Diffusion and transport phenomena in a collisional magnetoplasma with temperature anisotropy

*K C Baral & G Nath

*Department of Physics, Salipur College, Salipur, Cuttack 754 202
+Department of Engineering Physics, Dhaneswar Rath Institute of Engineering & Management Studies, Tangi, Cuttack 754 022

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The theory of collisional cross field diffusion and transport of a fully ionised singly charged plasma particles is analysed theoretically at the onset of an induced anisotropy \( T_{||} \neq T_{\perp} \) in plasma temperatures where \( T_{||} \) and \( T_{\perp} \) represent the components of temperature along the magnetic field direction and the direction perpendicular to it, respectively. The modified diffusion transport coefficients like the electrical resistivity \( \eta_{||} \) and thermal conductivity \( K \) are found to decrease significantly with increase in the thermal ratio \( T_{||}/T_{\perp} \) while both the \( \vec{E} \times \vec{B} \) drift velocity and the thermoelectric coefficient \( \lambda \) remain independent of the thermal anisotropy. However, in the limiting approximation \( \eta_{||} = T_{\perp} \), the results agree with the early isotropic works of Rosenbluth and Kaufmann, Phys Rev, 1 (1958) 109, Goldstone and Rutherford [IOP Publishing 1995, Braginskil (1965)].

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1 Introduction

Temperature anisotropy is an intrinsic feature in magnetospheric and space plasma including laboratory and fusion plasmas etc. Recent advancements in space probes including launching NASA’s space craft Odyssey and investigation of Rovers Space Vehicle in Mars resulting in finding of sediments of lava on its surface, study of thermal energy imbalance relating to the explosion of Columbia Space Shuttle in USA in February 2003, thermal energy variation in space and coronal surface of sun, study of the perpendicular ion flows at a sub solar magnetopause crossing by the polar satellite on 1st April 2001. [Mozer et al.] and other studies by the in-situ measurements in earth’s magnetosphere including space docking, space aviation technology and discovery of high temperature confinement devices etc. have warranted the study of the basic physics underlying the collisional kinetics of diffusion and transport of charged particles in crossed electric and magnetic field where there is an anisotropy in temperature. The studies are of use to ascertain the effects of thermal variations in solar corona, thermal belts, interplanetary belts, space and plasma belt and understanding of the impact of thermal anisotropy on various transport properties relevant to the datas provided by TRACE – Space craft analysis. Although substantive works have been done to study the collisional diffusion and transport in a two component magnetoplasma in non-relativistic regime of temperature both for isotropic plasma and a plasma having anisotropy in energy momentum distribution\(^{1-15}\). No attempt has yet been made to understand the significant effect of temperature anisotropy in cross field diffusion and transport process, especially, in a singly charged two component (e-i) plasma, commonly observed in magnetospheric space plasma coronal surfaces of sun and solar bursts and relevant laboratory simulation diagnostics. Hence, the novel aspects are emphasized in the present context to predict for the first time the dependence of various transport and diffusion coefficient on temperature anisotropy and variations.

2 Theory

To unravel the novel features of thermal anisotropy in collisional diffusion and transport process, the appropriate equilibrium Maxwellian distribution\(^{16}\) of the charged particles in a fully ionised singly charged electron ion (e-i) plasma is formulated as:
\[ f^\pm_2 = \frac{N}{2} \left( \frac{m^\pm}{2\pi kT_{\perp}} \right)^{1/2} \times \exp \left[ -\frac{m^\pm (V_x^2 + V_y^2)}{2kT_{\perp}} - \frac{m^\pm V_z^2}{2kT_{\parallel}} \right] \]  

... (1)

Where \( T_{\parallel} \) and \( T_{\perp} \) describe the temperature along the direction of the magnetic field and along the direction perpendicular to the direction of the applied magnetic field, respectively. The form and analysis involve properly the mechanism of temperature anisotropy (\( T_{\parallel} \neq T_{\perp} \)). Our collisional kinetic model relevant for a weakly coupled e-i plasma in an open confinement system under the impact of a strong magnetic field configuration is governed by the Boltzmann transport equation.

\[ \frac{\partial f^\pm}{\partial t} + \vec{V} \cdot \vec{\nabla} f^\pm + \frac{e}{m^\pm} \left( \pm \vec{E} + \frac{\vec{V} \times \vec{B}}{C} \right) \vec{\nabla} f^\pm = \left( \frac{\partial f^\pm}{\partial t} \right)_{\text{coll}} \]

where collision integrals \( C^{\pm}, C^{\mp}, C^{\perp}, C^{\parallel} \) for zeroth and perturbed orders defined in the usual way as:

\[ \frac{e}{m^\pm C} \epsilon_{ijk} \frac{\partial f^\pm}{\partial V_i} V_j B_k = C_{00}^{\pm} + C_{0\pm}^{\pm} \]  

