Combination of Beidou and GPS observations to establish regional Ionospheric map

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This paper uses dual-model observations of the 118 stations from October 11th to November 27th, 2016 to establish the quasi real time Beidou Regional Ionospheric Map (RIMC), GPS Regional Ionospheric Map (RIMG) and the combined Regional Ionospheric Map (RIMM). The precise weights of each Beidou and GPS observations are determined by using the Helmert variance component estimation method. Then the DCBs of GPS satellites are compared with that published by CODE. The grid points of the areas where the IPPs covers are selected to further analyze the accuracy of the RIMM by comparing with the RIMC, RIMG and CODE GIM of DoY 287-332, 2016. The experimental results show that the accuracy of the RIMM improves about 0.3 TECU after adding the GPS observations.

[Keywords: Beidou, Global Ionosphere Map, GPS, Total Electron Content]

Introduction

The ionosphere is a part of the upper atmosphere where the density of free electrons and ions is high to influence the propagation of electromagnetic radio frequency waves. Ionosphere is one of the dispersive mediums. It means that the velocities of the electromagnetic waves which travel through the ionosphere are frequency-dependent. So we can get the Total Electron Content (TEC) or Vertical Total Electron Content (VTEC) of the ionosphere by using the dispersion characteristics of it. It is known that Global Navigation Satellite Systems (GNSS) satellites have two different frequency sigals. So the ionospheric delays of GNSS observations can be easily obtained. These delays can be used to calculate the model of the ionosphere. The Global Ionospheric VTEC Map (GIM) is one of the famous global ionospheric models with a 2-hour time resolution and daily sets of GNSS satellite and receiver hardware differential code bias (DCB) values. In order to develop the GIM using GNSS observations, the International GNSS Service (IGS) initiated the special Ionosphere Working Group in 1998.

China is launching the Beidou Navigation Satellite System (Beidou) to improve the autonomy and security of satellite navigation and positioning. Because of the compatibility and interoperability between the Beidou and other navigation and positioning systems, users can use multi-system observational data simultaneously to greatly improve data availability, accuracy, integrity and reliability. Beidou has provided positioning, navigation and timing services for the Asia Pacific areas from December 27th, 2012. It is scheduled to provide global services before 2020. China started to build the Beidou foundation reinforcement system network (Beidou FRSN) since 2014. By the end of 2016, China has built the Beidou FRSN with 175 frame reference stations.

Beidou and GPS observations are used in this study to establish a combined Regional Ionospheric Map (RIM). We also considered the accuracy differences and systematic bias between Beidou and GPS observations. The precise weights of each observations are determined by using the Helmert variance component estimation method. The differences of DCBs that published by CODE and estimated in this paper are analyzed. We also analyzed the differences of the combined RIM and GIM published by CODE.

Methodology

Algorithm of modeling a regional ionospheric area with ground-based GNSS observations

Ionospheric VTEC from ground-based GNSS observations

GNSS is widely used to explore the ionosphere in past two decades with the advantages of low cost, global coverage, good continuity and large amount of observations. Dual-frequency observations of ground-
based GNSS can be used to obtain the ionospheric STEC through the following expression\textsuperscript{11}.

$$\text{STEC} = \frac{1}{40.28} f_1^2 - f_2^2 \cdot \left( \Delta P^i - c \cdot \Delta b^i - c \cdot \Delta b^i - \Delta c \right) \ldots (1)$$

Where $f_1$ and $f_2$ represent the carrier frequencies of GPSS; $P_1$ and $P_2$ represents the two pseudo-range observations of GNSS; $\Delta P^i$ is the pseudo-range differences between $P_1$ and $P_2$, while $\Delta P^i$ is the pseudo-range differences between GNSS receiver ‘$i$’ and satellite ‘$j$’; $\Delta b^j$ and $\Delta b^i$ are the DCBs of GNSS receiver ‘$i$’ and satellite; ‘$c$’ is the velocity of light spreads in vacuum; $\Delta c$ is the residual error.

