Ecological, exergetic efficiency and heating load optimizations for irreversible variable-temperature heat reservoir simple air heat pump cycles

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Thermodynamic optimization of an irreversible air heat pump with variable-temperature heat reservoirs and hot- and cold-side counter-flow heat exchangers has been studied. The expressions of the heating load, the exergetic efficiency and the ecological function of the heat pump cycle are derived. Performance comparisons among exergetic efficiency optimization, ecological optimization and traditional heating load optimization objectives are done. The effect of the pressure ratio of the compressor, the allocation of heat exchanger inventory and the heat capacity rate matching between the working fluid and the heat reservoirs on the optimal performance of the cycle has been investigated by detailed numerical examples. When the performance optimization of the cycle is carried out by selecting the pressure ratio, three optimization objectives give simultaneously attention to the coefficient of performance (COP). The pressure ratio should be the one that is little bigger than the optimum pressure ratio corresponding to maximum COP, however, the results of three optimization objectives are consistent by optimizing the allocation of heat exchanger inventory and optimizing the heat capacity rate matching between the working fluid and the heat reservoirs. The optimum allocations of heat conductance are close to each other, and they are all less than 0.5. The results may provide guidelines for the design and optimization of practical air heat pump plants.

Keywords: Thermodynamic optimization, Irreversible air heat pump, Variable-temperature heat reservoir, Heating load, Exergetic efficiency, Ecological function

1 Introduction

The development of air heat pump was very slow in the early days because the system energy efficiency was lower than a vapour compression type machine having a phase change process and the reciprocating compression and expansion machinery used rendered the systems inefficient. Attention in air heat pumps has been received because air heat pumps utilized the environment friendly air as refrigerant. Moreover, today’s highly efficient turbomachinery has enhanced the performance of the air heat pump. Nowadays, air heat pumps are currently being considered for variety of applications and the analysis for the air heat pump cycle has been paid more attentions\textsuperscript{1-5}.

The finite time thermodynamics (FTT) or entropy generation minimization (EGM) has become the premier method for analyzing and optimizing performance of thermodynamic processes and cycles\textsuperscript{6,7}. FTT analysis for the performances of air heat pump cycles were performed by some researchers\textsuperscript{3,5}. The analytical formulae for the heating load and the coefficient of performance (COP) for air heat pump cycle were obtained\textsuperscript{3,4,5}. As to the performance optimization for heat pump cycles, the heating load and the coefficient of performance (COP) were often taken as the optimization objectives\textsuperscript{5}. In recent years, the research combining classic exergy concept\textsuperscript{8} with FTT\textsuperscript{6,7} is becoming increasingly important. Angulo-Brown\textsuperscript{9} proposed an ecological criterion for endoreversible Carnot heat engines. The optimization of the ecological function represents a compromise between the power output and the lost power. Chen \textit{et al.}\textsuperscript{10} provided a unified ecological optimization objective for all thermodynamic cycles. Ecological optimization has been carried out for endoreversible simple Brayton power-cycle\textsuperscript{11}, endoreversible\textsuperscript{12} and irreversible\textsuperscript{13} regenerative intercooled Brayton power-cycles, and endoreversible Brayton refrigeration cycles\textsuperscript{14}. A similar approach can also be applied to investigate the performances of heat-pump cycles. The ecological optimal performances of irreversible Stirling and Ericsson\textsuperscript{15} and Carnot\textsuperscript{16} heat-pump cycles were investigated. Exergy output optimization and exergetic efficiency optimization for endoreversible cogeneration cycle\textsuperscript{17}, endoreversible and irreversible
Carnot refrigeration cycles\cite{18} and irreversible Brayton refrigeration cycle\cite{19} were also carried out.

In this paper, heating load, exergetic efficiency and ecological criterion are taken as optimization objectives for performance optimization of an irreversible simple air heat pump cycle with variable-temperature heat reservoirs. Performance comparisons among ecological optimization, exergetic efficiency optimization and traditional heating load optimization objectives are performed.

2 Exergy Analysis Method

The exergy analysis method that combines the first and second laws of thermodynamics and environment conditions is used as a tool to understand thermodynamic systems and improve their performance.