... (3)

with the Rutherford’s scattering cross section:

\[ \sigma^\pm (\Omega) = \frac{e^2}{4m^2 U^4 \sin^4 (\theta/2)} \]

where \( m \) represents the reduced mass of the system of interacting ions and electrons with masses \( m^+ \) and \( m^- \), respectively and \( \vec{u} \) represents their relative velocity. The elimination of the divergence of collisional integral for small angle deflection and appropriate inclusion of mean gyros radius in it to account for the effect of the strong magnetic field which are included as usual.

With a view to solve the collisional transport Eq. (2), we make use of the usual Chapman – Enskog type expansion of the distribution function in two small parameters \( \alpha = \left( \frac{\lambda_G}{N} \right) \left( \frac{dN}{dx} \right) \) and \( \beta = (\omega_e \tau)^{-1} \) as:

\[ f^\pm = f^\pm_0 + f^\pm_1 \]

and \( f^\pm_0 = f^\pm_0 + f^\pm_1 \) where \( \lambda_G \) represents the mean gyro radius of the particle species and the collision frequency \( \tau^{-1} \) is assumed to be much smaller than the gyration frequency \( \omega^\pm \). \( f^\pm_0 \) and \( f^\pm_1 \) are defined as the perturbed distribution to the first order in \( \alpha \), zero order in \( \beta \) and first order in both \( \alpha \) and \( \beta \), respectively. The consquential perturbed kinetic equation of various orders of perturbation are expressed as:

\[ \frac{e}{m^\pm C} \epsilon_{ijk} \frac{\partial f^\pm_{0\pm}}{\partial V_i} V_j B_k = C_{00}^{\pm} + C_{0\pm}^{\pm} \]

... (4)

\[ \frac{e}{m^\pm C} \epsilon_{ijk} \frac{\partial f^\pm_{1\pm}}{\partial V_i} V_j B_k = 0 \]  

... (5)

and

\[ \frac{e}{m^\pm C} \epsilon_{ijk} \frac{\partial f^\pm_{0\perp}}{\partial V_i} V_j B_k = C_{10}^{\pm} + C_{0\pm}^{\pm} \]

... (6)

As a sequel to the inclusion of the thermal anisotropy in the plasma species distribution, the normalized zero order equilibrium Maxwellian distribution function is introduced as:

\[ f_0^\pm = \frac{N}{2} \left( \frac{m^\pm}{2\pi kT_{\perp}} \right)^{1/2} \times \exp \left[ -\frac{m^\pm (V_x^2 + V_y^2)}{2kT_{\perp}} - \frac{m^\pm V_z^2}{2kT_{\parallel}} \right] \]  

... (7)

Now calling \( P \) to the zeroth order pressure, we estimate that:

\[ P = \sum s m^s \int \vec{V} \vec{V} f_0^s d^3 \vec{V} = \frac{1}{3} \left( 2 + \frac{T_{\perp}}{T_{\parallel}} \right) NkT_{\parallel} \]

... (8)
species separately, integrate the resulting equation over velocity space and add them together to obtain.

\[ \tilde{\nabla} P = \frac{1}{C} \tilde{J} \times \tilde{B} \]  

where we define the plasma current density as:

\[ \tilde{J} = \frac{1}{2} \text{Ne}(\tilde{u}^+ - \tilde{u}^-) \]  

\( \tilde{u}^\pm \) being the mean velocities of the respective components. It is worthwhile to note that in this model both the density and temperature gradients are taken along X-direction while the magnetic field \( \tilde{B} \) is chosen along Z-direction. The electric field \( \tilde{E} \) is induced owing to the motion of the diamagnetic plasma and to convince, it is chosen along Y-direction. With this configuration we can solve Eq. (5) to obtain the perturbed distribution function to zero order in \( \beta \) as:

\[ f_{10}^+ = \frac{1}{\omega_0^+} f_{0}^+ v_y \frac{\partial}{\partial x} \log f_{0}^+ = f_{0}^+ \phi^+ (\tilde{v}) \]

where \( \phi^+ (\tilde{v}) = \frac{1}{\omega_0^+} v_y \frac{\partial}{\partial x} \log f_{0}^+ = \frac{1}{m_0^+} m^+ v_y \]

\[ \times \left[ \frac{\partial}{\partial x} \log N T_{\perp}^{-3/2} + \frac{5}{2} m_0^+ (v_x^2 + v_y^2) \frac{d}{dx} \left( \frac{1}{k T_{\perp}} \right) \right] \]

