Laminar flow heat transfer in concentric equilateral triangular annular channels

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The problem of laminar flow heat transfer in channels of annular cross section formed by concentric equilateral triangles is studied. Numerical solutions for friction factor, and magnitude of minimum limiting Nusselt numbers for both constant heat input per unit length and constant wall temperature are obtained. The results obtained by the numerical method are compared with the available solutions for the limiting geometries of equilateral triangle and parallel plates. Experimental data for isothermal pressure drop and constant wall temperature boundary condition at the outer wall are presented. Test sections with length to equivalent diameter ratios of 13.85, 15.12 and 20.8 are employed in the present investigation. The Prandtl numbers are varied from 4 to 65 applying Glycerol-water mixture as coolant. Empirical correlations for isothermal friction factor and Nusselt number for constant wall temperature boundary conditions are presented.

Keywords: Laminar flow, Equilateral triangular annulus, Friction factor, Limiting Nusselt number

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Several researchers investigated the problem of laminar flow forced convection in ducts of various cross-sections during 1950 through 1970. Solutions for developing and developed velocity and temperature profiles, by various exact, analytical and approximate methods are available in the literature. Experimental results are also available for different flow geometries. An extensive review of these solutions is presented by Shah and London1. Several investigators are continuing studies on laminar flow convective heat transfer in non-circular ducts. Uzun and Unsal2 presented a numerical solution for laminar heat conduction in ducts of irregular cross-sections. The solutions of heat transfer to a power law fluid in arbitrary cross-sectional ducts was presented by Uzun3. Muzychka and Yovanovich4,5 reviewed the laminar flow friction and heat transfer in non-circular ducts and the same authors6 proposed a new model for predicting Nusselt numbers for laminar forced convection heat transfer in the combined entry region of non-circular ducts.

It is a common understanding that turbulent flow gives high heat transfer coefficients and hence is desirable in heat exchanger applications. With this understanding, most of the heat exchangers are designed in the turbulent flow region except in a few specific applications. But, the second law analysis shows that high efficiency heat exchangers must be non-turbulent with minimum temperature difference between two counter flowing streams2. Shah3 has shown theoretically that laminar flow in a heat exchanger can provide high heat transfer coefficients with reasonable pressure drops and thereby resulting substantial reduction in pumping power.

In recent times, with newer manufacturing technologies and materials of construction, a new range of highly compact heat exchange equipment, like laminar flow heat exchangers (surface area density $\beta$ above 3000 m$^2$/m$^3$), micro heat exchangers ($\beta$>10000), are being developed compared to the conventional shell and tube heat exchangers ($\beta$<500). In this process, so many new flow passage geometries have come into existence.

The new generation compact heat exchangers may need the application of new geometries, which may result in high heat transfer coefficients compared to presently known geometries. It thus appears worthwhile to investigate different possible flow cross sections. A survey of literature reveals that doubly connected non-circular ducts received little attention while the annular cross-sections formed by concentric circular ducts was extensively studied. An important class of flow geometries in these non-circular annular ducts is the flow through annular cross section of two concentric regular polygons. The first of the polygon series, the annular channel formed between two
concentric equilateral triangular ducts is considered in the present study.

The objectives of this study are (i) to determine laminar flow friction factors, limiting Nusselt number solutions for an annulus formed between two concentric equilateral triangular ducts for both constant heat input per unit of length and constant wall temperature, (ii) to compare these solutions with the two limiting cases for this geometry i.e. (a) equilateral triangle with the size of the inside triangle reducing to zero and (b) parallel plate geometry with the inside triangle size approaching the outside triangle, (iii) to compare the “Flow area goodness factor “$j/f$” values of the present geometry with circular annulus, (iv) to present the experimental data for friction factors and a correlation to predict the $fRe$ values for the present geometry, and (v) to present the experimental data for the laminar flow heat transfer in concentric equilateral triangular ducts with constant wall temperature at the outer wall over a wide range of Prandtl numbers and a correlation to predict the Nusselt numbers.