Exergy is defined as the maximum theoretical work that can be obtained by bringing a system into equilibrium with the reference environment. The basic equation of exergy flow rate balance for a single-stream (one-inlet, one-exit) steady-state control volume is:

\[
\sum_j (1 - \frac{T_0}{T_j}) \dot{Q}_j - \dot{W}_{cv} + \dot{m} \psi_{in} - \dot{m} \psi_{out} - \dot{\xi}_d = 0 \quad \ldots (1)
\]

where \( T_0 \) is the temperature of typical environment, and \( \dot{Q}_j \) is the rate of heat transfer at the boundary where the instantaneous temperature is \( T_j \). The term \( \dot{W}_{cv} \) is the net power output across the system boundary. The mass flow rate is represented by \( \dot{m} \) while \( \psi_{in} \) and \( \psi_{out} \), respectively, represent the specific flow exergies at the inlet and the outlet. The terms \( \dot{m} \psi_{in} \) and \( \dot{m} \psi_{out} \) represent the exergy flow rates at the inlet and outlet, respectively. The final term \( \dot{\xi}_d \) accounts for the rate of exergy destruction due to irreversibilities within the control volume.

When a whole heat pump cycle is considered, the two terms of flow exergy rates become zeros. The exergy input rate \( \dot{\xi}_{in} \), the negative value of net power output \( \dot{W}_{cv} \) that crosses the system boundary, gives:

\[
\dot{\xi}_{in} = -\dot{W}_{cv} \quad \ldots (2)
\]

The purpose of employing a heat pump system is to release heat for heating space. The rate of exergy output utilized is the negative value of exergy transfer rate accompanying heat \( \sum_j (T_0/T_j - 1) \dot{Q}_j \). It gives:

\[
\dot{\xi}_{out} = -\sum_j (1 - \frac{T_0}{T_j}) \dot{Q}_j \quad \ldots (3)
\]

It is also noted that the values of exergy output rates are different for equivalent heat transfer rates at various boundary temperatures. From the above, the following equation is also obtained:

\[
\dot{\xi}_{out} = \dot{\xi}_{in} - \dot{\xi}_d \quad \ldots (4)
\]

The exergetic efficiency is defined as the ratio of exergy output rate to exergy input rate:

\[
\eta_{ex} = \frac{\dot{\xi}_{out}}{\dot{\xi}_{in}} \quad \ldots (5)
\]

Based on the view of point of exergy analysis, Chen et al.\cite{10} provided a unified ecological optimization objective for all of thermodynamic cycles, that is:

\[
\dot{E} = A/\tau - T_0 \Delta S / \tau = A/\tau - T_0 \sigma \quad \ldots (6)
\]

where \( A \) is exergetic output, \( T_0 \) is environment temperature, \( \Delta S \) is entropy generation, \( \sigma \) is entropy generation rate and \( \tau \) is cycle period. Thus, \( A/\tau \) is the exergy output rate and \( T_0 \sigma \) is the exergy destruction rate, that is:

\[
\dot{\xi}_{out} = A/\tau, \quad \dot{\xi}_d = T_0 \sigma \quad \ldots (7)
\]

Eq. (6) represents the best compromise between the exergy output rate and the exergy loss rate (i.e. entropy production rate) of the thermodynamic cycle. For heat engine cycles, it is the best compromise between the power output and the lost power\cite{9,11-13}.

Combining Eqs (4), (7) with (6) gives the general ecological function of heat pump cycles:

\[
\dot{E} = 2\dot{\xi}_{out} - \dot{\xi}_{in} \quad \ldots (8)
\]

3 Irreversible Air Heat Pump with Variable-temperature Heat Reservoirs Model

A simple irreversible air heat pump and its cycle with variable-temperature heat reservoirs are shown in Figs 1 and 2, respectively. The following assumptions are made for this model:

(i) The working fluid flows through the system in a steady-state fashion. The cycle is a Brayton heat pump and consists of two isobaric processes (1-2, 3-4) and two nonisentropic adiabatic processes (2-3, 4-1).
The working fluid is an ideal gas having constant thermal capacitance rate (the product of mass flow rate and specific heat) $C_{wf}$.

