Levy index analysis in relativistic and ultrarelativistic nuclear collision – Evidence of non-thermal phase transition

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An analysis on Levy index of compound hadrons (pions + protons) emitted from $^{12}$C-AgBr and $^{24}$Mg-AgBr interactions both at 4.5 AGeV/c and $^{32}$S-AgBr interactions at 200 AGeV/c using the results of Takagi moment methodology in emission angle ($\cos \theta$) space and azimuthal angle ($\phi$) space has been presented. The results of our study reveal non-thermal phase transition at both relativistic and ultrarelativistic energy.

Keywords: Relativistic heavy ion interactions, Compound hadrons, Anomalous fractal dimensions, Levy index

1 Introduction

During the last few decades, various experiments have been performed with lepton-lepton, lepton-nucleus, hadron-hadron, hadron-nucleus and nucleus-nucleus interactions at relativistic and ultrarelativistic energies in order to reveal the underlying dynamics of multiparticle production process. The study of the production of relativistic particles (pions) has always been emphasized with the common belief that these particles are the most frequently produced particles and that the knowledge of pion production mechanism is essential for the understanding of main features of high energy interactions. Bialas and Peschanski\textsuperscript{1} introduced a new methodology to find out the non-statistical fluctuation in pseudorapidity space in place of analyzing the average features of multiparticle production. The idea of studying the intermittent pattern of particle production was introduced to explain the statistical significance of unusual events having sharp spikes in pseudorapidity interval. The statistical counting variable called the scaled factorial moment (SFM) $F_q$ is the main tool for studying intermittency. Scaled factorial moments are designed for extraction of non-statistical fluctuation after eliminating the statistical part. The concept of intermittency was first propounded by Bialas and Peschanski\textsuperscript{1} in high energy physics. It has been shown that the average SFM is equal to the moment of a true probability distribution of particle density without any statistical bias. The pioneers suggested that a growth of factorial moment $F_q$ following a power law with decreasing phase space interval size signals the onset of intermittency in the context of high energy interactions. The existence of self-similar nature in particle production process directly implies a connection between intermittency and fractality. The word fractality was first introduced by Mandelbrot\textsuperscript{2} in widely varying fields from galaxy distributions to coast line analysis. A fractal consists of a system in which more and more structure appears at smaller and smaller scales similar to the one at large scales. The theory of multifractals was developed in order to handle non-uniform fractals. For investigating the fractal structure in multiparticle data, various methods have been suggested. Hwa\textsuperscript{3} proposed the $G_q$ moment approach which has enriched the study of multifractality in multiparticle production. Hwa and Pan\textsuperscript{4} modified the old form of the $G_q$ moment by introducing a step function which can act as a filter for the low multiplicity events. Takagi\textsuperscript{5} proposed a novel method, $T_q$ moment for studying the fractal structure where the difficulties faced in the conventional method were overcome. Takagi\textsuperscript{5} pointed out that the experimental data do not show the expected linear behaviour in a log-log plot and this is partially due to the fact that most methods are unable to give the required mathematical limit: the number of points tends to infinity. $T_q$ moment approach is different from both $F_q$ moment and $G_q$ moment approach.

One of the properties of universal multifractals is that they can be classified by a parameter $\mu$($0<\mu<2$) called the Levy index, that indicates the degree of multifractality as well as estimates the cascading rate in self-similar branching process. The Levy index ($\mu$) can also be utilized to decipher possible mechanism...
of particle production. Such a characterization of multifractality is possible if the underlying density distribution can be described by a Levy stable law. The anomalous fractal dimension is related to the Levy law approximation. If \( \mu \) value is equal to 2, the Levy distribution is transformed into a Guassian one. Under this condition, one expects minimum fluctuation in the self-similar branching processes. On the other hand, \( \mu = 0 \) corresponds to monofractals and maximum fluctuation and might, therefore, be a signal of second order phase transition. \( \mu > 1 \) indicates the presence of wild singularities arising out of non-Poisson like fluctuations in density distribution.