Further, on employing the expansion to the first order in \( \beta \), we obtain, the quasilinearized form of the collision flux integral:

\[ C_{10}^+ (\tilde{x}, \tilde{v}, t) = -f_{0}^+ (\tilde{x}, \tilde{v}, t) d^3 \tilde{v} \tilde{v} \cdot \tilde{u} f_{0}^+ (\tilde{x}, \tilde{u}, t) d \Omega \sigma^{(+,-)} \]

\[ \times \left[ \phi^+ (\tilde{v}) + \phi^- (\tilde{u}) - \phi^+ (\tilde{v}') - \phi^- (\tilde{u}') \right] \]  

(11)

In a similar way, one can obtain an analogous equation for ion-ion collision \( c_{10}^{++} \) add them together and simplify the result to yield.

\[ C_{10}^{++} + C_{10}^{+-} = -f_{0}^+ (\tilde{V}) \frac{\omega_0^+}{v^+} v_y \psi \]  

\[ \times \{ f_0 (\tilde{u}) \int d \Omega \sigma^{(++,+)} (m^+ v^+ \tilde{v} - m^- \tilde{u} + f_{0}^+ (\tilde{u})) \}

\[ \times \{ f_0^+ (\tilde{u}) \int d \Omega \sigma^{(++,+)} (m^+ v^+ \tilde{v} - m^- \tilde{u} + f_{0}^+ (\tilde{u})) \}

\[ \times \{ f_0^+ (\tilde{u}) \int d \Omega \sigma^{(++,+)} (m^+ v^+ \tilde{v} + m^- \tilde{u}^2) \}

where \( \Delta A = A - A' \) and the subscript \( y \) standing for the y-component of the term in the bracket.

Next, using Eq. (12) in Eq.(6) for ion species and solving the resulting equation in the formal way in centre of mass and relative velocity frame, it is straightforward to obtain the resulting first order perturbed function in the form.

\[ f_1^+ = \frac{m^+ C}{k T_{\perp}} V_x \frac{E_y}{B} f_{0}^+ - \frac{v_x}{\omega_0^+} v_y m^+ [f_1^++ f_2^+] f_0^+ \]  

(14)

where

\[ F_1^+ = \frac{1}{m^+} \left[ \frac{5}{2} \frac{\partial}{\partial x} \left( \frac{1}{k T_{\perp}} \right) \int d^3 \tilde{u} \mid \tilde{v} - \tilde{u} \mid f_0 (\tilde{u}) \int d \Omega \sigma^{(++,+)} 2m \Delta u \]  

(15)

\[ F_2^+ = \frac{5}{4m} \left[ \frac{\partial}{\partial x} \left( \frac{1}{k T_{\perp}} \right) \int d^3 \tilde{u} \mid \tilde{v} - \tilde{u} \mid \left( \frac{N}{2} \right) \frac{m^+}{2 \pi k T_{\perp}} \right] \]

\[ \times \left[ \frac{m^+}{2 \pi k T_{\|}} \right]^{1/2} \exp \left[ \frac{\left( m^+ (v_x^2 + y^2) + m^+ v_y^2 \right)}{2k T_{\|}} \right] \]

\[ \times \{ f_1^+ \int d \Omega \sigma^{(++,+)} m^+ m^+ v^2 \Delta U \]  

(16)

\[ F_2^+ = \frac{5}{4m} \left[ \frac{\partial}{\partial x} \left( \frac{1}{k T_{\perp}} \right) \int d^3 \tilde{u} \mid \tilde{v} - \tilde{u} \mid \left( \frac{N}{2} \right) \frac{m^+}{2 \pi k T_{\perp}} \right] \]

\[ \times \left[ \frac{m^-}{2 \pi k T_{\|}} \right]^{1/2} \exp \left[ \frac{\left( m^- (v_x^2 + y^2) + m^+ v_y^2 \right)}{2k T_{\|}} \right] \]

\[ \times \{ f_1^+ \int d \Omega \sigma^{(++,+)} m^- m^- v^2 \Delta U \]  

(17)

It is instructive to note that \( F_1^{++} \) vanishes owing to momentum conservation. The drift diffusion velocity viz.,
\[ \dot{U}_d = \int f_{\parallel} \tilde{V} d^3 \tilde{v} - C \frac{E \times B}{B^2} - 3 \eta' \parallel B^2 \sqrt{v} p + \left( 10 + \frac{T_\parallel}{T_\perp} \right) \eta' \parallel \frac{1}{B^2} \nabla (k T_\perp) + k T_\parallel \eta' \parallel \frac{1}{B^2} \nabla N \]

which in the isotropic limit \((T_\parallel = T_\perp = T)\) reduces to the usual form:

\[ \dot{U}_d = C \frac{E \times B}{B^2} - 2 \eta' \parallel \frac{1}{B^2} \nabla p + 10 \eta' \parallel \frac{1}{B^2} \nabla (k T_\perp) \]

We find it with minor difference in numerical coefficients owing to various approximation in the evaluation of anisotropic collisional flux integrals. In the modified expression given in Eq. (19), there arises an additional density gradient term signifying the contribution of temperature anisotropy to the diffusion drift kinetics.