The rate of heat transfer ($\dot{Q}_H$) released to the heat sink, i.e., the heating load, the rate of heat transfer ($\dot{Q}_L$) supplied by the heat source, and the coefficient of performance (COP) of the heat pump cycle are, respectively, given by:

$$\dot{Q}_H = U_H[(T_3 - T_{Hout}) - (T_4 - T_{Hin})] / \ln[(T_3 - T_{Hout}) / (T_4 - T_{Hin})]$$

$$\times (T_4 - T_{Hin})] = C_H (T_{Hout} - T_{Hin})$$

$$= C_{H_{min}} I_{H1} (T_3 - T_{Hin}) = C_{wf} (T_3 - T_4) \quad \ldots (10)$$

$$\dot{Q}_L = U_L[(T_{Lin} - T_2) - (T_{Lout} - T_1)] / \ln[(T_{Lin} - T_2) / (T_{Lout} - T_1)] = C_L (T_{Lin} - T_{Lout})$$

$$= C_{L_{min}} I_{L1} (T_{Lin} - T_1) = C_{wf} (T_2 - T_1) \quad \ldots (11)$$

$$\beta = \frac{\dot{Q}_H}{\dot{Q}_L} \quad \ldots (12)$$

where $U$ is the heat conductance, $I$ is the effectiveness of the heat exchanger and $N$ is the number of heat transfer units. They are defined as:

$$I_{H1} = \{1 - \exp[-N_{H1}(1 - C_{H_{min}} / C_{H_{max}})]\} / \{1 - (C_{H_{min}} / C_{H_{max}})\} \times [1 - \exp[-N_{H1}(1 - C_{H_{min}} / C_{H_{max}})]\} \quad \ldots (13)$$

$$I_{L1} = \{1 - \exp[-N_{L1}(1 - C_{L_{min}} / C_{L_{max}})]\} / \{1 - (C_{L_{min}} / C_{L_{max}})\} \exp[-N_{L1}(1 - C_{L_{min}} / C_{L_{max}})]\} \quad \ldots (14)$$

where $C_{H_{min}}$ and $C_{H_{max}}$ are the minimum and maximum of $C_H$ and $C_{wf}$, respectively, and $C_{L_{min}}$ and $C_{L_{max}}$ are the minimum and maximum of $C_L$ and $C_{wf}$, respectively:

$$N_{H1} = U_{H1} / C_{H_{min}}, \quad N_{L1} = U_{L1} / C_{L_{min}}$$

$$C_{H_{min}} = \min\{C_H, C_{wf}\}, \quad C_{H_{max}} = \max\{C_H, C_{wf}\}$$

$$C_{L_{min}} = \min\{C_L, C_{wf}\}, \quad C_{L_{max}} = \max\{C_L, C_{wf}\} \quad \ldots (15)$$

The second law of thermodynamics requires $T_{out} T_{out} T_3 = T_2 T_4$. Combining Eqs (10)-(15) gives:

$$C_{wf} \eta_c C_{H_{min}} I_{H1} T_{Hin} (\eta_c x^2 - \eta_c + 1)$$

$$+ C_{L_{min}} I_{L1} T_{Lin} (C_{wf} - C_{H_{min}} I_{H1} \times (x + \eta_c x^2 - \eta_c + 1)$$

$$= C_{wf} \eta_e (C_{wf} - C_{H_{min}} I_{H1})(C_{wf} - C_{L_{min}} I_{L1})$$

$$\times (x + \eta_e x^2 - \eta_e + 1) \quad \ldots (16)$$

where $\eta_c$ is the compressor and expansion efficiencies are defined as:

$$\eta_c = (T_3 - T_2) / (T_3 - T_2), \quad \eta_e = (T_4 - T_1) / (T_4 - T_1) \quad \ldots (9)$$

(ii) The high-temperature (hot-side) heat sink is considered as having a finite thermal capacitance rate $C_H$. The inlet and outlet temperatures of the cooling fluid are $T_{Hin}$ and $T_{Hout}$ respectively. The low-temperature (cold-side) heat source is considered as having a finite thermal capacitance rate $C_L$. The inlet and outlet temperatures of the heating fluid are $T_{Lin}$ and $T_{Lout}$, respectively.

(iii) Because the counter-flow heat exchanger is optimal configuration, the hot- and cold-side heat exchangers are considered to be counter-flow heat exchangers, and their heat conductance (heat transfer coefficient-area product) are $U_H$ and $U_L$, respectively.
respectively:

\[ \eta = \frac{x + \eta_c - 1}[C_{sf} C_{min} T_{L1} T_{Lin} + C_{Hmin} I_{H1} T_{Lin}] \]

\[ \dot{Q}_H = \frac{\eta_c}{C_{Hmin} I_{H1} + C_{min} I_{L1} - C_{Hmin} C_{min} I_{H1} I_{L1} / C_{sf}} \]

