.59	41.90
400	80	21.67	31.85	92.01	109.66	15.00	373.57	472.80	302.17
500	90	25.00	74.93	198.25	240.51	17.50	815.18	862.28	616.65
600	100	28.33	362.70	422.66	408.03	20.00	1250.49	1272.13	977.10
700	110	31.67	681.05	704.49	612.24	22.50	1681.28	1684.60	1375.27
800	120	35.00	988.26	1001.38	853.12	25.00	2089.38	2096.16	1802.94
900	130	38.33	1300.44	1302.47	1130.69	27.50	2507.53	2506.14	2251.86
1000	140	41.67	1611.32	1604.70	1444.93	30.00	2923.29	2914.49	2713.80
1100	150	45.00	1921.13	1907.06	1795.86	32.50	3337.22	3321.36	3180.52
1200	160	48.33	2230.07	2209.15	2183.46	35.00	3749.70	3726.92	3643.77
1300	170	51.67	2538.31	2510.82	2607.75	37.50	4161.02	4131.31	4095.33
1400	180	55.00	2845.96	2812.03	3068.72	40.00	4571.40	4534.68	4526.95

where  $q$ , traffic volume (veh/h) for each approaching link;  $c$ , cycle time (s);  $w_p, w_p, \dots$ , corresponding weighting parameters. Eq. (9) is solved with Quasi-Newton method by minimizing sum of squared error (SSE) between theoretical delays resulting from TRANSYT and estimated delays. During solution of TDE, quasi-Newton method was used with solver facility in spreadsheet. Solver can be used to maximize or minimize value of a target worksheet cell by altering values of other selected changing cells in spreadsheet that influence value in the target cell. It also allows constraints to be placed on values of any cell. Thus, it is a general-purpose tool capable of solving constrained linear and nonlinear optimization problems. Form of SSE is given as

$$f(SSE) = \sum_i^n (d_{theoretical} - d_{TDE})^2 \quad i=1,2,\dots,n \quad \dots(10)$$

where,  $d_{theoretical}$ , measured delay (veh-h/h);  $d_{TDE}$ , estimated delay (veh-h/h);  $n$ , number of delay measurements. Weighting parameters of TDE was obtained in the following way:

For 4-leg with four stage signalized intersections, if traffic volumes for each approaching links is < 290 veh/h and cycle time is  $\leq 69$  s, then expression obtained was

$$TDE = 0.0008 \cdot q^2 - 0.0001 \cdot q + 0.0002 \cdot c^2 + 0.0001 \cdot c - 0.004 \cdot q \cdot c \quad R^2 = 0.99 \quad \dots(11)$$

If traffic volumes for each approaching links are > 300 veh/h and cycle time is  $\geq 70$  s, then expression obtained was

$$TDE = -0.0564 \cdot q^2 - 23.6323 \cdot q - 0.0014 \cdot c^2 - 6.0857 \cdot c + 0.624 \cdot q \cdot c \quad R^2 = 0.98 \quad \dots(12)$$

Similarly, for 3-leg with three stage signalized intersections, if traffic volumes for each approaching links is < 290 veh/h and cycle time is  $\leq 69$  s, then expressions obtained was

$$TDE = 0.0001 \cdot q^2 - 0.0002 \cdot q + 0.0006 \cdot c^2 + 0.0001 \cdot c + 0.0002 \cdot q \cdot c \quad R^2 = 0.99 \quad \dots(13)$$

If traffic volumes for each approaching links are > 300veh/h and cycle time is  $\geq 70$  s, then expressions obtained was

$$TDE = 0.0024 \cdot q^2 - 0.0013 \cdot q - 0.0279 \cdot c^2 - 0.0572 \cdot c - 0.0028 \cdot q \cdot c \quad R^2 = 0.99 \quad \dots(14)$$

**Results**

**Practical Use of TDE Models**

TDE models provide obtaining delays for given cycle time without calculating any complex set of

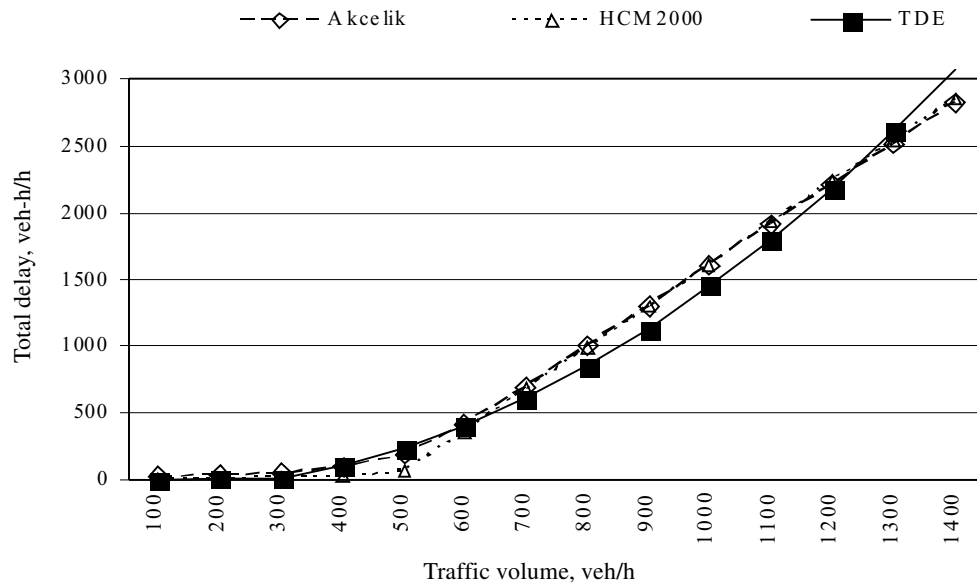


Fig. 5— Comparison of TDE models with Akcelik and HCM2000 models

equations. When link traffic volume reaches 500 veh/h per lane, values of degree of saturation on all links are 97% (Fig. 4). After that, degrees of saturation in all links exceed critical value (1). For light traffic conditions, there are no significant variations between UD and RO delay. When link flows are near to link capacity, RO delay is higher than UD. As reported<sup>18</sup>, UD takes place only for cyclic variations while RO component takes place when junction is getting congested.

Fig. 4 may provide practitioner to set up timing parameters and their corresponding total delays and/or components of delays. For example, if traffic volume on each approaching links is 900 veh/h, then UD is 135, RD is 1074, TD (total delay) is 1209 veh-h/h and corresponding cycle time is 130 s for 3-leg signalized intersections. Similarly, for 4-leg signalized intersection, if same amount of traffic volume approaches each leg, UD, RD and TD increase to 234, 2028 and 2263 veh-h/h, respectively.

TD resulting from HCM2000, Akcelik and TDE models for 3- and 4-leg signalized intersections (Table 1) indicates much difference between TDE, HCM2000 and Akcelik models for estimated delays at light traffic conditions. At congested traffic conditions, this difference gets small (Fig. 5).

## Conclusions

This study proposed TDE models to estimate junction delays at signalized intersection to set up

graphical delay calculations and to signal timing parameters for 3- and 4-leg signalized intersections. Proposed models were solved using quasi-Newton method with solver facility on spreadsheet. Estimation of delays was applied to typical 3- and 4-leg signalized intersections.

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