**Idealizations for minimum Nusselt number analysis**

The following idealizations are utilized in obtaining the minimum limiting Nusselt numbers. (a) Velocity and temperature profiles are fully developed; (b) the fluid properties ($\rho, c_p, \mu, k$) are constant; (c) heat conduction in the direction of flow is negligible; (d) conversion of mechanical energy to thermal energy due to friction is negligible and (e) natural convection effects are negligible.

**Differential equations and solutions**

**Velocity problem**

From the equations of change to describe a flow situation in rectangular coordinate system, one may deduce the hydrodynamic differential equation consistent with the forgoing idealizations.

\[
\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = \frac{1}{\mu} \frac{\partial p}{\partial x}
\]  

...(1)

For solving this equation Finite difference method is adopted and the resulting linear equation is solved by successive relaxation.

Figures 1a and 1b show the flow cross-section, which is formed by inserting concentrically an equilateral triangular rod or duct in another equilateral triangular channel.

A normalized square mesh with ‘n’ grid points is imposed on the flow cross section as shown in Fig. 2. The normalization is based on the property, that the height of the equilateral triangle is equal to $\sqrt{3}/2$ times the side. Hence, the height of the grid is calculated as follows such that the grid points coincide with the surface of the triangle.

\[
\Delta z = \frac{a}{n} \quad \ldots (2)
\]

\[
\Delta y = \Delta z \times \frac{\sqrt{3}}{2} \quad \ldots (3)
\]

In the finite difference form Eq. (1) becomes

\[
\frac{u_{i-1,j} + u_{i+1,j} - 2u_{i,j}}{\Delta y^2} + \frac{u_{i,j-1} + u_{i,j+1} - 2u_{i,j}}{\Delta z^2} = \frac{1}{\mu} \frac{\partial p}{\partial x}
\]

...(4)

Substituting the values of $\Delta y$ and $\Delta z$ Eq. (4) becomes:
The boundary condition at the wall is $u = 0$, i.e. no slip at the wall. The $u^*$ values are taken as zeros at all the grid points on the boundary and as a first approximation the $u^*$ values are taken as 0.01 at the grid points inside the flow area. Using Eq. (7) the $u^*$ values are calculated repeatedly at all the grid points inside the flow area until the maximum residue, the difference between the calculated $u^*$ and old $u^*$ is less than $1 \times 10^{-5}$.

After the velocity profile is obtained, the average $u^*$ is calculated as

$$u_m^* = \frac{1}{A_c} \int \int \int u^* dydz$$

and the integral is evaluated by means of a two dimensional extension of Simpson’s rule.

The $fRe$ values are calculated as follows:

$$f_{\mu}Re = -\frac{D_e^2 \frac{dp}{dx}}{2\mu u}$$

Substituting $u^*$ for $u$ and simplifying

$$f_{\mu}Re = \frac{1}{2u_m^*}$$

The $fRe$ values for the entrance region of constant cross sectional ducts can be estimated based on hydro-dynamically developed $f_{\mu}Re$ values and Incremental pressure drop number $k(\infty)$ by the method proposed by Shah.

$$fRe = 3.44(\infty)^{-0.5} \text{ for } \infty \leq 0.001$$

The $k(\infty)$ values are calculated by the following approximate analytical equation of Lundgren et al.

$$k(\infty) = \frac{2}{A_c} \int \int \left[ \left( \frac{u}{u_m} \right)^3 - \left( \frac{u}{u_m} \right)^2 \right] dA_e$$

The thermal differential equation, derived by equating an energy balance and rate equation on the same element is

$$u \frac{\partial t}{\partial x} = \left( \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2} \right)$$

For constant heat input boundary condition, $\frac{\partial t}{\partial x}$ is constant. Defining a dimensionless temperature function as:

$$t^* = \int \int \left[ \frac{D_e^2 \frac{dp}{dx}}{\mu \alpha} \frac{\partial t}{\partial x} \right]$$

Using the same grid, the energy equation is written as:

$$t_{i,j}^* = \left( \frac{3a^2}{D_e^2 n^2} u_{i,j}^* + 4u_{i-1,j}^* + 4u_{i+1,j}^* + 3u_{i,j-1}^* + 3u_{i,j+1}^* \right) / 14$$