where \( x = \eta_c \) is the isentropic temperature ratio of the working fluid, that is, \( x = T_n / T_2 = (P_2 / P_1)^{\gamma} \), where \( P_1 \) is the pressure ratio of the compressor, \( n = (k-1) / k \), and \( k \) is the ratio of specific heats. Combining Eqs (9)-(12), (16) and (17) gives the heating load (\( \dot{Q}_H \)) and the COP (\( \beta \)) of the irreversible cycle, respectively:

\[ \dot{Q}_H = \frac{C_{Hmin} I_{H1} C_{sf} (x + \eta_c - 1)[C_{min} I_{L1} T_{Lin} + T_{Lin} (C_{sf} C_{min} I_{L1})]}{C_{Hmin} I_{H1} C_{sf} (x + \eta_c - 1)[C_{min} I_{L1} T_{Lin} + \eta_c (x-1) - \eta + 1]} \]

\[ \dot{Q}_H = \frac{C_{Hmin} I_{H1} C_{sf} (x + \eta_c - 1)[C_{min} I_{L1} T_{Lin} + T_{Lin} (C_{sf} C_{min} I_{L1})]}{C_{Hmin} I_{H1} C_{sf} (x + \eta_c - 1)[C_{min} I_{L1} T_{Lin} + \eta_c (x-1) - \eta + 1]} \]

When \( \eta_c = \eta_t = 1 \) is satisfied, the irreversible cycle becomes endoreversible one, the heating load (\( \dot{Q}_H \)) and the COP (\( \beta \)) of the endoreversible cycle are, respectively:

\[ \dot{Q}_H = \frac{C_{Hmin} C_{min} I_{H1} I_{L1} (T_{Lin} x - T_{Hin})}{C_{Hmin} I_{H1} + C_{min} I_{L1} - C_{Hmin} C_{min} I_{H1} I_{L1} / C_{sf}} \]

\[ \beta = (1 - 1 / x)^3 = x / (x-1) = P_{in} / P_{out} \]

When the heat transfer areas of both heat exchangers tend to infinity, i.e. the overall heat conductance tends to infinity for both the heat exchangers, \( I_{H1} = I_{L1} = 1 / \lim_{x \to \infty} \) is satisfied. If \( \eta_c = \eta_t = 1 \) is also satisfied, that is an internally and externally reversible Brayton heat pump, the COP of the reversible Brayton heat pump is:

\[ \beta_{rev} = T_{Hin} / (T_{Hin} - T_{Lin}) \]

From Eqs (2) and (3), the exergy input rate and the exergy output rate are, respectively:

\[ \dot{E}_{in} = \dot{Q}_H - \dot{Q}_L \]

\[ \dot{E}_{out} = \int_{t_{Lin}}^{t_{Lin}} C_H (1 - T_0 / T) dT - \int_{t_{Lin}}^{t_{Lin}} C_L (T_0 / T - 1) dT \]

Combining Eqs (5), (10), (11), (16), (17), (23) and (24) gives the exergetic efficiency and the entropy-generation rate (\( \sigma \)) of the irreversible cycle, respectively:

\[ \eta_{ex} = 1 - \frac{C_{Hmin} I_{H1} C_{sf} (1 - \eta_c - 1)[C_{min} I_{L1} (x + \eta_c - 1) \eta_c (x-1) - \eta + 1]}{C_{Hmin} C_{min} I_{H1} I_{L1} / C_{sf}} \]

\[ \sigma = C_H \ln \left[ 1 + \frac{C_{Hmin} I_{H1} C_{min} I_{L1} (x + \eta_c - 1) \eta_c (x-1) - \eta + 1]}{C_{Hmin} C_{min} I_{H1} I_{L1} / C_{sf}} \right] \]

\[ \sigma = C_H \ln \left[ 1 + \frac{C_{Hmin} I_{H1} C_{min} I_{L1} (x + \eta_c - 1) \eta_c (x-1) - \eta + 1]}{C_{Hmin} C_{min} I_{H1} I_{L1} / C_{sf}} \right] \]

\[ \sigma = C_H \ln \left[ 1 + \frac{C_{Hmin} I_{H1} C_{min} I_{L1} (x + \eta_c - 1) \eta_c (x-1) - \eta + 1]}{C_{Hmin} C_{min} I_{H1} I_{L1} / C_{sf}} \right] \]

