Mathematical model for prediction of soil microbial functional attributes affected by pulp and paper industry effluent

S Kumar, K K Dube, M Saxena and J P N Rai
Department of Mathematics, Statistics & Computer Science, G B Pant University of Agriculture and Technology, Pantnagar 263 145

Received 02 April 2004; accepted 02 July 2004

A mathematical model to study the impact of pulp and paper industry effluent on some functional attributes of soil microbes is proposed and analyzed. In modeling the system, it is assumed that the microbial biomass and activity increase exponentially with decreasing load of pollution at distance from the source of pollutants' origin. The model developed is applied to soil microbial biomass C and N, respiration and dehydrogenase activity to obtain the projected data of agro-ecosystem soil fed with the effluent during crop irrigation.

Keywords: Pulp and paper industry, Soil microbial attributes, Mathematical model

IPCI Code: Int.Cl.7: C 02 F 3/34

Introduction

Though microorganisms are very tiny but they play significant role in ecosystem dynamics. The dynamics of soil ecosystem is primarily governed by the structural and functional attributes of microorganisms present over there, which also indicates the fertility status of the soil. However, estimation of structural attributes of soil microbes is often under scored owing to the limitation of estimation procedures. On contrary the functional attributes embodying microbial respiration and enzyme activities are relatively easier to measure precisely. Beside the estimation of total microbial biomass that acts as vital source of plant nutrient further owes relevance for determining the ecosystem dynamics and thus the fertility status of the soil.

Several workers have over-emphasized the alteration in soil microbial status leading to the reduction in soil fertility by point and non-point sources of pollution. It has also been assumed that the magnitude of soil microbial activity reduction does not remain uniform along with the distance from pollution source and much depends on the interactive impact of quality and quantity of pollutants and nature of soil microbes. Keeping this in view, microbial kinetics and dynamics have been taken as an index to depict the impact of pollution load and to predict the fate of pollutants with the help of mathematical models. However the studies leading to the impact analysis of point source industrial pollutants on spatial activity of soil microbes are few and far between. In the present study, attempt has been made to develop a model predicting the impact of pulp and paper industry effluent on soil microbial functional attributes for determining the fertility status of the soil along with the distance from the industry.

Materials and Methods

The study was conducted on the soil irrigated with the effluent of Century Pulp and Paper Mill Ltd, Ghanshyamdharam, Lalkuan (Uttaranchal) situated at 8 km, north-east of Pantnagar. Soil samples for study were collected from CPP effluent irrigated plots at zero site (in close vicinity of industry) and 1 km, 3 km, and 5 km, respectively away from zero site. The effluent used for irrigation was post methanated. The study was conducted in pre-monsoon season of the year 2000. On sampling date, five random surface (0-15 cm) soil samples of 1 kg each was drawn form the effluent fed plot and mixed to give a composite sample. Corresponding control soil samples were drawn from the adjoining area, which was not irrigated by CPP effluent. There were five replicates of composite samples of soil of a particular treatment.
Soil samples collected in polyethene bags were brought to the laboratory and stored at 4°C in a refrigerator until their further use.

The soil microbial biomass was determined by chloroform fumigation incubation technique\(^5\). Biomass C was calculated by dividing the flush of CO\(_2\) by a \(k_c\) factor of 0.41\(^9\) and biomass N was estimated by dividing the flush of mineral N by a \(k_n\) factor of 0.40\(^10\). It was assumed that the experimental soil has \(k_c\)=0.41 and \(k_n\)=0.40. The long-term soil respiration was assayed by measuring the CO\(_2\) evolution using the method of Macfadyen\(^11\).

Dehydrogenase activity of the soil fed with the effluent and control soil was estimated following the method given by Ross\(^12\).

**Mathematical Model**

**Assumptions**

It is assumed that the variation in the activity difference is due to the pollution by pulp and paper industry effluent. The effluent of pulp and paper industry containing heavy load of organic and inorganic compounds is often used for crop irrigation. It is likely that during irrigation a major portion of the effluent alters the soil environment and thus impairs microbial activity.

In this study, it is assumed that with increase in distance from the origin of pollutants, i.e., industry the concentration and/or efficacy of the pollutants decreases by dilution and/or exogenous addition of other compounds capable of nullifying the pollutants’ activity. As such, it is assumed that the altered pollutants in response to increasing distance from the source may influence the soil microbial activity determining fertility status, differently. It is also assumed that the soil pollution caused by industrial effluent cannot exceed beyond finite limit and is maximum in close vicinity of the pollutant source. The model developed would not be valid for non-point source pollution but will be applicable to point source pollution of industries of varied nature.

