FEATURE

Combating Epidemics–Pandemics Mathematically

Kumar Gandharv Mishra

COVID-19 pandemic has significantly changed today's lifestyle. With every passing day, people are adapting to new practices and getting accustomed to new sets of rules (at both social and personal level) in order to reduce the risk of getting oneself and others infected from with the SARS-COV-2 virus.

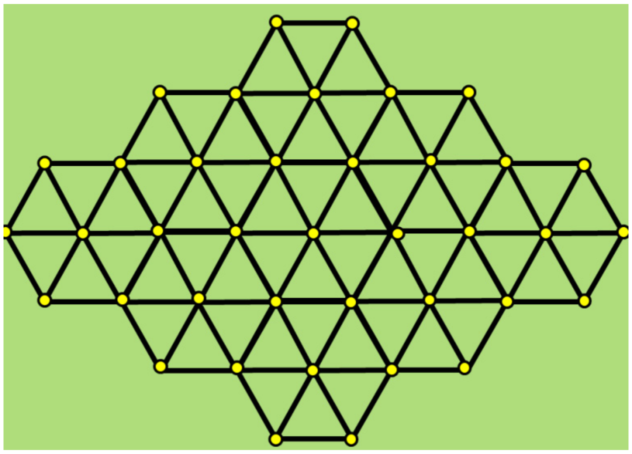
At the policy and administration level, officials have been involved 24x7 from exercising rules of the *Lockdown* to transforming and improvising these rules during *Unlock*. As fresh information and research studies regarding COVID-19 came out every day, some of the practices and rules also underwent frequent changes. The idea behind implementation and improvisation of such rules is to facilitate citizens along with proper social distancing, minimum crowd gathering and also to maintain hygiene so that the spread of infection can be reduced.

These situations reflect the importance and need for logic-based and practical decisions. So, how are these decisions regarding lockdown, closing and reopening of services made? These must be based on studies through various mathematical-statistical tools. From tackling the invisible virus biologically to contact tracing and adaptation of various rules and practices by common citizens in their behaviour, mathematics plays an integral role (both implicit and explicit).

This article presents a brief overview of mathematics embedded in various aspects of the situation created by the pandemic COVID-19. Though the article is based on COVID-19, the mathematics discussed here also applies to other epidemics in general.

Maths of Physical Distancing

One of the most effective and widely adopted steps for prevention from coronavirus infection is to maintain physical distance from each other. It has been suggested that individuals should meet or talk with each other by maintaining a distance of minimum 1 metre. Workplaces need changes, but this is a challenge for offices, theatres, schools, colleges and various centres.



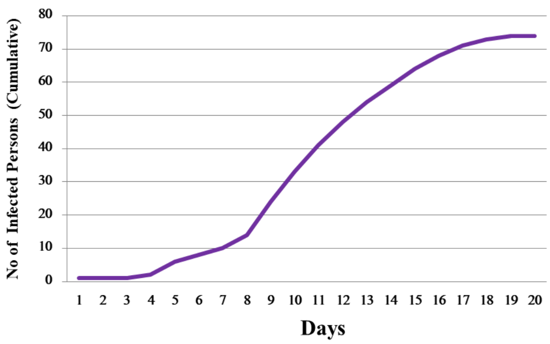
A Hexagonal arrangement (extension) of people in an open space with social distancing

It is easy to maintain a distance of 1 m among 2-3 persons, but it becomes complex as the number of persons increases in a given space. Seating arrangement transforms from triangular, square to hexagonal as the number of persons increases. The scenario may be treated as a geometric puzzle and can be solved by applying basic mathematics in different situations. However, it is tough to be followed because of various constraints in public places.

Graphs and Data Analysis

You would have often come across newspapers and other mediums depicting graphs, pie-charts or bar diagrams related to COVID-19. These graphs and other depictions also help policymakers and administrators to understand the effect and impact of infection and take timely decisions such as lockdown, economy packages, implementation of strict measures, opening and closing of various services, etc.

A study of these graphs through data sets and various mathematical-statistical models brings out relevant information about the spread of infection over a time period, over a geographical location, in different age groups, etc. A graph represents the relation between two quantities (say, day and number of infections) pictorially.



A graph depicting the relationship between total numbers of infected persons with each passing day (graph is for representative purpose only)

You would have observed COVID-19 related graphs in these formats: a) Linear or Logarithmic Scale and b) Cumulative or Daily basis.

