Radiation and mass transfer effects on an unsteady MHD convection flow past a semi-infinite vertical permeable moving plate embedded in a porous medium with viscous dissipation

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An unsteady, two-dimensional, hydromagnetic, laminar mixed convective boundary-layer flow of an incompressible, Newtonian, electrically-conducting and radiating fluid along a semi-infinite vertical permeable moving plate with heat and mass transfer is analyzed, by taking into account the effect of viscous dissipation. The plate moves with a constant velocity in the direction of fluid flow while the free stream velocity follows an exponentially increasing small perturbation law. The dimensionless governing equations for this investigation are solved analytically using two-term harmonic and non-harmonic functions. Numerical evaluation of the analytical results is performed and graphical results for velocity, temperature and concentration profiles within the boundary layer and tabulated results for the skin-friction coefficient, Nusselt number and Sherwood number are presented and discussed. It is observed that, when the radiation parameter increases, the velocity and temperature decrease in the boundary layer, whereas when thermal and solutal Grashof increase, the velocity increases.

Keywords: Radiation, Viscous dissipation, Heat and mass transfer

1 Introduction

Combined buoyancy-generated heat and mass transfer, due to temperature and concentration variations, in fluid-saturated porous media, have several important applications in a variety of engineering processes including heat exchanger devices, petroleum reservoirs, chemical catalytic reactors, solar energy porous wafer collector systems, ceramic materials, migration of moisture through air contained in fibrous insulations and grain storage installations, and the dispersion of chemical contaminants through water-saturated soil, superconverting geothermics etc. The vertical free convection boundary layer flow in porous media owing to combined heat and mass transfer has been investigated. Lai and Kulacki used the series expansion method to investigate coupled heat and mass transfer in natural convection from a sphere in a porous medium. Comprehensive reviews of porous media thermal/species convection have been presented by Nield and Bejan, Vafai and Pop and Ingham.

There has been a renewed interest in studying magnetohydrodynamic (MHD) flow and heat transfer in porous and non-porous media due to the effect of magnetic fields on the boundary layer flow control and on the performance of many systems using electrically conducting fluids. In addition, this type of flow finds applications in many engineering problems such as MHD generators, plasma studies, nuclear reactors, and geothermal energy extractions. Kim presented an analysis of an unsteady MHD convection flow past a vertical moving plate embedded in a porous medium in the presence of transverse magnetic field. Helmy presented an unsteady two-dimensional laminar free convection flow of an incompressible, electrically conducting (Newtonian or polar) fluid through a porous medium bounded by infinite vertical plane surface of constant temperature. Singh studied the effects of mass transfer on free convection in MHD flow of viscous fluid. Recent studies of MHD convection in porous media with various complex effects include those by Bég et al. and Takhar and Ram.

For some industrial applications such as glass production and furnace design, and in space technology applications such as cosmical flight aerodynamics rocket, propulsion systems, plasma physics and spacecraft re-entry aerothermodynamics which operate at higher temperatures, radiation effects can be significant. In view of this, Hossain and Takhar et al. analyzed the effect of radiation on mixed convection along a vertical plate with uniform surface temperature. Bakier and Gorla investigated the effect of radiation on mixed convection flow over horizontal surfaces embedded in a porous medium. Kim and Fedorov analyzed the transient mixed radiative convective flow of a micropolar fluid past a moving semi-
infinite vertical porous plate. Bestman and Adjepong presented unsteady hydromagnetic free convection flow with radiative heat transfer in a rotating fluid. Recently, Cooley et al. have investigated unsteady two-dimensional flow of a radiating and chemically reacting MHD fluid with time-dependent suction.

In most of the studies mentioned above, viscous dissipation is neglected. Gebhurt has shown the importance of viscous dissipative heat in free convection flow in the case of isothermal and constant heat flux at the plate. Gebhurt and Mollendorf considered the effects of viscous dissipation for external natural convection flow over a surface. Soundalgekar analyzed the viscous dissipative heat on the two-dimensional unsteady free convective flow past an infinite vertical porous plate when the temperature oscillates in time and there is constant suction at the plate. Israel Cooley et al. had investigated the influence of viscous dissipation and radiation on unsteady MHD free convection flow past an infinite heated vertical plate in a porous medium with time dependent suction.

The objective of the present paper is to analyze the radiation and mass transfer effects on an unsteady two-dimensional laminar mixed convective boundary layer flow of a viscous, incompressible, electrically conducting fluid, along a vertical moving semi-infinite permeable plate with suction, embedded in a uniform porous medium, in the presence of transverse magnetic filed, by taking into account the effects of viscous dissipation. The equations of continuity, linear momentum, energy and diffusion, which govern the flow field are solved by using a regular perturbation method. The behaviour of the velocity, temperature, concentration, skin-friction, Nusselt number and Sherwood number has been discussed for variations in the governing parameters.