**Model Describing the Activity Difference**

Let \(A\) be the activity difference (in soil sample of control water irrigation–effluent water irrigation) at distance \(s\) from the origin of the polluted effluent, i.e., the distance from the pulp and paper industry. Then the rate of change in activity difference is directly proportional to the activity difference at that distance.

\[
\frac{dA}{ds} \propto A
\]

Now we take the constant of proportionality, \(k\) then we get

\[
\frac{dA}{ds} = kA
\]

or

\[
\frac{dA}{A} = kds
\]

Integrating Eq. (1) both side we get,

\[
\ln A = ks + C
\]

where \(C\) is the constant of integration, to get the value of \(C\), we put the initial condition in Eq. (2) as when distance, \(s\) from the origin is zero, i.e., \(s = 0\); the activity difference \(A\) will be maximum, let it be \(Ao\). Then we have,

\[
\ln Ao = k \cdot 0 + C
\]

or \(C = \ln Ao\).

Putting the value of \(C\) in Eq. (2) we get,

\[
\ln A = ks + \ln Ao
\]

or

\[
\ln A - \ln Ao = ks
\]

or

\[
\ln(A/Ao) = ks
\]

Therefore,

\[
A/Ao = e^{ks}
\]

\[
A = Ao e^{ks}
\]

**Properties of the Model**

**Property 1**—The model is valid only for the point source pollution and cannot be applicable for non-point source.

**Property 2**—The model derived is valid only for \(0 < s\).

**Property 3**—The value of \(k\) depends upon the curve of the difference according to the distance, i.e. the value of \(k\) will be positive if the curve is increasing, otherwise negative.

**Method of Fitting the Model**

We have from Eq. (3),

\[
A = Ao e^{ks}
\]

then we have,

\[
A/Ao = e^{ks}
\]

or

\[
k = \{\ln(A/Ao)\}/s
\]
Now we take equidistant point, let these be \(s_1\), \(s_2\), and \(s_3\).

Then we calculate the value of \(k_i\), i.e \(k_i = \frac{\ln(A_i/A_0)}{s_i}\), where \(i = 1, 2, 3\).

Then we have \(k = \frac{\Delta k}{3}\), and by putting the value of \(k\) in the Eq.(3), we can predict the activity.

### Application of the Model for Activity Difference in Soil by Irrigation with the Effluent of Century Pulp and Paper Industry at Lalkuan

As an application of the model developed for predicting activity difference of soil microbes in response to effluent irrigation, we have considered situation prevailing in Tarai region of Uttarakhand more precisely near Lalkuan where one of largest Pulp and Paper Industry (CPP) emits pollutant loaded effluent. This effluent flows through 18 km long kachha nalla along with the agricultural fields and joins river Gola. On the way, beside submerging a large track of agricultural land, the effluent is used as crop irrigant and thus impairs the soil environment and thereby fertility status of Tarai agro-ecosystem. The present mathematical model is the outcome of data gathered on impact analysis of pulp and paper industry effluent on soil microbial attributes by Saxena\(^7\) (Table 1) where no mathematical relationship has been established yet. Tables 2-5 depict the impact of pulp and paper industry effluent irrigation on activity difference in terms of soil microbial biomass C and N, respiration and dehydrogenase activity which have been constructed from the original data given in Table 1. This shows the functional attributes of the soil samples collected at four different stations according to the decreasing load of pollution, \(i.e.,\) from up stream to downstream in pre-monsoon season.

For fitting of model to the observed data, method discussed in the above section was used. Since three-equidistant points are needed, observations corresponding to 1 km, 3 km, and 5 km were selected. From these observations the constant values \(k\) as given in Tables 2-5 were calculated.