\[ \sigma = C_H \ln \left[ 1 + \frac{C_{Hmin} I_{H1} C_{min} I_{L1} (x + \eta_c - 1) \eta_c (x-1) - \eta + 1]}{C_{Hmin} C_{min} I_{H1} I_{L1} / C_{sf}} \right] \]

where, \( \tau_s = T_{Hin} / T_{Lin} \) and \( \tau_s = T_{Hin} / T_0 \) are the inlet temperature ratio of the heat reservoirs and the ratio of hot-side heat reservoir inlet temperature to ambient temperature, respectively. When \( \eta_c = \eta_t = 1 \) is satisfied, the exergetic efficiency and the entropy-generation rate (\( \sigma \)) of the endoreversible cycle are, respectively:

\[ \eta_{ex} = 1 - \frac{C_{Hmin} I_{H1} + C_{min} I_{L1} - C_{Hmin} C_{min} I_{H1} I_{L1} / C_{sf} \sigma}{C_{Hmin} C_{min} I_{H1} I_{L1} / C_{sf} \sigma} \]
The ecological function of the irreversible cycle is obtained by combining Eqs. (8)-(11),(16)-(18), (23) and (24) as:

$$
E = \frac{C_{uf}C_{Lunm}I_{L1}(C_{uf}(x+\eta_c-1)(\eta_c x^3-\eta_c+1)\tau_c-C_{Lunm}E_{H1})}{C_{uf}C_{Lunm}I_{L1}(x+\eta_c-1)(\eta_c x^3-\eta_c+1)\tau_c + C_{uf}C_{Lunm}E_{H1}(C_{uf}(x+\eta_c-1) - C_{Lunm}E_{H1})}
$$

$$
E = \frac{(C_{uf} - C_{Lunm}E_{H1})(C_{uf} - C_{Lunm}E_{H1})}{(C_{uf} - C_{Lunm}E_{H1})(C_{uf} - C_{Lunm}E_{H1})} 
\times (x+\eta_c-1)(\eta_c x^3-\eta_c+1) - 2T_0 \sigma
$$

When $\eta_c = \eta_c = 1$ is satisfied, the ecological function of the endoreversible cycle is:

$$
E = \frac{C_{Hmin}C_{Lunm}I_{L1}(C_{Hmin}I_{L1} - T_{lin} - T_{lin} / P_{in})}{C_{Hmin}I_{L1} + C_{Lunm}I_{L1} - C_{Hmin}C_{Lunm}I_{H1}I_{L1} / C_{uf}} - 2T_0 \sigma
$$

To facilitate analysis and optimization, the dimensionless heating load and the dimensionless ecological function of the irreversible cycle are defined as, respectively:

$$
\tilde{Q}_H = \frac{\tilde{Q}_H}{(C_c T_{Hmin})} = \frac{C_{Hmin}C_{Lunm}I_{H1}I_{L1}/\tau_c - (P_{in} \tau_c - 1)}{C_{Hmin}C_{Lunm}I_{H1}I_{L1}/\tau_c - C_{Hmin}C_{Lunm}I_{H1}I_{L1} / C_{uf}}
$$

$$
\tilde{E} = \frac{\tilde{E}}{(C_c T_{Hmin})} = \frac{C_{Hmin}C_{Lunm}I_{H1}I_{L1}/\tau_c - (P_{in} \tau_c - 1)}{C_{Hmin}C_{Lunm}I_{H1}I_{L1}/\tau_c - C_{Hmin}C_{Lunm}I_{H1}I_{L1} / C_{uf}}
$$

$$
\tilde{E} = \frac{\tilde{E}}{(C_c T_{Hmin})} = \frac{-2\sigma}{\tau_c C_c}
$$

4 Performance Analysis and Optimization

Eqs. (19), (25), (31) and (32) indicate that when the heat reservoir temperature ratio and the ratio of hot-side heat reservoir temperature to ambient temperature are fixed, the exergetic efficiency ($\eta_{ex}$), the COP ($\beta$), the dimensionless heating load ($\tilde{Q}_H$) and the dimensionless ecological function ($\tilde{E}$) of the irreversible air heat pump cycle with variable-temperature heat reservoirs are dependent on the pressure ratio ($P_r$), the heat-transfer irreversibility ($I_{H1}$ and $I_{L1}$), the internal irreversibility ($\eta_c$, $\eta_e$) and the heat capacity rate of the working fluid and the heat reservoirs. Therefore, the performance optimization of the cycle can be carried out through selection of the pressure ratio, optimization of the heat-transfer or optimization of the heat capacity rate matching between the working fluid and the heat reservoirs.