2 Mathematical Analysis

An unsteady two-dimensional hydromagnetic laminar mixed convective boundary layer flow of a viscous, incompressible, electrically conducting and radiating fluid in an optically thin environment, past a semi-infinite vertical permeable moving plate embedded in a uniform porous medium, in the presence of thermal and concentration buoyancy effects has been considered. The $x'$-axis is taken in the upward direction along the plate and the $y'$-axis normal to it. A uniform magnetic field is applied in the direction perpendicular to the plate. The transverse applied magnetic field and magnetic Reynolds number are assumed to be very small, so that the induced magnetic field is negligible. Also, it is assumed that the there is no applied voltage, so that the electric field is absent. The concentration of the diffusing species in the binary mixture is assumed to be very small in comparison with the other chemical species which are present, and hence the Soret and Dufour effects are negligible. Further, due to the semi-infinite plane surface assumption, the flow variables are functions of normal distance $y'$ and $t'$ only. Now, under the usual Boussinesq’s approximation, the governing boundary layer equations are:

$$\frac{\partial y'}{\partial y} = 0 \quad \ldots (1)$$

$$\frac{\partial u'}{\partial t'} + v' \frac{\partial u'}{\partial y'} = -\frac{1}{\rho} \frac{\partial p'}{\partial x'} + v' \frac{\partial^2 u'}{\partial y'^2} + g\beta(T' - T'_\infty) + g\beta(C' - C'_\infty) - v' \frac{\sigma B_0^2 u'}{\rho} \quad \ldots (2)$$

$$\frac{\partial T'}{\partial t'} + v' \frac{\partial T'}{\partial y'} = \frac{k}{c_p} \left[ \frac{\partial^2 T'}{\partial y'^2} - \frac{1}{k} \frac{\partial q'}{\partial y'} \right] + \frac{v'}{c_p} \left( \frac{\partial u'}{\partial y'} \right)^2 \quad \ldots (3)$$

$$\frac{\partial^2 q'}{\partial y'^2} - 3a^2 q' - 16\sigma^* aT'_\infty \frac{\partial T'}{\partial y'} = 0 \quad \ldots (4)$$

$$\frac{\partial C'}{\partial t'} + v' \frac{\partial C'}{\partial y'} = D \frac{\partial^2 C'}{\partial y'^2} \quad \ldots (5)$$

where $u'$, $v'$ are the velocity components in $x'$, $y'$ directions respectively, $t'$ - the time, $p'$ - the pressure, $\rho$ - the fluid density, $g$ - the acceleration due to gravity, $\beta$ and $\beta'$ - the thermal and concentration expansion coefficients respectively, $K'$ - the permeability of the porous medium, $T'$ - the temperature of the fluid in the boundary layer, $v$ - the kinematic viscosity, $\sigma$ - the electrical conductivity of the fluid, $T'_\infty$ - the temperature of the fluid far away from the plate, $C'$ - the species concentration in the boundary layer, $C'_\infty$ - the species concentration in the fluid far away from the plate, $B_0$ - the magnetic induction, $\alpha$ - the fluid thermal diffusivity, $k$ - the thermal conductivity, $q'$ - the radiative heat flux, $\sigma^*$ - the Stefan-Boltzmann constant and $D$ the mass diffusivity. The third and fourth terms on the right hand side of the momentum Eq. (2) denote the thermal and concentration buoyancy effects respectively. Also, the second and third terms on right hand side of the energy Eq. (3) represent the radiative heat flux and viscous dissipation respectively.

It is assumed that the permeable plate moves with a constant velocity in the direction of fluid flow and the free stream velocity follows the exponentially increasing small perturbation law. In addition, it is assumed that the temperature and concentration at the wall as well as the suction velocity are exponentially varying with time. Eq. (4) is the differential approximation for radiation under fairly broad realistic assumptions. In one space coordinate
where \( u'_p \) is the plate velocity, \( T'_w \) and \( C'_w \) the temperature and concentration of the plate respectively. \( U'_a \) the free stream velocity, \( U_0 \) and \( n' \) the constants. From Eq. (1), it is clear that the suction velocity at the plate is either a constant or function of time only. Hence, the suction velocity normal to the plate is assumed in the form:

\[
v' = -V_0 (1 + \epsilon A e^{n'y'})
\]

where \( A \) is a real positive constant and \( \epsilon \) is small such that \( \epsilon \ll 1, \epsilon A \ll 1 \), and \( V_0 \) is a non-zero positive constant, the negative sign indicates that the suction is towards the plate.