### Table 1—Soil microbial biomass C and N, soil respiration and dehydrogenase activity as influenced by pulp and paper industry effluent irrigation \((n = 5, \pm S E)\)

<table>
<thead>
<tr>
<th>Soil microbial attributes</th>
<th>Treatment</th>
<th>Sampling sites</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>0 km</td>
</tr>
<tr>
<td>Biomass C (mg/kg)</td>
<td>Control</td>
<td>610±6.8</td>
</tr>
<tr>
<td></td>
<td>Treated</td>
<td>307±3.0</td>
</tr>
<tr>
<td>Biomass N (mg/kg)</td>
<td>Control</td>
<td>200±2.0</td>
</tr>
<tr>
<td></td>
<td>Treated</td>
<td>155±1.6</td>
</tr>
<tr>
<td>Respiration [10-d (CO_2)-C(g/kg)]</td>
<td>Control</td>
<td>9.8±0.6</td>
</tr>
<tr>
<td></td>
<td>Treated</td>
<td>4.92±0.3</td>
</tr>
<tr>
<td>Dehydrogenase activity (mg TPF / kg soil / h)</td>
<td>Control</td>
<td>180±4.0</td>
</tr>
<tr>
<td></td>
<td>Treated</td>
<td>68±1.2</td>
</tr>
</tbody>
</table>

### Table 2—Calculation of \(k\) for soil microbial biomass carbon

<table>
<thead>
<tr>
<th>Control (c)</th>
<th>Treated (t)</th>
<th>(c-t)</th>
<th>(s)</th>
<th>(A/A_0)</th>
<th>(\ln(A/A_0))</th>
<th>(k=\ln(A/A_0)/s)</th>
<th>Mean (k)</th>
</tr>
</thead>
<tbody>
<tr>
<td>610</td>
<td>307</td>
<td>303</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>-0.09267</td>
<td>-0.09267</td>
</tr>
<tr>
<td>621</td>
<td>352</td>
<td>269</td>
<td>1</td>
<td>0.887789</td>
<td>-0.11902</td>
<td>-0.11902</td>
<td>-0.11902</td>
</tr>
<tr>
<td>625</td>
<td>392</td>
<td>233</td>
<td>3</td>
<td>0.768977</td>
<td>-0.26269</td>
<td>-0.08756</td>
<td>-0.08756</td>
</tr>
<tr>
<td>617</td>
<td>405</td>
<td>212</td>
<td>5</td>
<td>0.69967</td>
<td>-0.35715</td>
<td>-0.07143</td>
<td>-0.07143</td>
</tr>
</tbody>
</table>

### Table 3—Calculation of \(k\) for soil microbial biomass nitrogen

<table>
<thead>
<tr>
<th>Control (c)</th>
<th>Treated (t)</th>
<th>(c-t)</th>
<th>(s)</th>
<th>(A/A_0)</th>
<th>(\ln(A/A_0))</th>
<th>(k=\ln(A/A_0)/s)</th>
<th>Mean (k)</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>155</td>
<td>45</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>-0.27493</td>
<td>-0.27493</td>
</tr>
<tr>
<td>203</td>
<td>169</td>
<td>34</td>
<td>1</td>
<td>0.755556</td>
<td>-0.2803</td>
<td>-0.2803</td>
<td>-0.2803</td>
</tr>
<tr>
<td>202</td>
<td>182</td>
<td>20</td>
<td>3</td>
<td>0.512821</td>
<td>-0.66783</td>
<td>-0.22261</td>
<td>-0.22261</td>
</tr>
<tr>
<td>201</td>
<td>192</td>
<td>9</td>
<td>5</td>
<td>0.2</td>
<td>-1.60944</td>
<td>-0.32189</td>
<td>-0.32189</td>
</tr>
</tbody>
</table>
Table 4—Calculation of $k$ for soil microbial respiration

<table>
<thead>
<tr>
<th>Control ($c$)</th>
<th>Treated ($t$)</th>
<th>$c-t$</th>
<th>$s$</th>
<th>$A/A_0$</th>
<th>$\ln(A/A_0)$</th>
<th>$k = \ln(A/A_0)/s$</th>
<th>Mean $k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.8</td>
<td>4.92</td>
<td>4.88</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>-0.22442</td>
</tr>
<tr>
<td>9.79</td>
<td>6</td>
<td>3.79</td>
<td>1</td>
<td>0.776639</td>
<td>-0.25277</td>
<td>-0.25277</td>
<td></td>
</tr>
<tr>
<td>9.82</td>
<td>7.08</td>
<td>2.74</td>
<td>3</td>
<td>0.561475</td>
<td>-0.57718</td>
<td>-0.192395</td>
<td></td>
</tr>
<tr>
<td>9.8</td>
<td>8.24</td>
<td>1.56</td>
<td>5</td>
<td>0.319672</td>
<td>-1.14045</td>
<td>-0.22809</td>
<td></td>
</tr>
</tbody>
</table>