Levy index analysis is very useful since it can classify different kinds of phase transitions during the cascading process: (a) \( 0 < \mu < 1 \), suggests a thermal phase transition (quark-gluon-plasma); (b) \( \mu > 1 \), suggests non-thermal phase transition. Various analyses have been reported in terms of produced pions. Although very little work has been done with the medium energy (30-400 MeV) knocked out target protons, which are also supposed to carry information about the inner dynamics of the particle production in high energy interactions. If the number of these fast target protons are combined with produced pions in a collision, a new parameter, named compound multiplicity \( n_c = n_g + n_s \) where \( n_c \) = compound multiplicity, \( n_g \) = number of grey tracks and \( n_s \) = number of shower tracks) is formed. Those particles (pions + fast target protons) will be called compound hadrons. It would be interesting to study the behaviour of the compound hadron data using the available tools since it may reveal more information about the inner dynamics of the particle production in high energy nuclear interactions. So far only limited attempts have been made to work with this parameter.

We have calculated the parameter \( \lambda_q = (\alpha_q + 1)/q \) from the intermittency exponent \( \alpha_q \) to look for possible non-thermal phase transition. But the analysis revealed no evidence of non thermal phase transition. The levy index analysis can give information about different kinds of phase transitions. In present paper, we have used the data of multifractal moments (Takagi) from our previous paper of compound hadrons over a wide range of energy from a few GeV to few hundred GeV (\(^{12}\text{C}-\text{AgBr} \) and \(^{24}\text{Mg}-\text{AgBr} \) interactions both at 4.5 AGeV/c and \(^{32}\text{S}-\text{AgBr} \) interactions at 200 AGeV/c) to study phase transition using Levy index. The study suggests a non-thermal phase transition in all interactions.

2 Experimental Details

The data sets used in this present analysis are obtained by exposing NIKFI-BR2 emulsion plates to \(^{12}\text{C} \) and \(^{24}\text{Mg} \) beam at incident momentum 4.5 AGeV/c at JINR Dubna, Russia, and G5 nuclear emulsion plates to \(^{32}\text{S} \) beam at incident momentum 200 AGeV/c at CERN SPS. The details of scanning and measurement have been described in our previous publications. For this study, we have taken 800 events of \(^{12}\text{C}-\text{AgBr} \) interactions both at 4.5 AGeV/c and 150 events of \(^{32}\text{S}-\text{AgBr} \) interactions at 200 AGeV/c. The emission angle \( \theta \) and azimuthal angle \( \phi \) with respect to beam direction were measured for each track by taking the readings of the coordinates of the interaction point \( (x_0, y_0, z_0) \), the coordinates \( (x_i, y_i, z_i) \) of a point on each secondary track and the coordinate \( (x_s, y_s, z_s) \) of a point on the incident beam. The emulsion technique possesses a very high spatial resolution, which makes it a very effective detector for studying the anomalous fractal dimension and Levy index.

3 Method of Study

Suppose in a multiparticle production process at some incident energy, the particle distribution is considered in a phase space \( x \). A single event contains \( K \) particles distributed in the interval \( x_{\text{min}} < x < x_{\text{max}} \). The multiplicity \( K \) changes from event to event according to the distribution \( P_K(\Delta x) \), where \( \Delta x = x_{\text{max}} - x_{\text{min}} \). The selected phase space interval of length \( \Delta x \) has been divided into \( M \) bins of equal size, \( \Delta x = \Delta x/M \). Then, the multiplicity distribution for a single bin is denoted as \( P_n(\Delta x) \) for \( n = 0, 1, 2, 3, \) etc. The particle distribution \( dn/dx \) is assumed to be constant and then \( P_n(\Delta x) \) is independent of the bin. The particles produced in \( \Omega \) independent events are distributed in \( \Omega M \) bins of size \( \Delta x \). Let \( N \) be the total number of compound hadrons produced in these \( \Omega \) events and \( n_{aj} \) the multiplicity of compound hadrons in the \( j \)-th bin of the \( a \)-th event. The theory of multifractals motivates one to consider the normalized density \( P_{aj} \) defined by:

\[
P_{aj} = n_{aj}/N \quad \text{...(1)}
\]
and to consider the quantity

\[ T_q(\delta x) = \ln \sum_{a=1}^{\Omega} \sum_{j=1}^{M} P_{aj}^{q} \text{ for } q > 0 \]  

...(2)

which behaves like a linear function of the logarithm of the resolution \( R(\delta x) \).

\[ T_q(\delta x) = A_q + B_q \ln R(\delta x) \]  

...(3)

where \( A_q \) and \( B_q \) are constants independent of \( \delta x \), \( q \) is the order number.