Outside the boundary layer, Eq. (2) gives:

\[
\frac{1}{\rho} \frac{\partial}{\partial x'} \frac{dU'_u}{d t'} = \frac{v}{K'_t} \frac{U'_u}{U'_w} + \frac{\sigma}{\rho} B'_0 \frac{U'_u}{U'_w} \quad ... (8)
\]

Since the medium is optically thin with relatively low density and \( \alpha \ll 1 \), the radiative heat flux given by Eq. (3), in the spirit of Cogley et al.24, becomes:

\[
\frac{\partial q}{\partial y'} = 4\alpha^2 (T' - T'_w) \quad ... (9)
\]

where \( \alpha^2 = \int_0^\infty \frac{\partial B}{\partial T} \), where \( B \) is Planck's function.

In order to write the governing equations and the boundary conditions in dimensionless form, the following non-dimensional quantities are introduced.

\[
u = \frac{u}{U_0}, \quad v = \frac{v}{V_0}, \quad y = \frac{V_0 y'}{v}, \quad U = \frac{U'_u}{U'_w}, \quad U = \frac{U'_u}{U'_w},
\]

\[
U_p = \frac{u'_p}{U_0}, \quad t' = \frac{V_0^2}{v}, \quad \theta = \frac{T' - T'_w}{T_w - T'_w}, \quad C = \frac{C'_w - C'_w}{C'_w - C'_w}, \quad n' = \frac{n' v}{V_0^2}, \quad K = \frac{K' Y'_0}{v}, \quad \rho = \frac{\rho}{V_0^2}, \quad M = \frac{\sigma B_0}{\rho V_0^2},
\]

\[
Gr = \frac{\nu \beta g (T'_w - T'_w)}{U_0 V_0^2}, \quad Gm = \frac{\nu \beta g (C'_w - C'_w)}{U_0 V_0^2}, \quad Ec = \frac{U_0^2}{c_p (T'_w - T'_w)}, \quad R^2 = \frac{\alpha^2 (T'_w - T'_w)}{\rho c_p k U_0^2}
\]

In view of Eqs (4) and (7-10), Eqs (2), (3) and (5) reduce to the following dimensionless form:

\[
\frac{\partial u}{\partial t} - (1 + \epsilon A e^{n'\theta}) \frac{\partial u}{\partial y} = \frac{d U_u}{d t} + \frac{\partial^2 u}{\partial y^2} + Gr \theta + Gm C + N (U_0 - u) \quad ... (11)
\]

\[
\frac{\partial \theta}{\partial t} - (1 + \epsilon A e^{n'\theta}) \frac{\partial \theta}{\partial y} = \frac{1}{Pr} \left[ \frac{\partial^2 \theta}{\partial y^2} - R^2 \right] + Ec \left( \frac{\partial u}{\partial y} \right)^2 \quad ... (12)
\]

\[
\frac{\partial C}{\partial t} - (1 + \epsilon A e^{n'\theta}) \frac{\partial C}{\partial y} = \frac{1}{Sc} \frac{\partial^2 C}{\partial y^2} \quad ... (13)
\]

where \( N = M^+(1/K) \) and \( Gr, Gm, Pr, R, Ec \) and \( Sc \) are the thermal Grashof number, solutal Grashof Number, Prandtl number, radiation parameter, Eckert number and Schmidt number, respectively.

The corresponding dimensionless boundary conditions are:

\[
u = U_p, \quad \theta = 1 + \epsilon e^{n'0}, \quad C = 1 + \epsilon e^{n'0} \quad \text{at} \quad y = 0
\]

\[
u = U_0, \quad \theta \to 0, \quad C \to 0 \quad \text{as} \quad y \to \infty \quad ... (14)
\]

### 3 Solution of the Problem

The Eqs (11-13) are coupled, non-linear partial differential equations and these cannot be solved in closed-form. However, these equations can be reduced to a set of ordinary differential equations, which can be solved analytically. This can be done by representing the velocity, temperature and concentration of the fluid in the neighbourhood of the plate as:

\[
u(y,t) = u_0(y) + \epsilon e^{n_0} u_1(y) + o(\epsilon^2) + ...
\]

\[
\theta(y,t) = \theta_0(y) + \epsilon e^{n_0} \theta_1(y) + o(\epsilon^2) + ...
\]

\[
C(y,t) = C_0(y) + \epsilon e^{n_0} C_1(y) + o(\epsilon^2) + ...
\]

Substituting Eq.(15) in Eqs (11-13) and equating the harmonic and non- harmonic terms, and neglecting the higher-order terms of \( o(\epsilon^2) \), we obtain:

\[
u_0' + Nu_0 = - Gr \theta_0 - Gm C_0
\]

\[
u_1' + (N + n)u_1 = -(N + n) - Au_0' - Gr \theta_1 - Gm C_1
\]

\[
u_1' + (N + n)u_1 = -(N + n) - Au_0' - Gr \theta_1 - Gm C_1
\]
The corresponding boundary conditions can be written as:

\[ u_0 = U_p, u_1 = 0, \theta_0 = 1, \theta_1 = 1, C_0 = 1, C_1 = 1 \text{ at } y = 0 \]

\[ u_0 = 1, u_1 = 1, \theta_0 \rightarrow 0, \theta_1 \rightarrow 0, C_0 \rightarrow 0, \]
\[ C_1 \rightarrow 0 \text{ as } y \rightarrow \infty \]