4.1 Heating load, COP, exergetic efficiency and ecological function versus pressure ratio

To see the effect of pressure ratio ($P_r$) on the heating load, the COP, the exergetic efficiency and the ecological function, detailed numerical examples are provided. The following data used in the numerical calculation come from the real example of Qin et al., so that the analytical results can be validated.

Figures 3 and 4 show the effects of the compressor and expansion efficiencies on the relations of $\beta$ versus $P_r$, $\tilde{Q}_H$ versus $P_r$, $\eta_{ex}$ versus $P_r$ and $\tilde{E}$ versus $P_r$ with $k=1.4$, $\tau_c=1$, $\tau_e=1.25$, $I_{H1}=I_{L1}=0.9$, $C_c=C_{uf}=1.0$ kW/K and $C_{uf}=0.8$ kW/K, respectively. $\eta_c = \eta_c = 1$ and $\eta_e = \eta_e = 0.8$ are given in Figs 3 and 4, respectively. From Fig. 3, one can see that for the above fixed parameters, both $\tilde{Q}_H$ and $\tilde{E}$ are monotonically increasing functions of $P_r$ while both $\beta$ and $\eta_{ex}$ are hyperbolic functions of $P_r$ and $\beta$ and $\eta_{ex}$ decrease...
monotonically with the increase of pressure ratio. Therefore, when the heating load is taken as the optimization objective, the COP should be simultaneously given attention and when the ecological function is taken as the optimization objective, the exergetic efficiency should be simultaneously given attention. Figure 4 shows that both $\bar{Q}_H$ and $\eta_{ex}$ are monotonically increasing functions of $P_r$, while $\bar{E}$ increases at first and then decreases with the increase of pressure ratio and the curve of $\beta$ versus $P_r$ is a parabolic-like one. That is, there exists a optimum pressure ratio ($P_{\text{opt}, \beta}$) which leads to maximum COP ($\beta_{\text{max}, P_r}$). Figures 5 and 6 show the effects of the effectiveness of the hot- and cold-side heat exchangers ($I_H$ and $I_L$), and the efficiencies of the compressor and expander ($\eta_c$ and $\eta_t$) on the optimum pressure ratio ($P_{\text{opt}, \beta}$) versus heat reservoir temperature ratio ($\tau_3$), respectively. Those figures indicate that $P_{\text{opt}, \beta}$ is monotonically increasing function of $\tau_3$ while $P_{\text{opt}, \beta}$ decreases with increases of $I_H$, $I_L$, $\eta_c$, and $\eta_t$.

For the fixed heat conductance of the hot- and cold-side heat exchangers, Figs 7-10 show the effects of heat exchanger effectiveness on $\beta$ versus $P_r$. $\bar{Q}_H$
versus $P_r$, $\eta_{ex}$ versus $P_r$ and $\overline{E}$ versus $P_r$, with $k = 1.4$, $\tau_3 = 1.25$, $\tau_4 = 1$, $\eta = \eta_i = 0.8$, $C_{w_f} = 0.8$ kW/K and $C_L = C_H = 1.0$ kW/K, respectively. The numerical results show that the heat exchanger effectiveness only affects quantitatively $\beta$ versus $P_r$, $\overline{Q_H}$ versus $P_r$, $\eta_{ex}$ versus $P_r$ and $\overline{E}$ versus $P_r$. It does not change the curve shape. $\beta$, $\overline{Q_H}$, $\eta_{ex}$ and $\overline{E}$ increase with the increases in $I_{HL}$ and $I_{L1}$.

4.2 Optimal distribution of heat conductance

If the heat conductances of the two heat exchangers are changeable, the dimensionless heating load, the COP, the exergetic efficiency and dimensionless ecological function may be optimized by searching the optimum distribution of heat conductance for the fixed total heat exchanger inventory. For the fixed heat exchanger inventory $U_T$, that is, for the constraint of $U_H + U_L = U_T$, defining the distribution of heat conductance $u = U_L / U_T$ leads to:

$$U_L = uU_T, \quad U_H = (1 - u)U_T$$

... (35)

The following data used in the numerical calculations come from the real example of Qin et al.\textsuperscript{20}, so that the analytical results can be validated.