The values of $t^*$ are taken as zero on the boundary and 0.1 inside the flow area. The same procedure as in the case of the velocity problem is repeated until the maximum residue is less than $1 \times 10^{-7}$. The $t_m^*$ value is calculated as

$$t_m^* = \frac{1}{u_m^*A_c} \int \int u^* t^* dydz$$

The Nusselt number $NuH$ is calculated as follows.

$$h = c_p \rho u_m A_c \frac{\partial t}{\partial x} dx / (\Delta t_m dA)$$

$$NuH = \frac{hD_e}{k} = c_p \rho u_m A_c \frac{\partial t}{\partial x} dxD_e / (k\Delta t_m dA)$$
By substituting \( u_m^* \) and \( \Delta t_m^* \) for \( u_m \) and \( \Delta t_m \) and simplifying
\[
Nu_H = A_e u_m^*/(3aD_e \Delta t_m^* )
\] ... (20)

**Constant wall temperature**

Seban\(^1\) introduced a generalized temperature distribution for a fully developed temperature profile which is expressed in the following form
\[
\frac{\partial}{\partial x} \left( \frac{t_w - t}{t_w - t_m} \right) = 0
\] ... (21)

and by carrying through the indicated differentiation of the equation,
\[
\frac{dt_w}{dx} - \frac{\partial t}{\partial x} \frac{(t_w - t)}{(t_w - t_m)} \left( \frac{dt_w}{dx} - \frac{dt_m}{dx} \right) = 0
\] ... (22)

for constant wall temperature boundary condition, \( \frac{dt_w}{dx} = 0 \), the above equation becomes
\[
\frac{\partial t}{\partial x} = \frac{(t_w - t)}{(t_w - t_m)} \frac{dt_m}{dx}
\] ... (23)

Substituting for \( \frac{\partial t}{\partial x} \) from the equation into the Eq. (14),
\[
\frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2} = \frac{u}{\alpha} \frac{(t_w - t)}{(t_w - t_m)} \frac{dt_m}{dx}
\] ... (24)

The right-hand side of this equation is multiplied with a factor \( (t_w - t_o) / (t_w - t) \) and after rearrangement,
\[
\frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2} = \frac{u}{\alpha} \frac{(t_w - t_o)}{(t_w - t_m)} \frac{dt_m}{dx} \frac{(t_w - t)}{(t_w - t_o)}
\] ... (25)

A dimensionless temperature term \( t' \) is defined as
\[
t' = \frac{t}{D_e^4 \frac{dp}{dx} (t_w - t_o) \frac{dt_m}{dx}} \frac{\alpha u}{dx} \frac{t_w - t_m}{dx}
\] ... (26)

The same grid network is used and the equation is written in the finite difference form as
\[
t_{i,j} = \frac{3a^2}{D_e^2 n^2} u_{i,j}^* \left( \frac{t_w - t}{t_w - t_o} \right) + 4t_{i-1,j}^* + 4t_{i+1,j}^* + \frac{3t_{i,j-1}^* + 3t_{i,j+1}^*}{14}
\] ... (27)

The temperature distribution for the previously obtained constant heat input is used as first approximation. The remaining procedure is the same as in the case of velocity problem and constant heat input problem. The values of \( t_m \) and \( Nu_T \) are also calculated in the same way.

For numerical calculations, a computer programme is developed in Quick BASIC. The results showed grid independency from \( 30 \times 30 \) onwards. A \( 48 \times 48 \) grid is used for the present analysis. The programme is first executed for empty triangular channel geometry and the results are found to be in good agreement with the values of \( fRe = 13.33, Nu_T = 2.47, Nu_e = 3.11 \) as reported by Shah and London\(^1\).

The values for \( fRe \) and \( k(a) \) for various \( D_e^* \) values are obtained and are shown in Fig. 3. The values of minimum limiting Nusselt numbers for various \( D_e^* \) values are obtained and are shown in Fig. 4.

**Comparison with equilateral triangular and circular annular channels**

The flow area goodness factor \( (j/f) \) is one method by which the performance of heat exchange surfaces made of different idealized passage geometries may be contrasted. This factor is defined as
\[
j/f = Nu \frac{Pr^{1/3}}{fRe}
\] ... (28)

The data for equilateral triangular and circular annular ducts is taken from Shah and London\(^1\) and \( j/f \)
factors are calculated for water at 20°C (Pr=7). The calculated $j/f$ factors are shown in Figs 5 and 6.