The Eqs (16-21) are still coupled and non-linear, whose exact solutions are not possible. So we expand \( u_0, u_1, \theta_0, \theta_1, C_0, C_1 \) in terms of \( Ec \) in the following form, as the Eckert number is very small for incompressible flows.

\[ F(y) = F_0(y) + EcF_1(y) + o(Ec^2) \]

where \( F \) stands for any \( u_0, u_1, \theta_0, \theta_1, C_0, C_1 \)

Substituting Eq. (23) in Eqs (16-21), equating the coefficients of \( Ec \) to zero and neglecting the terms in \( Ec^2 \) and higher order, we get the following equations:

The zeroth order equations are:

\[ u_{01}'' + u_{01}' - Nu_{01} = -N - Gr \theta_{01} - Gm C_{01} \]

\[ u_{02}'' + u_{02}' - Nu_{02} = -Gr \theta_{02} - Gm C_{02} \]

\[ \theta_{01}'' + Pr \theta_{01}' - R^2 \theta_{01} = 0 \]

\[ \theta_{02}'' + Pr \theta_{02}' - R^2 \theta_{02} = -Pr u_{01}'^2 \]

\[ C_{01}'' + Sc C_{01}' = 0 \]

\[ C_{02}'' + Sc C_{02}' = 0 \]

and the respective boundary conditions are:

\[ u_{01} = U_p, u_{02} = 0, \theta_{01} = 1, \theta_{02} = 0, C_{01} = 1, C_{02} = 0 \]

\[ u_{01} \rightarrow 1, u_{02} \rightarrow 0, \theta_{01} \rightarrow 0, \theta_{02} \rightarrow 0, C_{01} \rightarrow 0, C_{02} \rightarrow 0 \]

as \( y \rightarrow \infty \)

The first order equations are:

\[ u_{11}' + u_{11}' \rightarrow -(N + n)u_{11} = -(N + n) - Gr \theta_{11} \]

\[ -Gm C_{11} - Au_{01}' \]

\[ u_{12}' + u_{12}' \rightarrow -(N + n)u_{12} = -Gr \theta_{12} - Gm C_{12} - Au_{02}' \]

\[ \theta_{11}' + Pr \theta_{11}' - n Pr \theta_{11} = -Pr \theta_{01}' \]

\[ \theta_{12}' + Pr \theta_{12}' - N \theta_{12} = -Pr Au_{02}' - 2Pr u_{01}'u_{11}' \]

\[ C_{11}'' + Sc C_{11}' - n Sc C_{11}' = -AscC_{01}' \]

\[ C_{12}'' + Sc C_{12}' - n Sc C_{12}' = -AscC_{02}' \]

Solving Eqs (24-29) under the boundary conditions in Eq. (30) and Eqs (31-36) under the boundary conditions in Eq. (37), and using Eqs (23) and (15), we obtain the velocity, temperature, and concentration distributions in the boundary layer as:

\[ u(y, t) = P_3 e^{-\eta_0 y} + P_1 e^{-\eta_1 y} + P_2 e^{-\eta_2 y} + 1 \]

\[ + Ec \{ J_{10} e^{\eta_0 y} + J_1 e^{-\eta_1 y} + J_2 e^{-\eta_2 y} \}

\[ + J_3 e^{2\eta_2 y} + J_4 e^{2\eta_1 y} \}

\[ + J_5 e^{-\eta_1 y} + J_6 e^{-\eta_2 y} \}

\[ + J_7 e^{-2\eta_2 y} + J_8 e^{-2\eta_1 y} \}

\[ + J_9 e^{-\eta_1 y} + J_{10} e^{-\eta_2 y} \}

\[ + G_1 e^{-\eta_0 y} + G_2 e^{-\eta_1 y} + G_3 e^{-\eta_2 y} + G_4 e^{-\eta_2 y} \]

\[ + G_5 e^{-\eta_1 y} + G_6 e^{-\eta_2 y} + G_7 e^{-\eta_2 y} + G_8 e^{-\eta_2 y} \]

\[ + G_9 e^{-\eta_1 y} + G_{10} e^{-\eta_2 y} \}

\[ + Z_3 e^{-\eta_1 y} + Z_4 e^{-\eta_2 y} + Z_5 e^{-\eta_2 y} \]

\[ + Z_6 e^{-\eta_1 y} + Z_7 e^{-\eta_2 y} + Z_8 e^{-\eta_2 y} \]