Choosing \( R(\delta x) = \langle x \rangle \) and evaluating the double sum of Eq. (2), one can finally arrive at a simple linear relation between \( \ln(\sigma^q) \) and \( \ln(\langle x \rangle) \):

\[ \ln < n^q > = A_q + (B_q + 1) \ln\langle n \rangle \]

\[ = A_q + [(q-1)D_q + 1] \ln \delta x \]  

...(4)

The detailed of the Takagi moment methodology can be found in Refs (5, 9, 13, 20).

The generalized fractal dimension \( D_q \) can be obtained from the slope values using the relation:

\[ D_q = B_q/(q-1) \]  

...(5)

Thus, the anomalous fractal dimension \( d_q \) can be defined as:

\[ d_q = 1 - D_q \]  

...(6)

The ratios of higher order anomalous fractal dimensions with respect to the second order anomalous fractal dimension, can be written as:

\[ B_q/B_2 = (d_q/d_2)(q-1) \]  

...(7)

\[ \beta_q = (d_q/d_2)(q-1) \]  

...(8)

where \( \beta_q = B_q/B_2 \) represents the degree of multifractality:

\[ \beta_q \text{ is related to Levy index (µ) by the equation:} \]

\[ \beta_q = \frac{q^\mu - q}{2^\mu - 2} \]  

...(9)

### 4 Results and Discussion

For the present analysis, we have considered compound hadrons emitted from \(^{12}\text{C}-\text{AgBr}\) and \(^{24}\text{Mg}-\text{AgBr}\) interactions both at 4.5 AGeV/c and \(^{32}\text{S}-\text{AgBr}\) interactions at 200 AGeV/c and used the data of Takagi moment\(^\text{13}\) in emission angle (\(\cos \theta\)) space and azimuthal angle (\(\phi\)) space. The emission angle space for compound hadrons was divided into overlapping bins whose size is changed in steps of 0.1 around the central value 0 (zero) and the azimuthal angle space was also divided into overlapping bins whose size was increased symmetrically in steps of 18° around the central value 180°. In our earlier publications\(^\text{9,13,20}\), we showed linear behaviour of \(\ln(n^q)\) (for \(q = 2, 3, 4, 5\)) against \(\ln(\langle x \rangle)\) in \(\cos \theta\) and \(\phi\) space for the three interactions.

The generalized fractal dimensions \(D_1, D_2, D_3, D_4\) and \(D_5\) were calculated from the slopes of the best linear fits. The values of the generalized fractal dimensions for different order of moments are presented in Table 1 for the three interactions in both phase spaces. In the present paper, we have calculated anomalous fractal dimensions for different order of moments (\(q = 3, 4, 5\)) against \(\ln(\langle n^q \rangle)\) (for \(q = 2, 3, 4, 5\)) against \(\ln(\langle x \rangle)\) in \(\cos \theta\) and \(\phi\) space for the three interactions.

### Table 1 — Represents the values of generalized fractal dimensions \(D_q\) (\(q = 1, 2, 3, 4, 5\)) for compound hadrons emitted from \(^{12}\text{C}-\text{AgBr}\) and \(^{24}\text{Mg}-\text{AgBr}\) interactions both at 4.5 AGeV/c and \(^{32}\text{S}-\text{AgBr}\) interactions at 200 AGeV/c in both \(\cos \theta\) and \(\phi\) spaces