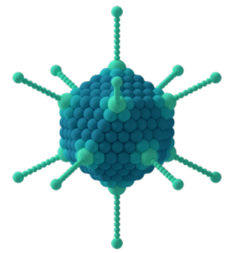
While the interval on one of the axes is fixed in the linear scale, the interval usually increases 10 times in a logarithmic scale. When the number of cases increases in thousands or millions, logarithmic scale is used. A glimpse of these graphs of different regions also helps get an idea of which regions are doing better in a certain period of time.

The cumulative cases and the corresponding graph can also be represented in the form of a function. Tools such as calculus (differentiation, differential equations and statistical tools) can be used to study these functions. For example the nature of function – whether it is increasing or decreasing – and the rate (positive or negative) with which it is increasing or decreasing can be found using first derivative and second derivative of the function. Based on such results, it can be known if the infection is spreading slowly or at a faster rate.

However, various factors are also considered and mathematical-statistical tools are applied for a precise investigation and conclusion. Indian institutions are using mathematical modelling and studying computational aspects of COVID-19 pandemic under MATRICS scheme of SERB (DST). These 11 projects undertake various aspects of COVID-19 such as migration, role of asymptomatic population, effects of lockdown, heterogeneity of population, etc. The studies will help provide insights into kinetics and management of infectious diseases *(https://dst.gov.in/serb-approves-funding-study-mathematical-simulation-aspects-covid-19)*.

Symmetry and Virus Structure

Vaccines and drugs are vital to protect lives from infections. The search for vaccines and drugs related to combat Sars-Cov-2 is on. Viruses need human cells to survive and multiply, so it is important for antiviral drugs to attack the virus without damaging human cells. A virus contains a protein shell called capsid which protects its nucleic acid – RNA or DNA. Capsid is responsible for viral infection by attacking living cells. These capsids are mostly icosahedron in shape.



A 3D schematic of Adenovirus
(Image source: Thomas Splettstoesser, https://upload.wikimedia.org/wikipedia/commons/6/6a/Adenovirus_3D_schematic.png)

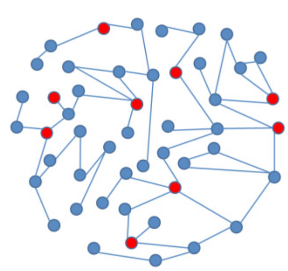
An Icosahedron
(Image source modified: DTR, https://en.wikipedia.org/wiki/Icosahedron#/media/File:Icosahedron.svg)

An Icosahedron structure consists of 20 equilateral triangular faces, 12 vertices and 30 edges connected with each other which appear like a sphere – making it highly symmetrical and stable. It has 120 symmetries. Symmetry gives rise to invariance in its view. Thus, one of the approaches in developing an antiviral drug is to understand the structure (symmetry) of the virus so that the drug can attack it and break or interfere with its structure to make the virus inactive.

Drugs are designed in a way to interfere or block the viral assembly i.e. a structure that can interfere with the symmetry structure of the virus. This requires a mathematical study of symmetries and structure of the virus. Such mathematical study of the structure of viruses was initiated in 1956 by James Watson and Francis Crick. With time, high-resolution X-ray crystallography and new techniques have provided insights into the structure of viruses and strategies for developing antiviral drugs. Symmetry is used as a tool in virology to understand and classify viruses and hence develop a drug that could attack its structure with effective orientation and symmetry.

Contact Tracing & Infection Trees

'Contact Tracing' has been one of the effective measures to combat the spread of coronavirus by tracing suspected cases. It involves tracing those persons who have come in contact with a COVID-19 positive person and testing-isolating them. In a way, contact tracing is a part of solving networks created with contact (travel) history of confirmed positive cases.

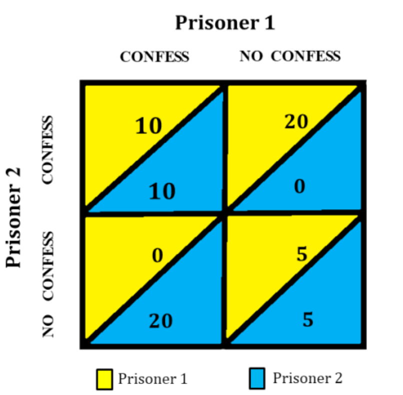


An example of a network: Dots represent nodes. Edges represent a contact between two persons. The red dots represent an infected person (population).

Scientists have been studying various network models based on epidemics. Take for example, Geographical Networks or Persons to Person Networks. In these networks, individuals or geographical locations can be treated as a node. Through the study of infection trees in networks, one can keep track of the situation and also plan the strategy for testing of suspected cases. These networks can be studied based on their structure. For example edges and nodes can be studied using various concepts of graph theory; the symmetry and design of networks can be studied to understand nature of spread in various geographical locations, the pattern of repetition of network across networks can also be investigated. Larger networks can be studied through the characteristics of smaller networks.