\[ + Z_9 e^{-\eta_1 y} + Z_{10} e^{-\eta_2 y} + Z_{11} e^{-\eta_2 y} \]
\[+Z_{12}e^{-(m_{1} + Sc)y} + Z_{13}e^{-(m_{2} + m_{3})y} + Z_{14}e^{-(m_{3} + m_{4})y} + Z_{15}e^{-(m_{1} + m_{5})y} + Z_{16}e^{-(m_{1} + m_{5})y} + Z_{17}e^{-(m_{1} + m_{5})y}\]

where,

\[m_2 = \frac{Pr + \sqrt{Pr^2 + 4R^2}}{2}, \quad m_3 = \frac{1 + \sqrt{1 + 4N}}{2},\]

\[m_4 = \frac{Pr + \sqrt{Pr^2 + 4N_1}}{2}, \quad m_5 = \frac{1 + \sqrt{1 + 4(N + n)}}{2}\]

\[P_1 = \frac{-Gr}{m_2 - m_2 - N}, \quad P_2 = \frac{-Gm}{Sc^2 - Sc - N}\]

\[P_3 = 1 - U_p + P_1 + P_2\]

\[J_1 = \frac{-GrS_1}{m_2^2 - m_2 - N}, \quad J_2 = \frac{-GrS_1}{4m_2^2 - m_2 - N}\]

\[J_3 = \frac{GrS_2}{4m_2^2 - m_2 - N}, \quad J_4 = \frac{GrS_1}{4Sc^2 - Sc - N}\]

\[J_5 = \frac{-GrS_4}{(m_1 + m_4)^2 - m_2 - m_3 - N}\]

\[J_6 = \frac{GrS_6}{(m_2 + Sc)^2 - m_2 - Sc - m_3 - N}\]

\[J_7 = \frac{-GrS_6}{(Sc + m_3)^2 - Sc - m_3 - N}\]

\[G_1 = \frac{-Gr}{m_3^2 - m_3 - (N + n)}, \quad G_2 = \frac{-Gr}{m_3^2 - m_3 - (N + n)}\]

\[G_3 = \frac{-Gr}{m_3^2 - m_3 - (N + n)}, \quad G_4 = \frac{-Gr}{m_3^2 - m_3 - (N + n)}\]

\[G_5 = \frac{-Gr}{m_2^2 - m_2 - (N + n)}, \quad G_6 = \frac{-Gr}{m_2^2 - m_2 - (N + n)}\]

\[G_7 = \frac{-Gr}{m_2^2 - m_2 - (N + n)}\]

\[G_{10} = -(G_1 + G_2 + G_3 + G_4 + 1)\]

\[Z_1 = \frac{2AJ_1m_3 - GrR_3}{4m_2^2 - 2m_2 - (N + n)},\]

\[Z_2 = \frac{2AJ_2m_2 - GrR_3}{4m_2^2 - 2m_2 - (N + n)},\]

\[Z_3 = \frac{2AJ_3m_2 - GrR_3}{4m_2^2 - 2m_2 - (N + n)}\]

\[Z_4 = \frac{2AJ_3m_2 - GrR_3}{4m_2^2 - 2m_2 - (N + n)}\]

\[Z_5 = \frac{2AJ_3m_2 - GrR_3}{4m_2^2 - 2m_2 - (N + n)}\]

\[Z_6 = \frac{2AJ_3m_2 - GrR_3}{4m_2^2 - 2m_2 - (N + n)}\]

\[Z_7 = \frac{2AJ_3m_2 - GrR_3}{4m_2^2 - 2m_2 - (N + n)}\]

\[Z_8 = \frac{2AJ_3m_2 - GrR_3}{4m_2^2 - 2m_2 - (N + n)}\]

\[Z_9 = \frac{2AJ_3m_2 - GrR_3}{4m_2^2 - 2m_2 - (N + n)}\]

\[Z_{10} = \frac{2AJ_3m_2 - GrR_3}{4m_2^2 - 2m_2 - (N + n)}\]

\[Z_11 = \frac{2AJ_3m_2 - GrR_3}{4m_2^2 - 2m_2 - (N + n)}\]

\[Z_12 = \frac{2AJ_3m_2 - GrR_3}{4m_2^2 - 2m_2 - (N + n)}\]

\[Z_13 = \frac{2AJ_3m_2 - GrR_3}{4m_2^2 - 2m_2 - (N + n)}\]

\[Z_14 = \frac{2AJ_3m_2 - GrR_3}{4m_2^2 - 2m_2 - (N + n)}\]

\[Z_15 = \frac{2AJ_3m_2 - GrR_3}{4m_2^2 - 2m_2 - (N + n)}\]

\[Z_16 = \frac{2AJ_3m_2 - GrR_3}{4m_2^2 - 2m_2 - (N + n)}\]

\[Z_17 = \frac{2AJ_3m_2 - GrR_3}{4m_2^2 - 2m_2 - (N + n)}\]
where,

\[ S_1 = -\frac{Pr m^2 P^2}{4m^2 - 2m - N}, \quad S_2 = -\frac{Pr m^3 P^2}{4m^2 - 2m - N}, \]