### Experimental setup

The schematic diagram (Fig. 7) shows the general arrangement of the equipment, pressure taps, heating device, the interconnecting piping, etc. The same experimental unit is used for obtaining pressure drop data as well as heat transfer data. A large reservoir is used for the liquid that is to be circulated through the test section by means of a centrifugal pump. The annulus is formed by inserting an equilateral triangular rod, with side of 0.0254 m (Channel I), 0.0317 m (Channel II) and 0.0508 m (Channel III), concentrically in an equilateral triangular channel with side of 0.1016 m. The cross section of the test section is shown in Fig. 1b. All the three test sections are iron castings of 0.0127 m thick and 0.6096 m length, with 0.0127 m thick flanges on the ends. To measure the mid plane temperature of the test section, of 0.00635 m deep and 0.00158 m diameter holes are drilled on the triangular test section. Six holes, three in each row are drilled lengthwise on each of the three sides. Eighteen copper-constantan thermocouples are used to measure the temperatures.

In order to minimize the end effects at the entrance and exit of the test section, four calming sections, three on the upstream and one on the downstream of the test section, are fixed with a thick heat insulating material in between them. These calming sections have the same geometrical shape, length and internal dimensions as that of the test section and when fixed they are in flush with the test section. The calming sections are not long enough to ensure fully developed flow in the test section. To measure the inlet and outlet temperatures of the liquid passing through the test section, two thermometers (mercury in glass) of 1/10°C accuracy are placed, one each at the entrance and exit of the coolant. The coolant to be heated is forced by a centrifugal pump through the upstream calming section, test section, downstream calming section and the heat exchanger in that order and then back to the reservoir from which the pump re-circulates the liquid.
Correlations

Friction factor

In order to correlate the experimental isothermal pressure drop data obtained in the present investigation for all the three test sections, the dependency of the fanning friction factors on Reynolds numbers, both calculated based on equivalent diameter, is observed and it is found that the friction factors are varying with \( Re^{-1} \). The transition observed at Reynolds numbers around 2000, 1800 and 1200 for the channels with \( D_e = 0.044, 0.0403 \) and 0.02926 m, respectively. The experimental \( fRe \) values are found to be 64, 48 and 26 and from the numerical analysis, the fully developed \( ffdRe \) values are 19.7, 20 and 21 for channels with equivalent diameters of 0.044, 0.0403 and 0.02926 m, respectively. An attempt is made to predict the \( fRe \) values in the developing region by the method proposed for concentric annular annulus by Shah\(^9\), using the fully developed \( ffdRe \) values and the incremental pressure drop number \( k(\propto) \) values. The \( fRe \) values calculated by this method are not in agreement with the experimental values. This may be because of the fact that the method was proposed for channels with no sharp corners and in the present investigation, the test sections are having sharp edges as well as corners.

To correlate the friction factor data empirically, and to obtain the functional relationship between these higher friction factors and the hydrodynamic developing length, the \( (fRe-ffdRe) \) values are plotted against their respective \( (L_h/D_e) \) ratios on a log-log graph as shown in Fig. 8 and the \( (fRe-ffdRe) \) values are found to vary as \( (L_h/D_e)^{-5.36} \). The values of \( (fRe-ffdRe) \) \( (L_h/D_e)^{-5.36} \) are plotted against the Reynolds numbers for all the three geometries and shown in Fig. 9. The data are well correlated within ±10% deviation and the correlating equation in the laminar region, for the concentric equilateral triangular annular channels is

\[
fRe = ffdRe + 2.1 \times 10^{10} \left( \frac{L_h}{D_e} \right)^{-5.36} \quad \text{...(29)}
\]

This correlation may not hold good in the immediate entrance region where the friction factor values are a strong function of Reynolds number.