Table 5—Calculation of $k$ for soil dehydrogenase activity

<table>
<thead>
<tr>
<th>Control ($c$)</th>
<th>Treated ($t$)</th>
<th>$c-t$</th>
<th>$s$</th>
<th>$A/A_0$</th>
<th>$\ln(A/A_0)$</th>
<th>$k = \ln(A/A_0)/s$</th>
<th>Mean $k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>180</td>
<td>68</td>
<td>112</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>-0.08452</td>
</tr>
<tr>
<td>182</td>
<td>79</td>
<td>103</td>
<td>1</td>
<td>0.919643</td>
<td>-0.0837699</td>
<td>-0.083769883</td>
<td></td>
</tr>
<tr>
<td>182</td>
<td>95</td>
<td>87</td>
<td>3</td>
<td>0.776786</td>
<td>-0.2525908</td>
<td>-0.08416918</td>
<td></td>
</tr>
<tr>
<td>178</td>
<td>105</td>
<td>73</td>
<td>5</td>
<td>0.651786</td>
<td>-0.4280394</td>
<td>-0.085607886</td>
<td></td>
</tr>
</tbody>
</table>

Table 6—Prediction of the activity difference for various soil microbial attributes

<table>
<thead>
<tr>
<th>Distance (km)</th>
<th>Microbial biomass C (mg/kg)</th>
<th>Microbial biomass N (mg/kg)</th>
<th>Microbial respiration [10 d CO$_2$-C (g/kg)]</th>
<th>Dehydrogenase activity (mg TPF / kg soil / h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>276.1828</td>
<td>34.18314</td>
<td>3.89902</td>
<td>102.9228</td>
</tr>
<tr>
<td>2</td>
<td>251.739</td>
<td>25.96638</td>
<td>3.115237</td>
<td>93.84635</td>
</tr>
<tr>
<td>3</td>
<td>229.4586</td>
<td>19.72472</td>
<td>2.48901</td>
<td>85.90473</td>
</tr>
<tr>
<td>4</td>
<td>209.1502</td>
<td>14.8339</td>
<td>1.988668</td>
<td>78.63516</td>
</tr>
<tr>
<td>5</td>
<td>190.6392</td>
<td>11.38176</td>
<td>1.588905</td>
<td>71.98076</td>
</tr>
<tr>
<td>6</td>
<td>173.7666</td>
<td>8.645876</td>
<td>1.269503</td>
<td>65.88949</td>
</tr>
<tr>
<td>7</td>
<td>158.3872</td>
<td>6.567626</td>
<td>1.014307</td>
<td>60.31368</td>
</tr>
<tr>
<td>8</td>
<td>144.369</td>
<td>4.988935</td>
<td>0.81041</td>
<td>55.20971</td>
</tr>
<tr>
<td>9</td>
<td>131.5915</td>
<td>3.789721</td>
<td>0.647501</td>
<td>50.53766</td>
</tr>
<tr>
<td>10</td>
<td>119.9449</td>
<td>2.878768</td>
<td>0.51734</td>
<td>46.26908</td>
</tr>
</tbody>
</table>

Fig. 1—Shows deviation between observed and estimated data of various parameters
From Tables 2-5, different constant values for the different soil microbial attributes were obtained. We used the constant values along with our model equation to predict the activity difference from 1 to 10 km, as shown in Table 6.

**Results and Discussion**

From Tables 2-5, it is observed that the microbial biomass and activity difference are decreasing exponentially with increasing distance, i.e. 0 to 5 km. The deviation of the estimated value of activity difference from that of observed value of various microbial activities at different sampling stations, as given in Fig.1, is least. However, with increase in distance from the industry the estimated value exceeds the observed value. This supports our assumption that with increase in distance from the origin of pollutants, i.e. industry the concentration and/or efficacy of the pollutants decreases by dilution and/or exogenous addition of other compounds capable of nullifying the pollutants’ activity.

**Conclusions**

Based on present investigation, it is concluded that the proposed model is useful in predicting the trend of pollution load exerted by pulp and paper industry effluent on soil microbial functional attribute at any distance from the source of origin of the pollution, which is quite helpful in rapid monitoring of pollution impact on microbial attributes determining the soil fertility status of agro-ecosystem of Uttaranchal Tarai. This model is equally applicable in predicting the soil microbial attributes influenced by the effluent of other industry as well.

**Acknowledgement**

Financial assistance received from Council of Scientific & Industrial Research, New Delhi, in the form of a research project is gratefully acknowledged.

**References**