\[ S_3 = -\frac{Pr Sc^2 P^2}{4Sc^2 - 2Sc - N}, \]

\[ S_4 = \frac{2Pr m^2 P^2}{(m + m) - m - m - N}, \]

\[ S_5 = \frac{2Pr m^2 P^2}{(m + Sc)^2 - m - Sc - N}, \]

\[ S_6 = \frac{2Pr m^2 P^2}{(m + Sc)^2 - m - Sc - N}, \]

\[ S_{10} = -(S_1 + S_2 + S_3 + S_4 + S_5 + S_6 + S_7) \]

\[ D_1 = \frac{m^2 Pr}{m^2 - m - Pr - N}, \quad D_2 = 1 - D_1 \]

\[ R_1 = \frac{Pr AS_{10}}{m^2 - m - N}, \quad R_2 = \frac{P_2 G^2 m^2 - 2 Pr Am_3 S}{4m^2 - 2m - N} \]

\[ R_3 = \frac{-(P^2 G^2 S^2 + 2 Pr AS mSc)}{Sc^2 - N}, \]

\[ R_4 = \frac{2Pr A S_4 (m^2 + m) - m^2 P^2 G^2 m^2 - P m^2 G m}{(m + m)^2 - (m + m) - N}, \]

\[ R_5 = \frac{m^2 P^2 G^2 S^2 - P m^2 G^2 S^2 - 2 Pr A S m (m + Sc)}{(m + Sc)^2 - (m + Sc) - N}, \]

\[ R_6 = \frac{P m^2 G S c - m^2 P^2 G S c + 2 Pr A S (m^2 + Sc)}{(m + Sc)^2 - (m + Sc) - N} \]

\[ R_7 = \frac{P G m^2 m^2}{4m^2 - 2m - N}, \]

\[ R_8 = \frac{P m^2 G m m^2}{(m + m)^2 - (m + m) - N} \]

\[ R_9 = \frac{P m^2 G m m^2}{(m + m) - (m + m) - N} \]

\[ R_{10} = \frac{P m^2 G m m^2}{(m + m) - (m + m) - N} \]

\[ R_{11} = \frac{P m^2 G m m^2}{(m + m) - (m + m) - N} \]

\[ R_{12} = \frac{P m^2 G m m^2}{(m + m) - (m + m) - N} \]

\[ R_{13} = \frac{P m^2 G m m^2}{(m + m) - (m + m) - N} \]

\[ R_{14} = \frac{P m^2 G m m^2}{(m + m) - (m + m) - N} \]

\[ R_{15} = \frac{P m^2 G m m^2}{(m + m) - (m + m) - N} \]

\[ R_{16} = \frac{P m^2 G m m^2}{(m + m) - (m + m) - N} \]

\[ R_{20} = -(R_1 + R_2 + R_3 + R_4 + R_5 + R_6 + R_7) + R_8 + R_9 + R_{10} + R_{11} + R_{12} + R_{13} + R_{14} + R_{15} + R_{16} \]

\[ C(y,t) = e^{-Scy} + e^{em} \left[ \left( 1 + \frac{ASc}{n} \right) + e^{-mny} \right] \]

where

\[ m = \frac{Sc + \sqrt{Sc^2 + 4nSc}}{2} \]

The skin-friction, Nusselt number and Sherwood number are important physical parameters for this type of boundary layer flow. Knowing the velocity field, the skin-friction at the plate can be obtained, which in non-dimensional form is given by:

\[ C_i = \frac{\tau'_i}{\rho U_0 V_0} = \left( \frac{\partial u}{\partial y} \right) _{y=0} = \left( \frac{\partial u}{\partial y} + e^{em} \frac{\partial u}{\partial y} \right) _{y=0} \]
Knowing the temperature field, the rate of heat transfer coefficient can be obtained, which in the non-dimensional form, in terms of the Nusselt number, is given by:

\[
Nu = -x \frac{\partial T}{\partial y} \bigg|_{y=0} \Rightarrow Nu Re_x^{-1} = -\left( \frac{\partial \theta}{\partial y} \right) \bigg|_{y=0}
\]

\[
= - \left( \frac{\partial \theta}{\partial y} + \varepsilon e^{n} \frac{\partial \theta}{\partial y} \right) \bigg|_{y=0}
= \left[ -m_2 + Ec \{ -S_{10} m_2 - 2S_y m_2 - 2S_z m_2 \} - 2S_y \delta - S_y (m_3 + m_3) - S_y (m_3 + S_c) + Ec \{ -G_m m_2 + G_m m_2 \} + G_m m_2 - G_m m_2 - G_y \delta + G_m m_2 \right]
\]