Figure 11 shows the COP ($\beta$), the dimensionless heating load ($\overline{Q_H}$), the exergetic efficiency ($\eta_{ex}$) and the dimensionless ecological function ($\overline{E}$) versus the...
distribution of heat conductance \((u)\) with \(k=1.4, \quad \tau_s=1.25, \quad \tau_a=1, \quad C_{of}=0.8\ kW/K, \quad U_f=5\ kW/K, \quad \eta_c=\eta_r=0.9\) and \(P_r=3\). The diagram indicates that the curves of \(b, \quad \overline{Q}_H, \quad \eta_{ex} \) and \(\overline{E}\) versus \(u\) are parabolic-like ones, which means there exist optimum allocations \((u_{\text{opt}}, \quad u_{\text{opt},Q_H}, \quad u_{\text{opt},\eta_r}, \quad u_{\text{opt},E})\) of heat conductance corresponding to maximum COP \((\beta_{\text{max},u})\), maximum dimensionless heating load \((\overline{Q}_{H\text{max},u})\), maximum exergetic efficiency \((\eta_{\text{ex,\max},u})\) and maximum dimensionless ecological function \((\overline{E}_{\text{max},u})\) for a fixed pressure ratio \((P_r)\), respectively.

Figures 12-15 show the effects of the compressor and expansion efficiencies on optimum allocations \((u_{\text{opt},\beta}, \quad u_{\text{opt},\overrightarrow{Q}_H}, \quad u_{\text{opt},\eta_r}, \quad u_{\text{opt},E})\) of heat conductance with \(k=1.4, \quad \tau_s=1.25, \quad \tau_a=1, \quad C_L=C_H=1.0\ kW/K, \quad C_{of}=0.8\ kW/K\) and \(U_f=5\ kW/K\), respectively. The optimum allocations \((u_{\text{opt},\beta}, \quad u_{\text{opt},\overrightarrow{Q}_H}, \quad u_{\text{opt},\eta_r}, \quad u_{\text{opt},E})\) of heat conductance are close to each other, and they are all less than 0.5. When \(\eta_r=\eta_t=1\), the cycle is endoreversible one, \(u_{\text{opt},\overrightarrow{Q}_H}, \quad u_{\text{opt},\eta_r}\) and \(u_{\text{opt},E}\) identically equal to 0.5 while \(\beta\) is independent to the distribution of heat conductance \((u)\).

### 4.3 Optimal thermal capacity rate matching between the working fluid and heat reservoirs

The effects of thermal capacity rate matching \((c=C_{of}/C_H)\) between the working fluid and the heat reservoir on the optimization objective functions are analyzed by numerical examples.

**Fig. 11** — The COP, dimensionless heating load, dimensionless heating load density, exergetic efficiency and dimensionless ecological function versus distribution of heat conductance.

**Fig. 12** — Effect of efficiencies of the compressor and expander on the optimum distribution of heat conductance versus pressure ratio.

**Fig. 13** — Effect of efficiencies of the compressor and expander on the optimum distribution of heat conductance versus pressure ratio.

**Fig. 14** — Effect of efficiencies of the compressor and expander on the optimum distribution of heat conductance versus pressure ratio.
Fig. 15 — Effect of efficiencies of the compressor and expander on the optimum distribution of heat conductance versus pressure ratio.

Fig. 16 — The COP, dimensionless heating load, dimensionless heating load density, exergetic efficiency and dimensionless ecological function versus thermal capacity rate matching between the working fluid and the heat reservoirs. In the calculations, \( k = 1.4 \), \( C_e = 1.0 \text{ kW/K} \), \( u = 0.5 \), \( P_r = 5 \), \( \tau_s = 1.25 \) and \( \tau_a = 1 \) are set.

Figure 16 shows the relations of the COP(\( \beta \)), the dimensionless heating load (\( \bar{Q}_H \)), the exergetic efficiency (\( \eta_{ex} \)) and the dimensionless ecological function (\( \bar{E} \)) versus thermal capacity rate matching between the working fluid and heat reservoirs (\( c \)) with \( C_H = 1.0 \text{ kW/K} \) and \( \eta_c = \eta_t = 0.99 \). Figure 16 indicates \( \beta \) is monotonically decreasing function of \( c \) while \( \bar{Q}_H \) versus \( c \), \( \eta_{ex} \) versus \( c \), and \( \bar{E} \) versus \( c \) are parabolic-like shaped curves. That is, there exist optimum heat capacitance rate matching between the working fluid and heat reservoirs \( c_{opt,\bar{Q}_H} \), \( c_{opt,\eta_{ex}} \) and \( c_{opt,\bar{E}} \) corresponding to the maximum dimensionless heating load (\( \bar{Q}_{H,\text{max,c}} \)), the maximum exergetic efficiency (\( \eta_{ex,\text{max,c}} \)) and the maximum dimensionless ecological function (\( \bar{E}_{\text{max,c}} \)), respectively.

