Study on prediction of snowmelt using energy balance equations and comparing with regression method in the Eastern part of Turkey

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Received 28 July 2004; revised 16 March 2005; accepted 16 May 2005

This paper presents two snowmelt models: i) Energy balance model (EBM); and ii) Linear regression model (LRM). To decide suitable model for the basin, daily mean flow data of Kırkgöze discharge gauging station of State Hydraulic Works (DSİ) was applied to Karasu-Kırkgöze mountainous basin that has 233.2 km² watershed drainage basin and elevation range of 1830-2854 m in the eastern part of Turkey. EBM was applied during snow melting period, in March-May for 1987-1995. Hourly temperature ($T$), wind velocity ($V$), shortwave radiation ($Rd$), relative humidity ($RH$) and intensity of rainfall ($Y$) were used as input parameters. Intervals of constants in EBM that are snow surface conductance ($K_S$), snow surface saturated conductance ($K_{Sat}$), liquid holding capacity of snow ($L_k$), fresh snow visible band reflectance ($\alpha_{vo}$) and fresh snow near infrared band reflectance ($\alpha_{iro}$) were determined for each period. The coefficients of correlation ($R$) between snow melting data calculated by EBM and gauging data were in the range of 0.88-0.98 for each year ($R^2=0.77-0.96$). Moreover, LRM is established for the only period of 1987 using observed discharges of basin and meteorological variables. The computed coefficient of correlation ($R$) between regression model including five predictor variables ($T$, $Rd$, $RH$, $V$ & $Y$) and gauging data was obtained as 0.87 ($R^2=0.757$). Two models are then compared in terms of coefficient of correlations. EBM was found more representative than LRM to predict snowmelt in eastern part of Turkey due to high coefficient of correlation.

Keywords: Energy balance, Mass balance, Snow, Snowmelt model, Regression, Hydrology

Introduction

Snow is a complicated and rapidly evolving material, which may have significant vertical and horizontal variations in its thermal, hydraulic and mechanical properties. Snowmelt is significant surface water input of importance to many aspects of hydrology including water supply, erosion and flood control. Modeling snowmelt in a hydrological model is especially problematic because an incorrectly simulated melt event not only incorrectly predicts flow on that day, but also on the day when the actual melt occurs.

Snowmelt is driven primarily by energy exchanges at the snow-air interface. The rate of snowmelt depends on the availability of energy to snowpack and is usually dominated by net radiation and temperature. Cold snowpacks (< 0°C) have a negative energy balance. Warming causes the pack to become isothermal (=0°C) and additional energy results in appositive energy balance and melt. Estimating watershed runoff in areas with seasonal snow cover requires a snowmelt algorithm be part of the modeling system. Generally, computation of snowmelt from a watershed is made using either an energy balance approach or some index approach. The energy balance, employed in the present work, requires information radiation energy, sensible and latent heat, energy transferred through rainfall over snow and heat conduction from ground to snowpack.

An extensive review on this topic is available. HEC-1, a single-event model, uses either a degree-day temperature index method or a simplified energy balance approach. SSARR (Streamflow Synthesis and Reservoir Regulation) was developed by the North Pacific Division (NPD) Corps of Engineers in 1956 to provide hydrologic simulations on snowmelt-dominated river systems for planning, design, and operation of water control works. The Generalized Streamflow Simulation System, frequently identified as the Sacramento Catchment Model, is a major component of the National Weather Service River Forecast System (NWSRFS). The snow accumulation and ablation routines used in NWSRFS are based primarily on the efforts of Anderson, who developed a combined energy and temperature index.
method that used only temperature and precipitation as input meteorology. A difference from other temperature index methods is that each physical process is represented separately, rather than using a single melt index. Processes are conceptually modeled and include snowpack accumulation, heat exchange at the air/snow interface, areal extent of snow cover, heat storage within the snowpack, liquid water retention, lagged transmission of melt through the pack, and heat exchange at the ground/snow interface. SNTHERM, a one-dimensional mass and energy balance model that considers the processes occurring within a multi-layered snowpack, includes snow accumulation, compaction, grain growth, melt, condensation melt, advection, pore water retention, and water flow through the pack. SNTHERM, being the most comprehensive of the models reviewed, has the most diverse scientific validation.