Nusselt number

In the present investigation, heat transfer by laminar flow forced convection in short passages of concentric triangular annular cross sections for the constant temperature boundary condition at the outer wall using distilled water and glycerol-water mixtures of 25, 50 and 75 percent glycerol as coolants is studied. The heat transfer data obtained, in each of the three channels, are plotted as Nusselt numbers versus Reynolds numbers, for all the four coolants, on log-log graph and are presented in Figs 10-12. It is observed that the Nusselt numbers are showing an increasing trend with increasing Reynolds numbers with a small slope. The effect of turbulence is not noticed even up to Reynolds numbers of 12,000. These trends are in accordance with the data reported earlier for various singly connected non-circular channels, Rectangular\(^12\), Trepezium\(^13\), Triangle\(^14\) and Ellipse\(^15\). The Nusselt numbers are also increasing with Prandtl numbers. The experimental Nusselt numbers are higher than the theoretically calculated minimum limiting Nusselt numbers, evidentially due to the partially developed velocity and developing temperature profiles.
The data of Clark et al. with hydro-dynamically developed velocity profile and developing temperature profile in square and rectangular tubes, show a similar trend and their correlation shows that Nusselt numbers are functions of Reynolds number, Prandtl number and \( \frac{L}{D_e} \). Lévêque suggested the following equation for the circular tubing and parallel plates to calculate the local Nusselt numbers.

\[
Nu_{X,T} = 0.427 \left( fRe \right)^{0.13} \left( X^* \right)^{-0.13} \quad \ldots(30)
\]

The same equation can be used to approximate the Nu values in the thermal entrance region for rectangular and square channels. Lundberg et al. calculated the Nusselt numbers based on the Lévêque type solution for the concentric circular annular channels and concluded that the same equation is applicable for the family of concentric circular annular channels.

The principle idealization in the Lévêque type approximation is that the thermal boundary layer is very thin compared to the momentum boundary layer in the thermal entry layer. Since the \( fRe \) factor is proportional to the velocity gradient at the wall, the \( fRe \) values are used to account for the velocity profile. In the present investigation, the velocity profile at the thermal entrance is not a fully developed profile; hence, the experimental \( fRe \) values are used to describe the velocity profile. From the literature, it is observed that the Nusselt number varies as \( \left( \frac{L}{D_e} \right)^{-0.13} \) and the same is taken as a correlating parameter for the present correlation to describe the effect of developing temperature profile. To study the effect of the Prandtl number on Nusselt number, the Nusselt numbers at a Reynolds around 500 are plotted against their respective Prandtl numbers on a log-log graph, presented in Fig. 13. It is found that Nusselt numbers are varying as \( \left( Pr \right)^{0.13} \). To account for the temperature dependent viscosity, the Sieder-Tate viscosity correction factor \( \left( \frac{\mu_{av}}{\mu_w} \right)^{0.14} \) is also applied. Finally the function 

\[
Nu = 1.646 \left( fRe \right)^{0.13} \left( Pr \right)^{0.13} \left( L/D_e \right)^{-0.13} \left( \frac{\mu_{av}}{\mu_w} \right)^{0.14}
\]

\ldots(31)

Values calculated by the above correlation showed an average deviation of 6% and a maximum deviation of 15% from the experimental data.
To verify the applicability of the present method of correlation to other non-circular short passages, an attempt is made to correlate the experimental data of Trapezoidal\textsuperscript{13} and Elliptical\textsuperscript{15} channels. The data are well correlated with an average deviation of less than 10%. The correlating equations are,

for trapezoidal channel

\[ \text{Nu} = 3.56(Pr)^{1/3}(Re)^{1/3}(L/De)^{-1/3}(Re)^{0.1}(\mu_{av}/\mu_{w})^{0.14} \]  \hspace{1cm} \text{...}(32)

and for elliptical channel

\[ \text{Nu} = 0.65(Pr)^{1/3}(L/De)^{1/3}(Re)^{0.27}(\mu_{av}/\mu_{w})^{0.14} \]  \hspace{1cm} \text{...}(33)

Conclusions

The results and conclusions of the foregoing numerical and experimental investigation may be summarized as follows:

(i) The relaxation method is utilized to solve the hydrodynamic and thermal differential equations of laminar flow of an incompressible fluid of constant physical properties through Concentric Equilateral Triangular annular duct with fully developed velocity and temperature profiles. Both constant heat input and constant wall temperature boundary conditions are considered.

