Convective effects in air layers bound by cellular honeycomb arrays

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Rayleigh-Benard convection in a fluid (air) layer in presence of transparent cellular structure (honeycomb) is reviewed for application to the engineering design of transparent insulation materials devices. The explicit numerical computations of Rayleigh numbers of the characteristic equation at higher wave numbers corresponding to the onset of instability are carried out. Results highlight a new facet of theory that it can indeed be used to estimate the critical Rayleigh number, which corresponds to the layer bound by vertical walls of cellular structure. A relationship between wave number and aspect ratio of honeycomb structure is evolved. Subsequently the stability of inclined fluid layer is considered and base flow existence and the need for its consideration in terms of Nusselt number is discussed. An outline of governing equations of the square celled air temperature, vorticity, stream functions and velocity are presented and it is found that the Nusselt number has a value in the range 1-1.5 for a wide range of Rayleigh numbers in near critical Rayleigh regime. This range may be considered as a trade off region for the engineering design of honeycomb devices. Convection can be suppressed by varying the aspect ratio of honeycomb cell. Honeycomb structure of aspect ratio 10-15 seems suitable for suppression of convection in air layer of depth 5-20 cm for temperature difference (∆T) of 20 to 120°.

Keywords: Critical Rayleigh number, Stability theory, Convection suppression, Honeycomb, Transparent insulation material

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Introduction

The air layers have been used as an insulating material for engineering applications such as nuclear reactors and building elements (double glazed windows, solar collectors, integrated collector storage systems). In such applications, it is important to suppress natural convection in the air layer because the thermal transfer coefficient across the layer increases with convective mass transport. Honeycomb cellular structures have been suggested as a means to suppress the convection in air layers heated from below. The air layer bound by transparent honeycomb are solar transparent; yet provide good thermal insulation and are often referred to as transparent insulation material in solar energy context. The fluid mechanical treatment of this type of problem for square cells has been given by several researchers¹-⁶ and for rectangular cell by Koutsoheras & Charters⁷. According to these investigations, when a cellular matrix (Fig. 1) is introduced in a fluid layer of infinite horizontal extent and finite vertical depth (L) contained between two isothermal bounding surfaces and heated at the bottom, the vertical walls provide extra viscous resistance to the onset of convection and effectively raise the value of critical Rayleigh numbers (Rc), which depend upon physical shape and aspect ratio (A=L/d) of honeycomb cell. In this paper, an alternative approach for the evaluation of critical Rayleigh numbers of air layers in presence of honeycomb structures has been developed. The approach illustrates a new facet, mathematical and physical, of general theory of hydrodynamic stability of fluid layer of horizontal infinite extent heated from below (Rayleigh Benard problem). It involves numerical solution for higher wave numbers of transcendental characteristic equations of normal mode analysis⁸.

Furthermore, inclined air layer bound by cellular structures have many applications in advanced glazing of solar thermal system. In inclined layers, Hollands et al⁹-¹⁰ have pointed out the inevitable existence of base flow and need for consideration in terms of Nusselt number. In this paper, Nusselt number has been examined as a function of Rayleigh number near the critical Rayleigh number in inclined
air layer bound by cellular matrix. The objective is to investigate trade-offs for the engineering design of honeycomb devices.

**Convective Effects in Horizontal Air Layer with Honeycomb**

Pellew & Southwell in 1940 were first to develop the numerical solution for critical Rayleigh regime stability problem for two rigid boundaries in a horizontal fluid (air) layers heated from below. In this context, two methods were used: 1) Energy method; and 2) Normal mode technique. Chandrasekhar gave a detailed analysis of Rayleigh Benard problem based on normal mode technique. The following two assumptions were made during the formulations of the governing equations:

1. *Boussinesq Approximation*—Density of the fluid is constant except in buoyancy term,

2. *Fourier’s Law of volume expansion*—The change in density is related with the change in temperature given by \( \delta \rho = -\alpha \rho_0 (T - T_0) \), which gives \( \rho = \rho_0 [1 - \alpha (T - T_0)] = \rho_0 [1 - \alpha \gamma y] \); where \( \alpha \) is the coefficient of volume expansion, \( T_0 \) is the initial temperature, \( \rho_0 \) is the initial density and \( \gamma = \frac{T - T_0}{y} \frac{\partial T}{\partial y} \) is adverse temperature gradient.