Lockdown, Human Behaviour & Game Theory

Stockpiling during lockdown: Game theory is the study of how individuals behave during the collective decision making situations. It is related to decision making in competitive situations. Let's try to understand Game theory through a famous example of Prisoners' Dilemma.



The matrix shows imprisonment (in years) for each prisoner based on combination of their choices

The prisoner's dilemma represents a social dilemma where decisions of individuals compete with decisions of another group or individual. It is assumed that individuals try to maximize their own outcome. Here, two prisoners are interrogated separately. The matrix represents the outcome for each prisoner based on combination of 'confession' and 'non-confession' by both. The maximum imprisonment for their crime is 20 years (say). A choice of 'reduced sentence' is given to whoever confesses. If both prisoners don't confess (both cooperate with each other), both get fixed imprisonment due to lack of any additional information related to the incident. However, if P1 confesses and shares information against P2 while P2 doesn't confess, then in this situation, P1 can be set free and P2 gets maximum imprisonment because of additional information and vice versa if P2 confesses and P1 doesn't.

In another scenario, if both prisoners confess about their involvement in the incident, they get reduced imprisonment compared to maximum imprisonment (according to the conditions of interrogation).

Now, let's see how Game theory gets redefined during the pandemic. A nationwide lockdown came into effect from 25 March 2020. Several regions again went into lockdown due to sudden rise in cases. You would have observed a rush to buy products from markets as well as stockpiling of products in different parts of the country, a day before the lockdown or as soon as the lockdown was announced. The situation can be related with aspects of Game theory, where every individual thinks he/she can quickly fetch some products before the lockdown starts and so does everyone, which ultimately creates chaos at the same time. It leads to unfavourable situations: (i) Breach of social distancing, (ii) Stockpiling and (iii) Rise in prices of products.

This game can be seen in these following contexts:

i. When customers act as players

ii. When shopkeepers and customers act as the two players

In such a situation, if you don't go but others go shopping, products would run out of stock and you will lose. This is what everyone thinks and goes to shop; consequently, crowd gathering takes place and social distancing breaks (customer-customer game). Now let's say, an individual buys 3-5 packets of a product instead of 1. Experiencing these, shopkeepers raise the price of these products (customer-shopkeeper game).

In such a situation, sometimes the administration also acts as a player to counter the behaviour of shopkeeper and customer by restricting human movement or by raising the prices of some products to reduce stockpiling so that everyone can get access to these products. Cooperative decisions by individuals can be helpful in controlling such chaos. However, it is often challenging because of human behaviour.

Human behaviour: Human behaviour during the times of an epidemic or pandemic can also be investigated in the context of Game theory. Risk perception and behaviour play a role here.

A study *'Risk perception and effectiveness of uncoordinated behavioral responses in an emerging epidemic'* (Winner of the Bellman Prize 2015 for the best paper in 2012 in the journal *Mathematical Biosciences*) applied Game theory to study the behaviour model. According to the researchers, the biggest changes in behaviour happen when there is an equal division of *reactive* and *unreactive* people; people who change behaviour are less likely to be infected and at a certain point in the epidemic those people have an advantage in the population, and others begin to copy their behaviour *(https://www.elsevier.com/connect/story/elsevier/awards/using-game-theory-to-predict-peoples-behavior-in-an-epidemic)*.

The study also suggests that people are more responsive when the symptoms of the disease are more evident, like coughing and sneezing. Reducing the number of people who can come in contact with an individual, even by a small amount, can make a difference to the spread of the disease. In the recent past, ICMR communicated that 80% of persons infected from SARS-Cov-2 were asymptomatic (or mild). If the majority of the people are asymptomatic, i.e. symptoms are not visible or evident, the population can become less responsive, may take the pandemic lightly and the spread of infection can increase. During these times, individuals often don't wear masks when they see people around them not wearing mask. If people wear mask, most people do. You don't want to hoard soaps, sanitizers or other food products but you may do so seeing others. Administration and policymakers may bring various guidelines, advisories and rules in times of an epidemic or pandemic, but everything depends on the behaviour of common people to make it successful.

Mathematical temperament can play an important role in times of an epidemic-pandemic. An appreciation and understanding of such mathematics among policymakers, administrators and common citizens can strengthen the efforts against epidemics or a pandemic like COVID-19.

Mr Kumar Gandharv Mishra works in the area of mathematics education. He can be reached at mishrakumargandharv@gmail.com