Knowing the concentration field, the rate of mass transfer coefficient can be obtained, which in the non-dimensional form, in terms of the Sherwood number, is given by:

\[
Sh = -x \frac{\partial C'}{\partial y} \bigg|_{y=0} \Rightarrow Sh Re_x^{-1} = -\left( \frac{\partial C}{\partial y} \right) \bigg|_{y=0}
\]

\[
= -\left( \frac{\partial C}{\partial y} + \varepsilon e^{n} \frac{\partial C}{\partial y} \right) \bigg|_{y=0}
\]

4 Results and Discussion

The formulation of the problem that accounts for the effects of radiation and viscous dissipation on the flow of an incompressible viscous fluid along a semi-infinite, vertical moving porous plate embedded in a porous medium in the presence of transverse magnetic field was accomplished. Following Cogley et al., 24 approximation for the radiative heat flux in the optically thin environment, the governing equations of the flow field were solved analytically, using a perturbation method, and the expressions for the velocity, temperature, concentration, skin-friction, Nusselt number and Sherwood number were obtained. In order to get a physical insight of the problem, the above physical quantities are computed numerically for different values of the governing parameters viz., thermal Grashof number Gr, the solutal Grashof number Gm, Prandtl number Pr, Schmidt number Sc, the plate velocity \( U_p \), the radiation parameter R and the Eckert number Ec.

In order to assess the accuracy of this method, we have compared our results with accepted data for the velocity and temperature profiles for a stationary vertical porous plate corresponding to the case computed by Helmy 10 and to the case of moving vertical porous plate as computed by Kim 9. The results of these comparisons are found to be in very good agreement.

Figure 1 shows the typical velocity profiles in the boundary layer for various values of the thermal Grashof number. It is observed that an increase in Gr, leads to a rise in the values of velocity due to enhancement in buoyancy force. Here, the positive values of Gr correspond to cooling of the plate. In addition, it is observed that the velocity increases rapidly near the wall of the porous plate as Grashof number increases and then decays to the free stream velocity. For the case of different values of the solutal Grashof number, the velocity profiles in the boundary layer are shown in Fig. 2. The velocity distribution attains a distinctive maximum value in the vicinity of the plate and then decreases properly to approach a free stream value. As expected, the fluid velocity increases and the peak value becomes more distinctive due to increase in the buoyancy force represented by Gm. For different values of the radiation parameter R, the velocity and temperature profiles are shown in Fig. 3(a and b). It is noticed that an increase in the radiation parameter results a decrease in the velocity and temperature within the boundary layer, as well as decreased the thickness of the velocity and temperature boundary layers.
Figure 4(a and b) show the effects of Schmidt number on the velocity and concentration, respectively. As the Schmidt number increases, the concentration decreases. This causes the concentration buoyancy effects to decrease yielding a reduction in the fluid velocity. Reductions in the velocity and concentration distributions are accompanied by simultaneous reductions in the velocity and concentration boundary layers.

The effects of the viscous dissipation parameter i.e., the Eckert number on the velocity and temperature are shown in Fig. 5(a and b). Greater viscous dissipative heat causes a rise in the temperature as well as the velocity.

Figure 6(a and b) show the behaviour velocity and temperature for different values of Prandtl number. The numerical results show that the effect of increasing values of Prandtl number results in a decreasing velocity. From Fig.6 (b), it is observed that an increase in the Prandtl number results a decrease of the thermal boundary layer thickness and in general lower average temperature with in the boundary layer. The reason is that smaller values of Pr are equivalent to increase in the thermal conductivity of the fluid and therefore, heat is able to diffuse away from the heated surface more rapidly for higher values of Pr. Hence, in the case of smaller Prandtl numbers as the thermal boundary layer is thicker and the rate of heat transfer is reduced. For various values of the magnetic parameter M, the velocity profiles are shown in Fig.7. It is obvious that existence of the magnetic field decreases the velocity. Fig.8 shows the velocity profiles for different values of the permeability.
Fig. 3(a) — Effect of radiation on velocity.

Fig. 3(b) — Effects of radiation on temperature.

Fig. 4(a) — Effect of Sc on velocity.

Fig. 4(b) — Effect of Sc on concentration.
Fig. 5(a) — Effect of Ec on velocity.

Fig. 5(b) — Effect of Ec on temperature.

Fig. 6(a) — Effect of Pr on velocity.

Fig. 6(b) — Effect of Pr on temperature.
Fig. 7 — Effect of magnetic parameter on velocity. Clearly, as $K$ increases the peak values of the velocity tend to increase.

Fig. 8 — Effect of permeability on velocity. Although we have different initial plate velocities, the velocity decreases to the constant value for given material parameters.

Fig. 9 — Effect of $U_p$ on velocity. Table 9 shows the variation of the velocity distribution across the boundary layer for different values of the plate velocity $U_p$ in the direction of the fluid flow. Although we have different initial plate velocities, the velocity decreases to the constant value for given material parameters.