Applying these assumptions to the governing equations of horizontal air layer heated from below bound by surfaces \( y = \pm L/2 \), the time dependent governing equations after neglecting the product of perturbations and their powers (higher than one) reduce to the form:

\[
\frac{\partial u}{\partial t} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial z} = 0. \tag{1}
\]

\[
\frac{\partial u}{\partial t} - \frac{1}{\rho_0} \frac{\partial \rho}{\partial x} + \nu \frac{\partial^2 u}{\partial y^2} = 0. \tag{2}
\]
\[
\frac{\partial v}{\partial t} = -\alpha \bar{g} \frac{1}{\rho_0} \frac{\partial p}{\partial y} + \nu \nabla^2 v. \quad \cdots (3)
\]
\[
\frac{\partial w}{\partial t} = -\frac{1}{\rho_0} \frac{\partial p}{\partial z} + \nu \nabla^2 w. \quad \cdots (4)
\]
\[
\frac{D\bar{T}}{Dt} = \kappa \nabla^2 \bar{T}. \quad \cdots (5)
\]
For rigid surfaces \((y=\pm 1/2)\); the boundary conditions may be taken as
\[
V = DV = (D^2 - h^2)^2 V = 0 \text{ at } y = \pm 1/2. \quad \cdots (14)
\]
Assuming \(V = A\cos q_y y + B\cosh q_y y + \bar{B}\cosh q_y y\) as even solution and \(V = A\sin q_y y + B\sinh q_y y + \bar{B}\sinh q_y y\) as odd solution for Eq. (13), where \(A, B\) are arbitrary constants, \(q^2\) is a root of the Eq. \((q^2 - h^2)^3 = -Rh^2\) and is given by \(q = q_1 + iq_2\) such that
\[
q_1 = \frac{1}{\sqrt{2}} h \sqrt{(1+n^2)} + 1 + \frac{1}{2} n, \quad q_2 = \frac{1}{\sqrt{2}} h \sqrt{(1+n^2)} - 1 - \frac{1}{2} n
\]
and \(q_0^2 = h^2 (n-1)\) \quad \cdots (15)

The characteristic Eqs for even and odd solutions of Eq. (13) obtained by using Eq. (15) are as:

Even characteristic equation:
\[
\text{im}\{(\sqrt{3} + i) q \tanh \frac{q}{2} + q_0 \tan \frac{1}{2} q_0 = 0, \quad \cdots (16)
\]
Odd characteristic equation:
\[
\left(\frac{\sinh q_0 - \text{isinh} q_2}{\cosh q_0 - \cos q_2}\right) = 0, \quad \cdots (17)
\]

The transcendental Eqs (16) and (17) are solved iteratively to obtain characteristic values of Rayleigh number as a function of wave number \(h\). Chandrasekhar\(^8\) reported results of such solutions for the values of wave numbers (0-8) and used the criteria that minimum Rayleigh number would correspond to critical Rayleigh number at the wave number of minima. From the plot of Rayleigh numbers, at which the instability sets in, the critical Rayleigh numbers of \(R_c = 1707.762\) at wave number \(h = 3.117\) and \(R_c = 17610.39\) when \(h = 5.365\) were obtained. In above analysis for Rayleigh number, the even perturbations are more adverse to the stability than the odd perturbations (Fig. 2).

The wave number \(h\) is a measure of non-dimensional width of the fluid cell. Holland\(^11\) correlated wave number with aspect ratio of cellular...
The computed values of $R_c$, corresponding to $m=0.75^{13}$, exhibit close agreement with experimentally measured values of Heitz & Westwater. Therefore, critical Rayleigh number curve of present analysis corresponding to the onset of instability as a function of wave number is compared with the analysis of Edward & Catton\(^{12}\) (Fig. 3). Results of Edward & Catton\(^{12}\) belong to the fluid mechanical treatment of fluid layer with honeycomb structure. The similarity in representation (Fig. 3) concludes that Table 1 for odd solution may be used for critical Rayleigh number as a function of wave number. The critical Rayleigh number for wave numbers, not given in Table 1, may be calculated iteratively by solving Eq. (17) for required $h$.