The maximum errors of formula (1) are the DCBs of receivers and satellites. Because the DCBs are stable, so they can be regarded as constant in one day\textsuperscript{12}. In this paper, the DCBs are parameters and estimated together with the coefficients of ionospheric model by least square method. In order to improve the accuracy of the results, several algorithms such as detection and repairing of cycle slip, phase smoothing pseudo range are also used\textsuperscript{13-15}. The ionospheric model is based on the assumption of Single Layer Model (SLM) which assumes that all the free electrons concentrate in an infinitesimally thin layer above the earth’s surface. The height of the SLM is usually set slightly above the height of the highest electron density is expected. In this paper, the height is 450 km. The points on the ionospheric layer where the GNSS signals transmit from the satellites to the receiver’s intersect the layer are called the ionospheric pierce points (IPPs).

The relationship of STEC and VTEC at the IPP can be expressed as a mapping function which is as following:

$$VTEC = mf \cdot STEC \ldots (2)$$

where $mf$ is the mapping function, it can be written as formula (3)\textsuperscript{(ref.16)}:

$$mf = \frac{1}{\sin(E + \theta)} = \sec \left[ \sin^{-1} \left( \frac{R}{R + H} \right) \cos E \right] \ldots (3)$$

In formula (3), $E$ is the satellite azimuth, $\theta$ is the field angle that GNSS receiver and IPP relative to the geo-center, $R = 6378137$ m is the mean earth radius, and $H$ is the height of SLM.

Regional ionospheric modeling based on ground-based GNSS

The Spherical Harmonic (SH) expression is adopted to establish the Regional Ionospheric VTEC Map (RIM) in this paper. The RIM is represented as a function of longitude, latitude and time as shown in formula (4). In this paper, a SH expression of four orders is chosen for modeling the RIM.

$$VTEC(\beta, s) = \sum_{n=0}^{n_{\text{max}}} \sum_{m=-n}^{n} \hat{a}_n^m \sin(\beta) (\cos(s) \hat{b}_n^m + \sin(s)) \ldots (4)$$

Where, $\beta$ is the latitude of IPP, ‘$s$’ is the sun angle of IPP under the sun-fixed coordinate; $n_{\text{max}}$ is the maximum order of the SH expression; $VTEC(\beta, s)$ is VTEC in TECU; $\hat{b}_n^m = N_{nm}^m P_{nm}$ is normalized legendre function from degree $n$ and order $m$; $N_{nm}$ is normalizing function; $P_{nm}$ is classical legendre function; $\hat{a}_n^m$ and $\hat{b}_n^m$ is unknown coefficients of the SH expression.

Helmert variance component estimation method

Helmert variance component estimation method is famous for solving the problem of weight determination in a system with different, but independent types of observations\textsuperscript{17,18}. In this paper, the ground-based Beidou and GPS observations are two different types of observations. They are also different, but independent types of observations. So, Helmert variance component estimation method is used to estimate the weights of each observations.

It assumes that the observations of Beidou and GPS

$$L = \begin{bmatrix} L_1 \\ L_2 \end{bmatrix}, \quad P = \begin{bmatrix} P_1 \\ 0 \end{bmatrix}$$

are $n_1$ and $n_1$, their prior weights are $n_1 n_1$ and $n_1 n_1$. it can be written as that

$$L = \begin{bmatrix} L_1 \\ L_2 \end{bmatrix}, \quad P = \begin{bmatrix} P_1 \\ 0 \end{bmatrix}$$

the error equations are as follows:

$$\begin{bmatrix} V_1 = A_1 X - L_1 \\ V_2 = A_2 X - L_2 \end{bmatrix} \ldots (5)$$

Then the error equations can be obtained from formula (5) and it can be written as formula (6):

$$N X = W \ldots (6)$$

In formula (6), $N = A_1^T P_1 A_1 + A_2^T P_2 A_2$,

$$W = A_1^T P_1 L_1 + A_2^T P_2 L_2$$

According to the law of covariance propagation, we could get the following formulas

$$D(V_i) = (P_i^{-1} + A_i N^{-1} N^{-1} A_i^T) \delta_{ij}^2 + \sum_{j \neq i} A_i N^{-1} A_j \delta_{ij}^2 \ldots (7)$$
From formula (8), the Helmert variance component estimation could be written as formula (9):
\[
S \hat{\delta}^2 = W \quad \ldots (9)
\]

In formula (9),
\[
S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}, 
S_{0j} = n_j - 2\tau(N^-N_j) + \tau(N^-N_j)^2,
\]

The results of formula (9) are as follows:
\[
\hat{\delta}^2 = S^{-1}W \quad \ldots (10)
\]