(ii) The solutions for friction factor and incremental pressure drop number \( k(x) \) are presented in Fig. 3.

(iii) The minimum limiting Nusselt numbers, which are approached in laminar flow as tube length is increased or as Reynolds number is decreased, are presented in Fig. 4.

(iv) The \( j/f \) factors for the present geometry are higher than the concentric circular annular channel for constant wall temperature boundary condition and are lower for constant heat input boundary condition and the results are presented in Figs 5 and 6. The annular geometries formed by regular polygons may be considered for further detailed investigation as these new configurations can provide more compact arrangement than circular annular channels.

(v) The experimental laminar isothermal friction data for the concentric triangular annular channels could be correlated by the following empirical equation

\[ fRe = f_{id}Re + 2.1 \times 10^{10} \left( L/h/De \right)^{-5.36}. \]

(v) The heat transfer data obtained under laminar flow conditions with all the coolants whose Prandtl numbers range from 4 to 65, run through short passages of concentric equilateral triangular annular cross-sections, heated from outside under constant wall temperature could be correlated empirically by the following equation

\[ \text{Nu} = 1.646(Pr)^{1/3}(L/De)^{1/3}(Re)^{0.047}(\mu_{av}/\mu_{w})^{0.14} \]

Nomenclature:

- \( a \) : side of the outer triangular channel, m
- \( A_e \) : flow cross sectional area, m\(^2\)
- \( b \) : side of the inner triangular rod or channel, m
- \( D_e \) : equivalent (hydraulic) diameter, m
\( D_{e}^* \): ratio of the equivalent diameter of inner triangular rod or channel to the equivalent diameter of outer triangular channel
\( f \): fanning friction factor
\( f_{fd} \): fanning friction factor in fully developed flow
\( g \): acceleration due to gravity, m/s²
\( G \): mass velocity, Kg/m²s
\( \Theta \): thermal boundary condition referring to constant wall heat flux, both axially and peripherally
\( h \): film coefficient of heat transfer, Watts/m²-k
\( j \): \( \left( \frac{h}{c_p G} \right) \left( \frac{c_p \mu_{av}}{k_{av}} \right)^{\frac{2}{3}} \left( \frac{\mu_s}{\mu_{av}} \right)^{0.14} \)
\( k(\alpha) \): incremental pressure drop number for fully developed flow
\( k_{av} \): thermal conductivity of the liquid at \( t_{av} \), Watts/m-k
\( Nu \): Nusselt number, \( \frac{hD_e}{k_{av}} \)
\( Pr \): Prandtl number, \( \frac{c_p H_{av}}{k_{av}} \)
\( P \): Pressure, N/m²
\( Re \): Reynolds number, \( \frac{D_G}{\mu_{av}} \)
\( \Theta \): thermal boundary condition referring to constant wall temperature, both axially and peripherally
\( t \): temperature, ºC
\( t_{av} \): average bulk temperature of the liquid, ºC
\( t_s \): temperature at the mid-plane of the heat transfer surface, ºC
\( t'* \): dimensionless temperature defined by the Eq. (15)
\( t' \): dimensionless temperature defined by the Eq. (26)
\( u \): fluid velocity component in x-direction, m/s
\( u* \): dimensionless velocity defined by the Eq. (6)
\( x, y, z \): flow directions in Cartesian coordinates
\( x^* \): axial coordinates for the hydrodynamic entrance region, \( x/D_eRe \)
\( x^* \): axial coordinates for the thermal entrance region, \( x/D_ePe \)
\( \beta \): surface area density, m²/m³
\( \Delta \): prefix denoting the difference
\( \mu_{av} \): viscosity of the liquid at \( t_{av} \), kg/m-s
\( \mu_s \): viscosity of the liquid at \( t_s \), kg/m-s
\( \rho_{av} \): density of the liquid at \( t_{av} \), kg/m³

**Subscripts**
- \( H \): Constant heat flux boundary condition
- \( i,j \): \( i^{th} \) and \( j^{th} \) positions in \( y \) and \( z \) directions
- \( m \): mean
- \( 0 \): initial value
- \( T \): Constant temperature boundary condition
- \( w \): fluid at the wall

**References**