In the present paper, an Energy Balance Model (EBM) and Linear Regression Model (LRM) for snowmelt in eastern part of Turkey are developed. Considering energy balance for amount of snow corresponding to unit area, two differential equations are obtained. Resulting differential equation system is solved numerically by using the Runge-Kutta method. LRM is established using least squares method. Both models are compared to decide which model is suitable for Kirkgöze basin.

**Basin and Data**

The models are applied to Karasu-Kirkgöze basin heavy snowfall region (area, 233.2 km²; elevation, 1830-2854 m), which reaches annual mean snow depth of 2 m during winter. Karasu- Kirkgöze basin has mountainous and forestless area and has been derived by 7 tributaries and Kirkgöze River, which is one of the major contributors of water. Snowmelt from mid-March to June contributes 65-70 percent of the annual runoff of Karasu basin. Data of this model was obtained from Erzurum Meteorology Station that is elevated 1869 m and located on 39° 57’ latitude, 41° 10’ longitude. Precipitation data was obtained from Şenyurt Meteorology Station that is elevated 2160 m and located on 40° 11’ latitude, 41° 29’ longitude. Energy contents and water equivalence values were taken from Güzelyayla snow obtaining station that is elevated 2070 m and located on 40° 12’ latitude, 41° 29’ longitudes (Fig 1).

**Energy Balance Model**

The energy balance or the heat budget of snowpack governs the productions of melt water. This method involves an accounting of incoming energy, outgoing energy and the change in energy storage for a snowpack for a given time. The energy is then expressed as the heat equivalent of snowmelt. The presence of cloud cover and vegetation cover
significantly affects the energy balance of a snow surface. The snowpack is characterized by: Water equivalence, \( W \) (m); Energy contents, \( U \) (kJ m\(^{-2}\)); and, Age of snow, which is only used for albedo calculations. Water equivalence includes any liquid water present in the snowpack. The energy content \( U \) is relative to a reference state of water at 0\(^{\circ}\)C in the ace phase. \( U > 0 \) means the snowpack (if any) is isothermal with some liquid content and \( U < 0 \) can be used to calculate the snowpack average temperature \( T \) (ºC). The model is designed following inputs: Air temperature, \( T_a \) (ºC); Wind speed, \( V \) (m/s); Relative humidity, \( RH \) (%); Precipitation, \( Y \) (m/hr); and Incoming solar radiation; \( Q_{si} \) (kJ m\(^{-2}\) h\(^{-1}\)). Given the state variables \( U \) and \( W \), their evolution in time is determined by solving the following energy and mass balance equations:

\[
\frac{dU}{dt} = \sum Q_i + \sum Q_o + \sum Q_e + \sum Q_s + Q_l - Q_n \quad \ldots (1)
\]

\[
\frac{dW}{dt} = Y - M_r - E \quad \ldots (2)
\]

where \( Q_i \) net shortwave radiation; \( Q_{in} \) incoming longwave radiation; \( Q_{ou} \) outgoing longwave radiation; \( Q_p \) advected heat from precipitation; \( Q_s \) ground heat flux; \( Q_l \) latent heat flux due to sublimation/ condensation; and \( Q_m \) advected heat removed by melt water. In Eq. (2) (all in m h\(^{-1}\) of water equivalence); \( Y=\)precipitation rate, \( M_r=\)meltwater outflow from snowpack and \( E=\)sublimation from the snowpack. A computer code was made based on Runge-Kutta method to solve set of non-linear Eqs (1) and (2). Time step was chosen as one hour in the numerical solution.

**Shortwave Radiation**

Net shortwave radiation is calculated as:

\[
Q_{si} = Q_{si} (1 - \alpha) \quad \ldots (3)
\]

where \( Q_{si} \) is the incoming shortwave radiation that is directly measured and \( \alpha \) is snow albedo, which is calculated\(^{\text{10}}\) as a function of snow surface age and solar illumination angle. The age of the snow surface is retained as a state variable, and is updated with each time step, dependent on snow surface temperature and snowfall. Reflectance is computed for two bands; visible (< 0.7 \( \mu \)m) and near infrared (> 0.7 \( \mu \)m). Then albedo is taken as the average of the two reflectances:

\[
\alpha = \frac{\alpha_{sd} + \alpha_{ird}}{2} \quad \ldots (4)
\]