If \( \delta_{0j}^2 \) and \( \delta_{ij}^2 \) were not equal or had great differences, it means that the weights were not reasonable. So it needed to recalculate \( \delta_{0j}^2 \) and \( \delta_{ij}^2 \) according to formula (11) until they were equal or the difference was small.

\[
\begin{align*}
\mathbf{p}_{k+1} & = \mathbf{p}_k \\
\mathbf{p}_{k+1} & = \frac{\delta_{ij}^2}{\delta_{0i}^2} \mathbf{p}_k \\
\end{align*} \quad \ldots (11)
\]

**Results and Discussion**

**Data sources and processing**

Beidou FRSN has a total of 175 frame reference stations which are equipped with Beidou/GPS dual mode receivers and meteorological instruments. This paper uses observations of the 118 stations taken from October 11th to November 27th, 2016 (DoY 287-333, a total of 46 days). The sampling intervals of the Beidou and GPS observations are 1 second, and the cut off angle of the observations are 10 degrees.

The data is processed and the RIM is given at the end of the processing. The covering areas of the RIM are as follows: longitude direction 70 – 145° E with the interval is 5°, latitudinal direction 7.5 – 55° N with the interval is 2.5°. In order to improve the accuracy of the results, the observation arcs that are less than 20 minutes are eliminated. At the same time, the first and the last 5 minutes observations of each arc are also eliminated.

**Distribution of IPP**

The coefficients of the SH expression are often estimated with a specific period of data. Because there are more Beidou FRSN frame reference stations in the eastern areas of China, so it can be seen that the IPPs are also concentrated in these areas. At the end of 2016, there are 23 Beidou navigation satellites that has been launched including five GEO, five inclined geosynchronous orbit (IGSO) satellites and three medium orbit (MEO) satellites. The Beidou satellites are less than GPS, so the IPPs of GPS are more than that of Beidou at the same period.

**Analysis of the GPS satellites DCBs**

Although GPS Satellites DCBs are the intermediate results, they can reflect the accuracy and reliability of the data processing. In this study, the satellites and receivers DCBs of GPS and Beidou system are all estimated once a day. The GPS satellites DCBs are also compared with the results published by CODE.

Figure 1 shows that the DCBs published by CODE and estimated in this study showed small differences of about -0.5 to 0.3 nsec. Figure 2 shows the mean and RMS differences of DoY 287 to 333 between DCBs of GPS satellites published by CODE and that estimated in this paper. The mean differences of 32 GPS satellites are about -0.5 to 0.5 nsec. The RMS differences of 32 GPS satellites are about 0 to 0.6 nsec.

**Analysis of accuracy of RIMM by comparing with RIMG, RIMC**

Three different experimental strategies were taken to establish the regional ionospheric map: (1) only Beidou observations (RIMC), (2) only GPS observations (RIMG), and (3) Beidou and GPS dual mode observations (RIMM).

A set of coefficients of the SH expressions were estimated every 10 minutes, and the standard IONEX files were produced with a grid range in latitude 5° - 55° N and longitude 70°-145° E. The DCB parameters of both satellites and receivers are
considered constant and estimated every day. The satellite orbit and clock errors are corrected using the Beidou precise orbit clock corrections provided by the IGS center of Wuhan University.

Figure 3 shows the RIMC, RIMG and RIMM at UTC 12:00:00 of DoY 287, 2016. It can be seen that VTEC of the RIMC, RIMG and RIMM are consistently well in mid-latitude areas. But the maximum value of RIMC is larger than that of the RIMG and RIMM.

Differences of the same grids between the RIMM and RIMC/RIMG at UTC 12:00:00 of DoY 287 are calculated and the results are shown in Figure 4. It can be seen that the differences between RIMM and RIMC are about -20 to 30 TECU. In the mid-latitude areas, the differences are about 0 to 5 TECU. The maximum and minimum differences between RIMM and RIMC appear in the northeast and south edge of the areas where the IPPs are almost blank. It can be also found that the differences between RIMM and RIMG are about -5 to 7 TECU. In the mid-latitude areas, the differences are about -2 to 2 TECU. The maximum and minimum differences between RIMM

Analysis of accuracy of RIMM by comparing with CODE GIM

The IGS analysis Center for Orbit Determination in Europe (CODE) publishes daily global ionosphere map (GIM), which corresponds to the middle day of a 3 days combination analysis using both GPS and GLONASS observations. The GIM errors are within the range of ±2 to ±8 TECU. CODE GIM is selected to compare with the RIMM. Differences between CODE GIM and RIMM at UTC 12:00:00 of DoY 287 are calculated and the results are shown in Figure 5. It can be seen that the differences are about 0 to 10
TECU in the mid-latitude areas. While in northeast and southwest edge of the areas where the IPPs are almost blank, the differences are larger than that of other areas.