Evidently, the first term in Eq. (18) contributes to the well known \(E \times B\) drift and concomitant diamagnetism, the other terms specifying different diffusion transport coefficients with perpendicular resistivity i.e., inverse of electrical conductivity which in the limit of isotropy gives the usual expressions:

\[ \eta' = \frac{8}{3} \left( \frac{\pi}{2} \right)^{1/2} e^2 \frac{m^{1/2}}{(k T_\perp)(k T_\parallel)^{1/2}} \log \left( \frac{2}{0} \right) \left( \frac{2 T_\parallel}{2 T_\parallel + T_\perp} \right) \]

Next, we evaluate various transport coefficients from Eq. (18). The drift diffusion velocity \(v_d\) involves the well known \(E \times B\) drift term:

\[ v_d = C \frac{E \times B}{B^2} \]

which reveals that it remains independent of the presence of an anisotropy in plasma temperature. As far as the evaluation of the analytical expression for the cross field thermal conductivity is concerned, one can multiply \(B\) from the left of Eq. (18) and rearrange it further to get:

\[ \frac{B \times \dot{U}_d}{C \dot{E}} = 3 \eta' \parallel C \dot{J} + \left( 10 + \frac{T_\parallel}{T_\perp} \right) \eta' \parallel \frac{1}{B^2} \frac{1}{N^2} \nabla (k T_\perp) + k T_\parallel \eta' \parallel \frac{1}{B^2} (B \times \dot{V} N) \]

On further simplification, Eq (22) leads to the derivation of the quasi-linear current density term which is given by:

\[ \dot{J} = \frac{1}{3 \eta' \parallel} E - \lambda \nabla (k T_\parallel) \times \vec{B} - D (\vec{V} N \times \vec{B}) \]

with \( \vec{E}' = \vec{E} + \frac{1}{C} \vec{U}_d \times \vec{B} \)

The resulting ohmic part of the current density is given by:

\[ \frac{\vec{E}}{\eta} \text{ where } \eta = 3 \eta' \parallel \]

called the electrical resistivity. In the isotropic limit, the expression given in Eq. (25) almost reduces to the early results.\(^{6,8}\)

The second term on the RHS of Eq. (23) contains the thermo-electric coefficient given by:

\[ \lambda = \frac{1}{3} \left( 10 + \frac{T_\parallel}{T_\perp} \right) \frac{N^2}{B^2} \]

In addition, Eq. (23) contains the expression for diffusion coefficient:

\[ D = \frac{1}{k T_\parallel} \frac{C}{B^2} \]

which indicates that the current density in Eq. (23) is modified owing to an anisotropy in \(E \times B\) and concomitant diamagnetism, the other terms specifying different diffusion transport coefficients with perpendicular resistivity i.e., inverse of electrical conductivity which in the limit of isotropy gives the usual expressions.

\[ \dot{Q} = \sum f_{\parallel} \frac{1}{m} (\vec{v} - \vec{u}) \sum f_{\parallel} (\vec{v} \times \vec{B}) d^3 \vec{v} \]

which on evaluation yields:

\[ \dot{Q} = -k \nabla (k T_\perp) + \lambda k T_\perp (E \times B) \]

The modified thermal conductivity is, thus, obtained as:

\[ K = \frac{2}{3} \left( 11 + \frac{T_\parallel}{T_\perp} \right) \left( \frac{2 m^{1/2}}{m} \right) \frac{N^2}{B^2} \frac{C}{k T_\perp} \eta' \parallel \]

The above coefficients are evaluated quantitatively with the following set of parameters.
$N = 10^{13}$ cm$^{-3}$, $B = 10^3$ gauss and $E = 1$ stat V/cm. The electrical resistivity $\eta_\perp$ is found to be significantly affected by the temperature anisotropy owing to the factor:

\[
\left(\frac{27T_\parallel}{2T_\parallel + T_\perp}\right)^{1/2} \left(\frac{kT_\perp}{(kT_\parallel)^{1/2}}\right)
\]

instead of the usual expression containing $(kT)^{3/2}$ law for isotropic case. This novel aspect reveals that $\eta_\perp$ decreases significantly with increasing thermal energy.