Where:

\[
\alpha_{sd} = (1 - C_0 F_{age}) \alpha_{vo} \quad \ldots (5)
\]

\[
\alpha_{ird} = (1 - C_i F_{age}) \alpha_{iro} \quad \ldots (6)
\]

\( \alpha_{sd} \) and \( \alpha_{ird} \) represent diffuse reflectance in the visible and near infrared bands respectively. \( F_{age} \) is function to account for aging of the snow surface. \( C_0=(0.2) \) and \( C_i=(0.5) \) are parameters that quantify the sensitivity of respective band albedo to snow surface aging (grain size growth), and \( \alpha_{vo} \) and \( \alpha_{iro} \) are fresh snow reflectance in each band.

\[
F_{age} = \Delta \tau / (1 + \Delta \tau) \quad \ldots (7)
\]

where \( \tau \) is a non-dimensional snow surface age that is incremented at each time step by quantity designed to emulate the effect of growth of surface grain sizes:

\[
\Delta \tau = \frac{r_1 + r_2 + r_3}{\tau_0} \Delta t \quad \ldots (8)
\]

Here \( \Delta t \) is the time step in seconds with \( \tau_0=10^6 \). The \( r_1 \) is a parameter dependent on snow surface temperature, \( T_s \) (ºK), which represents the effect of grain growth to vapor diffusion.

\[
r_1 = \exp \left[ 5000 \left( \frac{1}{273.16} - \frac{1}{T_s} \right) \right] \quad \ldots (9)
\]

\( r_2 \) represents the additional effect near and at freezing point due to melt and refreeze:

\[
r_2 = \min(r_1^{10}, 1) \quad \ldots (10)
\]

\( r_3=0.03 \), which represents the effect of dirt and soot.

**Longwave Radiation**

Incoming long wave radiation is intended to be a model input. However, where this is not available it is estimated based on air temperature \( T_a \) in ºK) using the Stefan-Boltzmann equation with air emissivity \( (\varepsilon_a) \) based on air vapor pressure \( (e_a \text{ in Pa}) \), air temperature and cloud cover.
\[ Q_{li} = \varepsilon_a \sigma T_a^4 \]  

... (11)

Outgoing long wave radiation is

\[ Q_{lo} = \varepsilon_s \sigma \epsilon_s T_s \]  

... (12)

where \( \varepsilon_s \) (0.97-1, here 0.99) is emissivity, \( \sigma \) the Stefan Boltzmann constant and \( T_s \) is snow surface temperature (K).

**Adveced Heat from Precipitation**

Measured precipitation rate \( Y \) (=\( Y_y + Y_k \)), is partitioned into rain \( Y_y \) \( (T_a \geq T_y = 3^\circ C) \), and snow \( Y_k \) \( (T_a \leq T_y = -1^\circ C) \), (both in terms of water equivalence depth) using the following rule based on air temperature \( T_a \), where \( T_y \) is a threshold air temperature, above which all precipitation is rain, and \( T_k \) a threshold air temperature, below which all precipitation is snow.  

The temperature of rain is taken as the greater of the air temperature and freezing point and the temperature of snow is the lesser of air temperature and freezing point. The advected heat is the energy required to convert this precipitation to the reference state (0°C ice phase).

\[ Q_p = Y_s \left[ h_a \rho_a + C_a \rho_a \max(T_a,0) \right] + Y_k C_a \rho_a \min(T_a,0) \]  

... (13)

**Ground Heat Flux**

Since ground heat flux is not known, it is taken to be zero.

**Sensible Heat Flux**

Sensible heat flux is given as follows.

\[ Q_h = K \rho_a C_a (T_a - T_s) \]  

... (14)

Sensible heat flux between the snow surface and air above is modeled using the concept of flux proportional to temperature gradients. Considering a unit volume of air, the heat content is \( \rho_a C_a T_a \), where \( \rho_a \) is air density (determined from atmospheric pressure and temperature), \( C_a \) air specific heat capacity, and \( V \) wind speed.

\[ \rho_a = \frac{P_a}{R_a T_a} \]  

... (15)

\( K \), turbulent transfer coefficients are functions of surface roughness and nominal measurement height for air temperature and wind speed. On comparing the usual expressions for turbulent transfer in a logarithmic boundary layer profile for neutral condition, the following expression is obtained:

\[ K = \left( \frac{k}{\ln(Z/z_o)} \right)^2 \]  

... (16)

where \( Z \) is height that is measure wind speed; \( z_o \) is roughness height at which the logarithmic boundary layer profile predicts zero velocity; and \( k \) is von Karman’s constant.