In order to further compare the accuracy of the RIMM and CODE GIM, the average and the root mean square (RMS) differences with 144 periods products of DoY 287 are calculated and are shown in Figure 6. The average differences are approximately -5 to 20 TECU. The RMS differences are approximately 0 to 25 TECU. It can be seen that the average and the RMS differences of the areas where the IPP is less are greater than other areas. The average difference reaches 20 TECU in south edge of the areas, and the RMS differences reaches 25 TECU in southeast edge of the areas. In the mid-latitude areas, the average differences are about 5 TECU and the RMS differences are about 2 TECU.

Figure 7 shows the average and RMS differences between CODE GIM and RIMC/RIMM/RIMG of DoY 287-333, 2016. The red, blue and cyan bar represents the average and RMS differences between CODE GIM and
RIMCMRIMC/RIMG respectively. Above diagram of Figure 6 displays average differences between CODE GIM and RIMC/RIMM/RIMG. In the rest of this paper, ACRIMM/ACRIMC and ACRIMG are used to represent the average differences between CODE GIM and RIMM/RIMC/RIMG, respectively. It can be seen that ACRIMM is less than that of the ACRIMC and ACRIMG. Mean of the ACRIMM is 2.90 TECU, mean of the ACRIMG is 3.30 TECU and mean of the ACRIMC is 3.25 TECU.

Figure 7 also shows the RMS differences between CODE GIM and RIMC/RIMM/RIMG. Similar to above, RCRIMM/ RCRIMC and RCRIMG are used to represent the average differences between CODE GIM and RIMM/RIMC/RIMG, respectively in rest of this paper. It also can be seen that RCRIMM is less than that of RCRIMC and RCRIMG. Mean of RCRIMM, ACRIMG and ACRIMC is 7.36, 6.50 & 7.30 TECU, respectively.

From above analyses, it is known that the accuracy of the RIMM is higher in areas where IPPs cover is more. So the accuracy of these areas is further analyzed in the next section. There are total of 124 points which are selected to further analyze the accuracy of the RIMM. The red symbols of ‘*’ in Figure 8 show the locations of the selected points.

In Figure 9, the red, blue and cyan bar represents the average and RMS differences between CODE GIM and RIMCMRIMC/RIMG respectively. In rest of this paper, ACRIMM/ ACRIMC and ACRIMG are used to represent the average differences between CODE GIM and RIMM/RIMC/RIMG respectively. It can be seen that ACRIMM is less than that of the ACRIMC and ACRIMG. Mean values of ACRIMM, ACRIMG, ACRIMC, were 3.55, 3.68 and 3.82 TECU, respectively.

Figure 9 also observed the comparison of RMS differences between CODE GIM and RIMM/RIMC/RIMG. Similar to above, RCRIMM/ RCRIMC and RCRIMG are used to represent the
average differences between CODE GIM and RIMM/RIMC/RIMG, respectively and it can be seen that RCRIMM is less than that of the RCRIMC and RCRIMG. Mean of the RCRIMM is 1.62 TECU, mean of the ACRIMG is 1.69 TECU and mean of the ACRIMC is 1.76 TECU. So the accuracy of the RIMM improved about 0.3 TECU after adding GPS observations.

Conclusions
This study uses dual-model observations of Beidou FRSN to establish the quasi real time RIMM/RIMC/RIMG. In order to improving the accuracy of the RIMM, the precise weights of each Beidou and GPS observations are determined by using the Helmert variance component estimation method taking into account of the different accuracy of them. Then the DCBs of GPS satellites are compared with that published by CODE. While the RIMC, RIMG and CODE GIM of DoY 287-332, 2016 are also selected and the grid points of the areas where the IPPs covers are selected to further analyze the accuracy of the RIMM. The experimental results show that the accuracy of the combined RIMM improves about 0.3 TECU after adding the GPS observations.

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