The values of critical Rayleigh number may be used for the engineering design of honeycomb devices. The Rayleigh number of a fluid layer heated from below may be expressed as:

$$R_c = \frac{(a_0^2 + 23.9)^3}{(a_0^2 + 7.97)}$$

where $a_0 = m\pi \sqrt{A}$ and $0.75 \leq m \leq 1$ \hspace{1cm} (19)
\[ Ra = \frac{g \alpha \Delta T^3}{\kappa \nu} \]  \hspace{1cm} \text{... (20)}

So, for convection suppression \((Ra < Rc)\)

\[ (\Delta T)_{\text{max}} = \frac{Rc \kappa \nu}{g \alpha L^3} \]  \hspace{1cm} \text{... (21)}

For air, one has

\[ g = 9.8 \text{m/s}^2, \kappa = 2.67 \times 10^{-5} \text{m}^2/\text{s}, \nu = 1.921 \times 10^{-5} \]  \hspace{1cm} \text{... (22)}

Therefore

\[ \frac{\kappa \nu}{g \alpha} = 17.622 \times 10^{-9} = \xi \]  \hspace{1cm} \text{(say)}  \hspace{1cm} \text{... (23)}

Hence

\[ (\Delta T)_{\text{max}} = \frac{\xi Rc}{L^3} \]  \hspace{1cm} \text{... (24)}

where \( \xi \) is a number, which has different values for different materials. The computed \((\Delta T)_{\text{max}}\) values of air layer obtained from Eq. (24) are illustrated in Figs 4a and 4b respectively. It shows that cell width (8-10 mm) is required to suppress convection for temperature difference (20-120°C) for 10-15 cm depth of air layer with honeycomb.

**Convective Effects in Inclined Air Layer with Honeycomb**

If an air layer is inclined from horizontal and heated from below, the fluid can never be stationary. Since the unbalanced buoyancy force always creates some motion, the flow, for which \( Ra < 1708 / \cos \beta \), is the base flow. The motion consists of a single cell with fluid rising from the hot surface and falling along the cold one and turning at upper and lower extremes of layer. The flow of convective heat across the layer will be normal to the plates and in middle region the heat transfer across the layer is only by conduction. This base flow fluid movement must be considered in the study of convective heat transfer through the inclined fluid layer bound by honeycomb cellular panel. Nusselt number (the dimensionless parameter characteristic of the ratio of convective heat flow to the conduction) must therefore, be considered in addition to critical Rayleigh number. Mathematical model of convective stability of inclined fluid (air) layer bound by square honeycomb was given by Koutsoheras & Charters, Arulanatham et al etc.

The critical Rayleigh as well as post-critical Rayleigh Regimes are examined through Nusselt number. Following assumptions have been made to study the convective effects in inclined air layer bound by square honeycomb (Fig. 5).

i) The density of the fluid remains constant except in buoyancy terms;

ii) Zero heat flux through the side walls i.e. \( \frac{\partial \theta}{\partial x} = 0 \);

iii) Linear variation in temperature, i.e. \( \theta = \theta_h + (\theta_h - \theta_l) \frac{Y}{L} \);

iv) The thickness and conductivity of the walls are finite.