Figures 17-19 show the effects of the efficiencies of the compressor and expander (\( \eta_c \) and \( \eta_t \)) on the dimensionless heating load (\( \bar{Q}_H \)), the exergetic efficiency (\( \eta_{ex} \)) and the dimensionless ecological function (\( \bar{E} \)) versus thermal capacity rate matching between the working fluid and heat reservoirs with \( C_H = 1.0 \text{ kW/K} \) and \( U_T = 5 \text{ kW/K} \). In the figures, \( \eta_c = \eta_t = 1 \) corresponds to endoreversible cycle, when...
The expressions of the ecological function and the exergetic efficiency of the simple air heat pump cycles with variable-temperature heat reservoirs are derived. The influences of various parameters on the optimal performance of the cycle are investigated by detailed numerical examples. Performance comparisons among ecological optimization, exergetic efficiency optimization and traditional heating load optimization objectives are carried out. The effects of the nonisentropic losses in the compression and expansion processes on the performance of an irreversible air heat pump cycle are analyzed.

The results show that there exist optimum distributions of heat conductance and optimum thermal capacity rate matching between the working fluid and heat reservoir corresponding to the maximum heating load, the maximum exergetic efficiency and the maximum ecological function, respectively. For an irreversible air heat pump cycle, when the performance optimization of the cycle is carried out by selecting the pressure ratio, three optimization objectives give simultaneously attention to the COP. The pressure ratio should be the one that is little bigger than the optimum pressure ratio corresponding to maximum COP. The optimum allocations of heat conductance are close to each other, and they are all less than 0.5. The optimum thermal capacity rate matching between the working fluid and heat reservoir is affected by the efficiencies of the compressor and expander. The work in this paper may provide some theoretical guidelines for the design of practical air heat pump plants. Constructing a composite objective function that combines more than one performance parameter for optimization mentioned in this paper will be a further step to obtain Pareto solution for the multi-objective optimization problem.

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Nomenclature
A Exergetic output (kJ)
C Thermal capacitance rate (kW/K)
c Thermal capacity rate matching between the working fluid and the heat reservoir
\(\dot{E}\) Ecological function (kJ)
\(\dot{E}\) Dimensionless ecological function
I Effectiveness of the heat exchanger
$k$ Ratio of specific heats
$m$ Mass flow rate (kg/s)
$N$ Number of heat transfer units
$n$ $(k-1)/k$
$P_r$ Pressure ratio
$\dot{Q}$ Rate of heat transfer (kW)
$\dot{Q}_H$ Heating load (kW)
$\bar{\dot{Q}}_H$ Dimensionless heating load
$\Delta S$ Entropy generation (kJ/K)
$T$ Temperature (K)
$U$ Heat conductance (kW/K)
$u_l$ Distribution of heat conductance $U_l/U_T$
$\dot{W}_{rev}$ Net work transfer rate across the system boundary (kW)
$x$ Isentropic temperature ratio of the working fluid

**Greek symbols**

$\beta$ Coefficient of performance (COP)
$\sigma$ Rate of entropy generation (kW/K)
$\Psi$ Specific flow exergy (kJ/kg)
$\eta$ Efficiency
$\tau$ Cycle period (s)
$\tau_1$ Heat reservoir temperature ratio
$\tau_2$ Ratio of hot-side heat reservoir temperature to ambient temperature
$\dot{\varepsilon}$ Rate of exergy (kW)

**Subscripts**

$c$ Compressor
d Destruction
ex Exergy
$\bar{E}$ Dimensionless ecological function point
$H$ Hot-side heat reservoir
$in$ Inlet or input
$L$ Cold-side heat reservoir
$\max$ Maximum value
$max_{max}$ Double maximum value
$opt$ Optimal value
$out$ Outlet or output
$P_r$ Pressure ratio

$\bar{Q}_H$ Maximum dimensionless heating load point
$T$ Total
t Expansion machine
$wf$ Working medium
$\eta_{ex}$ Maximum exergy efficiency point
$0$ Environment

**References**