<table>
<thead>
<tr>
<th>Interactions and energy</th>
<th>Phase space</th>
<th>(D_1)</th>
<th>(D_2)</th>
<th>(D_3)</th>
<th>(D_4)</th>
<th>(D_5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(^{12}\text{C}-\text{AgBr}) (4.5 AGeV)</td>
<td>(\cos \theta)</td>
<td>0.774 ± 0.030</td>
<td>0.760 ± 0.030</td>
<td>0.750 ± 0.032</td>
<td>0.745 ± 0.033</td>
<td>0.744 ± 0.034</td>
</tr>
<tr>
<td>(^{24}\text{Mg}-\text{AgBr}) (4.5 AGeV)</td>
<td>(\cos \theta)</td>
<td>0.550 ± 0.060</td>
<td>0.520 ± 0.061</td>
<td>0.505 ± 0.070</td>
<td>0.503 ± 0.071</td>
<td>0.503 ± 0.072</td>
</tr>
<tr>
<td>(^{32}\text{S}-\text{AgBr}) (200 AGeV)</td>
<td>(\phi)</td>
<td>0.805 ± 0.031</td>
<td>0.781 ± 0.027</td>
<td>0.763 ± 0.027</td>
<td>0.751 ± 0.030</td>
<td>0.745 ± 0.030</td>
</tr>
<tr>
<td>(^{12}\text{C}-\text{AgBr}) (4.5 AGeV)</td>
<td>(\phi)</td>
<td>0.638 ± 0.034</td>
<td>0.586 ± 0.038</td>
<td>0.569 ± 0.038</td>
<td>0.562 ± 0.038</td>
<td>0.562 ± 0.037</td>
</tr>
<tr>
<td>(^{32}\text{S}-\text{AgBr}) (200 AGeV)</td>
<td>(\phi)</td>
<td>0.895 ± 0.009</td>
<td>0.877 ± 0.011</td>
<td>0.862 ± 0.03</td>
<td>0.851 ± 0.009</td>
<td>0.844 ± 0.010</td>
</tr>
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</table>
Table 2 — Represents the values of degree of multifractality, \( \beta_q \) for different order of moments, \( q (q = 3, 4 \text{ and } 5) \) and Levy index, \( \mu \) for \(^{12}\text{C}-\text{AgBr} \) interactions at 4.5 AGeV/c in both \( \cos \theta \) and \( \phi \) spaces

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</table>
| \(^{12}\text{C}-\text{AgBr}\) (4.5 AGeV) | \( \cos \theta \) | 3 | 2.083 \( \pm \) 0.062 | \n
| \( \phi \) | 4 | 3.186 \( \pm \) 0.063 | 160 \( \pm \) 0.064 |
| 5 | 4.267 \( \pm \) 0.064 | 1.097 \( \pm \) 0.133 |

Table 3 — Represents the values of degree of multifractality \( \beta_q \) for different order of moments, \( q (q = 3, 4 \text{ and } 5) \) and Levy index, \( \mu \) for \(^{24}\text{Mg}-\text{AgBr} \) interactions at 4.5 AGeV/c in both \( \cos \theta \) and \( \phi \) spaces

<table>
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</table>
| \(^{24}\text{Mg}-\text{AgBr}\) (4.5 AGeV) | \( \cos \theta \) | 3 | 2.164 \( \pm \) 0.054 | \n
| 4 | 3.411 \( \pm \) 0.057 | 1.330 \( \pm \) 0.057 |
| 5 | 4.657 \( \pm \) 0.057 | 1.140 \( \pm \) 0.075 |

| \( \phi \) | 4 | 2.082 \( \pm \) 0.076 | \n
| 5 | 4.232 \( \pm \) 0.075 | \n
Table 4 — Represents the values of degree of multifractality, \( \beta_q \) for different order of moments, \( q (q = 3, 4 \text{ and } 5) \) and Levy index, \( \mu \) for \(^{32}\text{S}-\text{AgBr} \) interactions at 200 AGeV/c in both \( \cos \theta \) and \( \phi \) spaces

<table>
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<tr>
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<th>Phase space</th>
<th>Order of moments</th>
<th>( \beta_q )</th>
<th>( \mu )</th>
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</table>
| \(^{32}\text{S}-\text{AgBr}\) (200 AGeV) | \( \cos \theta \) | 3 | 2.244 \( \pm \) 0.041 | \n
| 4 | 3.634 \( \pm \) 0.020 | 1.210 \( \pm \) 0.021 |
| 5 | 5.073 \( \pm \) 0.021 | 1.430 \( \pm \) 0.032 |

| \( \phi \) | 4 | 2.627 \( \pm \) 0.028 | \n
| 5 | 6.614 \( \pm \) 0.032 | \n
\(^{24}\text{Mg}-\text{AgBr} \) interactions and in Fig. 3(a-b) for \(^{32}\text{S}-\text{AgBr} \) interactions in \( \cos \theta \) and \( \phi \) space. The \( \beta_q \) values gradually increase with the order of moment, \( q (q = 3, 4 \text{ and } 5) \) for all interactions in both phase spaces. We have then calculated Levy index \( \mu \) by fitting \( \beta_q \) versus \( q \) curve in the form given in Eq. (9). The values of \( \mu \) for the three interactions in both phase spaces have been presented in Tables 2-4. It is also interesting to observe that for all the interactions in both phase spaces \( \mu > 1 \), which suggest the evidence of non-thermal phase transition for all the three interactions in both phase spaces. The results are extremely interesting and useful for proper understanding of particle production in high and ultrahigh energy.
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References