Non-dimensional governing equation of continuity, momentum and energy equations are given by

\[ \frac{\partial u}{\partial \tau} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -R_s P T \sin \beta \frac{\partial p}{\partial x} + P \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \]  \hspace{1cm} \text{... (26)}

The two dimensional conduction Eq for the wall is given by

\[ \frac{\partial \theta_w}{\partial t} = \kappa_w \left[ \frac{\partial^2 \theta_w}{\partial X^2} + \frac{\partial^2 \theta_w}{\partial Y^2} \right] \]  \hspace{1cm} \text{... (29)}

\( \kappa_w \) is the thermal diffusivity of the wall material.
Fig. 4a—Variation of $(\Delta T)_{\text{max}}$ for honeycomb with its depth ($L$) and cell size ($d$)

Fig. 4b—Variation of $(\Delta T)_{\text{max}}$ with aspect ratio ($A$) for honeycomb of depth ($L$)
The average Nusselt number is given by

$$Nu_y = \frac{Q L}{K \Delta T}$$  \hspace{1cm} \text{(31)}$

After non-dimensionalisation of \((Q_y)\) and substituting it in \(Nu_y\), Nusselt number is given by

$$Nu_y = A \int_0^{1/A} \left[ \nu \left(T - \frac{1}{2}\right) \frac{\partial T}{\partial y} \right] dx$$ \hspace{1cm} \text{(32)}

Arulanathan et al.
 developed an algorithm for the calculation of Nusselt number using Eq. (32). It has been used here to investigate effect on Nusselt number of tilt angle and Rayleigh number (Figs 6 & 7). For a wide range of Rayleigh number, Nusselt number was from 1 to 1.5, which implies that the convection is very weak and almost suppressed. This range of Nu can be regarded as the trade off range for engineering design of honeycomb with non-convective air cell. It can also be considered as theoretical basis of \(Nu =1.2\) used by Holland in experiments on convection suppression.

The relations based on correlations for the Nusselt number of a square celled honeycomb given by Smart et al. and Cane et al. are as follows:

$$Nu = 1.0 + 0.89 \cos \left( \frac{\beta - \pi}{3} \right) \left( \frac{Ra}{2420A^4} \right)^{2.88-1.64 \sin \beta}$$ \hspace{1cm} \text{(33)}

The range of validity is \((Ra/A^4) < 6000, 30 < \beta < 90^\circ\) and \(A \geq 4\).

Hollands considered the base flow and recommended the engineering design of honeycomb with Nusselt number of 1.2. The minimum aspect ratio \(A\) that is required just to suppress the convection is given by:

$$A = F(\beta) \left( \frac{Ra}{2420} \right)^{1/4}$$ \hspace{1cm} \text{(34)}

$$Q_y = \rho C_p \int_0^D vT dx - K \int_0^D \frac{\partial T}{\partial y} dx$$ \hspace{1cm} \text{(30)}
where \( F(\beta) = (4.45 \cos (\beta - 60))^{(11.52 - 6.56 \sin \beta)} \), \( 30^\circ \leq \beta \leq 90^\circ \) … (35)

And the value of \( Ra \) used here is defined by Eq.

\[
Ra = 2737 (1 + 2 \beta_1)^2 \beta_1^4 \Delta T (100L)^3 p^2
\]

where \( \beta_1 = 100/Tm, Tm(\text{in}K) = (Th+Tc)/2, \Delta T = Th-Tc \), and \( p \) is the atmospheric pressure for mean temperature, \( 280K \leq Tm \leq 500K \).

By analytical observations, above expression is also valid for \( \beta = 0 \), if \( F(\beta) \) is taken as 1.072 and linear interpolation of values (0 - 30°) is also permitted. For air, at atmospheric pressure and moderate temperature (280 K \( \leq Tm \leq 370 \) K), minimum required cell width is to just suppress the convection is

\[
d = \frac{(100L)^{1/4}}{100c(\beta)(1+2\beta_1)^{1/2} \beta_1 (\Delta T)^{1/4}}
\]

\[
m = \frac{(100L)^{1/4}}{c(\beta)(1+2\beta_1)^{1/2} \beta_1 (\Delta T)^{1/4}} \text{ cm}
\]

where \( L \) is in m, \( \beta = \frac{100}{Tm} \) is in K\(^{-1} \), \( \Delta T \) is in K. The function \( c(\beta) \) is given by

\[
c(\beta) = 1.03F(\beta) \text{ for } 30^\circ \leq \beta \leq 90^\circ = 1.1 + 0.25 \sin \beta,
\]

for \( 0 \leq \beta \leq 30^\circ \) … (38)

The results are computed for the minimum cell width required to suppress the convection for the angle of inclination of 10, 40 and 70° based on the above correlations for honeycomb cell depth (2-16 cm) and \( \Delta T \) (20-120°C), which is the range of interest in solar thermal applications (Figs 8a - 8c). It is found that cell width (7-19 mm) is sufficient to suppress the convection.