$kT_\perp$ for varying thermal ratio $\left(\frac{T_\parallel}{T_\perp}\right)$. Fig. 1 shows the functional relationship between $\eta_\perp$ and $\left(\frac{T_\parallel}{T_\perp}\right)$ for varying thermal regimes. The calculation reveals that $\eta_\perp$ is found to be about 9 times greater than that obtained in early isotropic works. However, in the isotropic limit ($T_\parallel = T_\perp$), the early results are recovered. The thermoelectric coefficient ($\lambda$) is found to remain independent of temperature, thereby, conforming to the early isotropic results. As regards the thermal conductivity ($K$), it is interesting to note that the thermal conductivity ($K$) is significantly affected by the temperature anisotropy owing to the appearance of the terms containing $\left(\frac{T_\parallel}{T_\perp}\right)$ in the expression given in Eq. (30). Both qualitative and quantitative estimates reveal that $K$ significantly decreases with increase in the temperature ratio $\left(\frac{T_\parallel}{T_\perp}\right)$. Fig. 2 shows the variation of $K$ for different temperature ratio $\left(\frac{T_\parallel}{T_\perp}\right)$. As usual, the thermal conductivity ($K$) sharply decreases up to $kT_\perp=250$ eV and beyond this thermal range the variation is negligibly small. However, marked diminuation of $K$ is noticed for increasing temperature ratio $\left(\frac{T_\parallel}{T_\perp}\right)$. In all the cases, the curves resembles rectangular hyperbola as one observed in early isotropic works. In

![Graph showing the variation of electrical resistivity ($\eta_\perp$) with varying thermal energy ($kT_\perp$) for varying temperature ratios ($T_\parallel / T_\perp$)](image)

Fig. 1—Schematic variation of electrical resistivity ($\eta_\perp$) with varying thermal energy ($kT_\perp$) for varying temperature ratios ($T_\parallel / T_\perp$)
addition to the above transport coefficients, we account for the novel aspect of the density diffusion coefficient ($D$) as derived in Eq. (27). The current density $j$ is diminished by a factor.

$$D = \frac{1}{3}kT_\perp \frac{C}{B^2} (\nabla \times \mathbf{B})$$

Owing to the induced anisotropy in temperature ($T_\parallel \neq T_\perp$), though $D$ is dependent on the temperature ratio $\left(\frac{T_\parallel}{T_\perp}\right)$ term but is directly dependent solely on the temperature ($T_\parallel$) along the direction of the magnetic field lines. Obviously, it is clear that the diffusion coefficient ($D$) increases significantly with increase in $\left(\frac{T_\parallel}{T_\perp}\right)$. A schematic variation of $D$ with $kT_\perp$ for various values of $\left(\frac{T_\parallel}{T_\perp}\right)$ is shown in Fig. 3.

The diffusion coefficient $D$ does not appear in early isotropic results.

3 Results

In this paper, the theory of binary collisional kinetics is generalised for a singly charged fully ionised two component magnetoplasma under the impact of an anisotropy in temperature ($T_\parallel \neq T_\perp$) where $T_\parallel$ and $T_\perp$ are the temperature along the direction of magnetic field and $T_\perp$ is the temperature along the direction perpendicular to the magnetic field.
It is found that both the electrical resistivity ($\eta_{\perp}$) and the thermal conductivity ($K$) decrease significantly with increase in temperature ratio $\left(\frac{T_{\parallel}}{T_{\perp}}\right)$ as shown in Figs 1 and 2, respectively. However, for higher thermal regimes the variations are insignificant. In the limit of vanishing thermal anisotropy, these results conform to the early isotropic results. It is further found that the $\mathbf{E} \times \mathbf{B}$ drift remains independent of the temperature ratio $\left(\frac{T_{\parallel}}{T_{\perp}}\right)$ while the thermoelectric coefficient ($\lambda$) increases very slightly with increase in temperature ratio $\left(\frac{T_{\parallel}}{T_{\perp}}\right)$. It is further interesting to note that the current density decreases by a factor $D(\nabla N \times \mathbf{B})$ from the usual value where $D = \frac{1}{3} kT_{\parallel} \frac{C}{B^2}$ represent the density diffusion coefficient as shown in Fig. 3. It is dependent on the temperature ratio $\left(\frac{T_{\parallel}}{T_{\perp}}\right)$. It increases significantly with increase of temperature ratio $\left(\frac{T_{\parallel}}{T_{\perp}}\right)$.

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