Tables 1-5 present the effects of the thermal Grashof number, solutal Grashof number, radiation parameter, Schmidt number and Eckert number on the skin-friction $C_f$, Nusselt number $Nu$ and Sherwood number $Sh$. From Tables 1 and 2, it is observed that as $Gr$ or $Gm$ increases, the skin-friction coefficient increases. However, from Table 3, it can be seen that as the radiation parameter increases, the skin-friction decreases and the Nusselt number increases. From Table 4, it is noticed that an increase in the Schmidt number reduces the skin-friction and increases the Sherwood number. Finally, it is observed from...
and concentration buoyancy effects were enhanced and solutal Grashof numbers were increased, the thermal and concentration level was decreased resulting in decreased fluid velocity. In addition, it was found that when thermal and concentration buoyancy effects while it decreased due to increase in either radiation parameter or the Schmidt number. However, the Nusselt number increased as radiation parameter increased and the Sherwood number also increased as Schmidt number increased. Increase in Eckert parameter increased and the Sherwood number also increased. Reference values as in Fig. 2

Table 5 — Effects of $Ec$ and $NuRe_s^{-1}$. Reference values as in Fig. 5 (a and b).

<table>
<thead>
<tr>
<th>$Ec$</th>
<th>$C_f$</th>
<th>$NuRe_s^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2.5693</td>
<td>0.8552</td>
</tr>
<tr>
<td>0.01</td>
<td>2.6326</td>
<td>0.5629</td>
</tr>
<tr>
<td>0.03</td>
<td>2.6960</td>
<td>0.2706</td>
</tr>
</tbody>
</table>

Table 2 — Effects of $Gm$ on $C_f$. Reference values as in Fig. 2

<table>
<thead>
<tr>
<th>$Gr$</th>
<th>$C_f$</th>
</tr>
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<tr>
<td>0</td>
<td>1.9741</td>
</tr>
<tr>
<td>1</td>
<td>1.6565</td>
</tr>
<tr>
<td>2</td>
<td>1.3571</td>
</tr>
<tr>
<td>3</td>
<td>1.0718</td>
</tr>
<tr>
<td>4</td>
<td>0.8083</td>
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</tbody>
</table>

Table 3 — Effects of radiation on $C_f$ and $NuRe_s^{-1}$. Reference values as in Fig. 3 (a and b).

<table>
<thead>
<tr>
<th>$R$</th>
<th>$C_f$</th>
<th>$NuRe_s^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2.6871</td>
<td>0.6818</td>
</tr>
<tr>
<td>0.5</td>
<td>2.5123</td>
<td>1.1183</td>
</tr>
<tr>
<td>1.0</td>
<td>2.4326</td>
<td>1.3853</td>
</tr>
<tr>
<td>2.0</td>
<td>2.3474</td>
<td>1.7591</td>
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</tbody>
</table>

Table 4 — Effects of $Sc$ on $C_f$ and $ShRe_s^{-1}$. Reference values as in Fig. 4 (a and b).

<table>
<thead>
<tr>
<th>$Sc$</th>
<th>$C_f$</th>
<th>$ShRe_s^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.30</td>
<td>2.6088</td>
<td>0.3006</td>
</tr>
<tr>
<td>0.60</td>
<td>2.5123</td>
<td>0.6010</td>
</tr>
<tr>
<td>0.78</td>
<td>2.4677</td>
<td>0.7813</td>
</tr>
<tr>
<td>0.94</td>
<td>2.4340</td>
<td>0.9416</td>
</tr>
</tbody>
</table>

Table 5 as Eckert number increases the skin-friction increases, and the Nusselt number decreases.

5 Conclusions

The governing equations for unsteady MHD convective heat and mass transfer flow past a semi-infinite vertical permeable moving plate embedded in a porous medium with radiation was formulated. Viscous dissipation effects were also included in the present work. The plate velocity is maintained at constant value and the flow was subjected to a transverse magnetic field. The resulting partial differential equations were transformed into a set of ordinary differential equations using two-term series and solved in closed-form. Numerical evaluations of the closed-form results were performed and graphical results were obtained to illustrate the details of the flow and heat and mass transfer characteristics and their dependence on some physical parameters. It was found that when thermal and solutal Grashof numbers were increased, the thermal and concentration buoyancy effects were enhanced and thus, the fluid velocity increased. However, the presence of radiation effects caused reductions in the fluid temperature, which resulted in decrease in the fluid velocity. Also, when the Schmidt number was increased, the concentration level was decreased resulting in decreased fluid velocity. In addition, it was found that the skin-friction coefficient increased due to increase in thermal and concentration buoyancy effects while it decreased due to increase in either radiation parameter or the Schmidt number. However, the Nusselt number increased as radiation parameter increased and the Sherwood number also increased as Schmidt number increased. Increase in Eckert number leads to an increase skin-friction and decrease in Nusselt number.

References