**Conclusions**

The Rayleigh-Benard convection in a fluid (air) layer bound by transparent cellular structure (honeycomb) is examined for application to the engineering design of transparent insulation materials devices. The derivation of characteristic equation of thermo hydrodynamic instability of horizontal layer
Fig. 8b—Honeycomb cell width required for convection suppression (tilt angle $\beta=40^\circ$)

Fig. 8c—Honeycomb cell width required for convection suppression (tilt angle $\beta=70^\circ$)
heated from below, based on normal mode technique, is presented. The explicit numerical computations of Rayleigh numbers of the characteristic equation at higher wave numbers corresponding to the onset of instability are carried out. The results highlight a new facet that the resultant Rayleigh numbers can be used to estimate the critical Rayleigh number for fluid layer with honeycomb structure. Furthermore, convection can be suppressed by a suitable choice of the aspect ratio of honeycomb cell. The honeycomb structure of aspect ratio 10-15 seems suitable for suppression of convection in air layer of depth 5-15 cm for the temperature difference (ΔT) in the range of 20 to 120°. Subsequently, critical and post critical Rayleigh regime convective stability of inclined air layer bounded by cellular structures were investigated. The consideration of base flow is essential in inclined layers. The base flow can be taken into account through the consideration of Nusselt number in addition to the critical Rayleigh numbers. The governing equations of square celled air temperature, vorticity, stream functions and velocity are presented. The effect was studied of tilt angle and Rayleigh number on Nusselt number(1-1.5), which was of practical interest in solar energy application. The range 1-1.5 of Nusselt number corresponds to the almost nonconvective state of fluid. This range may be considered as the trade off region for the engineering design of convection suppression devices. It may, therefore, be used for the engineering design of non convective honeycomb cell.

References

Nomenclature
A Aspect Ratio of cell  \( \frac{L}{d} \)
A w Aspect Ratio of wall  \( \frac{L}{\delta} \)
C p Specific heat
\( d \) Honeycomb cell width
\( g \) Acceleration due to gravity
\( K w \) Thermal conductivity of wall
\( K f \) Thermal conductivity of fluid
L Depth of honeycomb cell
\( p \) Dimensionless pressure  \( \frac{pL^2}{\rho \kappa^2} \)
P r Prandtl Number  \( \frac{\nu}{\kappa} \)
\( R \) Rayleigh Number  \( \frac{g \alpha \gamma L^4}{\kappa \nu} = \frac{g \alpha \Delta \theta L^4}{\kappa \nu} \)
\( R c \) Critical Rayleigh Number
\( RK \) wall conductivity \( \frac{K w}{K f} \)

Fluid conductivity \( K f \)
\[ \frac{\tilde{\theta} - \theta}{\theta - \theta_c} = T \]

Perturbation in temperature

\( u \) Dimensionless velocity\( = \frac{UL}{\kappa_f} \)

\( v \) Dimensionless velocity\( = \frac{VL}{\kappa_f} \)

\( x \) Dimensionless distance\( = \frac{X}{L} \)

\( y \) Dimensionless distance\( = \frac{Y}{L} \)

\( \alpha \) Coefficient of volume expansion

\( \beta \) Tilt angle

\( \gamma \) Adverse temperature gradient

\( \delta \) Wall width

\( \rho \) Density of the wall at time \( t \)

\( \rho_0 \) The initial density

\( \nu \) Kinematic viscosity\( = \frac{\mu}{\rho} \)

\( \kappa \) Thermal diffusivity of the fluid

\( \tau \) Dimensionless time\( = \frac{\kappa_f t}{L^2} \)

\( t \) Time