**Latent Heat Flux**

Latent heat fluxes between the snow surface and air are modeled using the concept of flux proportional to vapor pressure gradients as follows:

\[ Q_e = K V \frac{0.622 h_d}{R_a T_a} (e_a - e_s) \]  

... (17)

where \( e_s \) is vapor pressure at snow surface, snow assumed saturated at \( T_s \) and calculated using a polynomial approximation; \( e_a \) is air vapor pressure, \( R_d \) is dry gas constant, and \( h \), latent heat of sublimation.

\[ e_s = 0.6108 e^{(17.27 T_s)/(T_s+237.3)} \]  

... (18)

\[ e_a = e_s \cdot RH \]  

... (19)

**Snow Surface Temperature**

Since snow is a relatively good insulator, \( T_s \) is in general different from \( T \). This difference is accounted for using an equilibrium approach that balances energy fluxes at the snow surface. Heat conduction into the snow is calculated using the temperature gradient and thermal diffusivity of snow.

\[ Q = K_s \rho_s C_s (T_s - T_a) \]  

... (20)

where \( K_s \) and termed snow surface conductance are analogous to the heat and vapor conductance. A value of \( K_s \) is obtained by assuming a constant.

Then assuming equilibrium at the surface, the surface energy balance gives:

\[ Q = Q_{al} + Q_{hi} + Q_{lo} + Q_{li} + Q_{ho} + Q_{ho}(T_a) + Q_h(T_s) - Q_e(T_s) \]  

... (21)

Analogous to the derivation of Penman equation for evaporation, the functions of \( T_s \) in this energy balance equation are linearized about a reference temperature \( T^* \), and equation is solved for \( T_s \):
where $\Delta = \frac{de}{dT}$ and all temperatures are absolute. This equation is used in an iterative procedure with an initial estimate $T^* = T_i$, in each iteration replacing $T^*$ by the latest $T_i$. The procedure converges to a final $T_f$, which, if less than freezing, is used to calculate surface energy fluxes. If the final $T_f$ is greater than freezing, it means that the energy input to the snow surface cannot be balanced by thermal conduction into the snow. Surface melt will occur and the infiltration of melt water will account for the energy difference and $T_f$ is then set to $0^\circ C$.

### Adveected Heat Removed by Melt Water

The energy content state variable $U$ determines the liquid content of the snowpack. This result, together with Darcy’s law for flow through porous media, is used to determine the outflow rate.

$$M_f = K_{sat}S^3$$  

... (23)

where $K_{sat}$ is the snow saturated hydraulic conductivity and $S$ is the relative saturation in excess of water retained by capillary forces. $S$ is given by:

$$S = \left( \frac{L_i}{1-L_i} - L_r \right) \left( \frac{\rho_{w}}{\rho_{s}} - \frac{\rho_{w}}{\rho_{s}} - L_i \right)$$  

... (24)

$$L_f = U h_j W$$  

... (25)

where $L_f$ denotes the mass fraction of total snowpack (liquid and ice) that is liquid, $L_r$ the capillary retention as a fraction of the solid matrix water equivalence and $\rho_i$ the density of ice.

$$Q_m = \rho_{w} h_j M_f$$  

... (26)

### Results of the Energy Balance Model

EBM was run for nine seasons (1 March - 31 May) of 1987-1995, the snowmelt seasons for this area. Intervals of EBM constants that are snow surface conductance ($K_s$), snow surface saturated conductance ($K_{sat}$), liquid holding capacity of snow ($L_k$), fresh snow reflectance in visible band ($\alpha_v$), and diffuse reflectance in the near infrared bands ($\alpha_{dvi}$), are determined for each period. Values of intervals of $K_s$, $K_{sat}$, $L_k$, $\alpha_v$, $\alpha_{dvi}$, are calibrated with daily mean flow data of Kirkgöze discharge gauging station of State Hydraulic Works (DSI) for all periods (Table 1).

The curves of simulated and observed daily discharges (Figs 2 & 3) are similar but time lag are determined as a three day discharge due to particularly catchments routing, water retention and movement, hydraulic conductivity, refrozen snow and soil conditions that changes from point to point in the area.

High coefficients of correlations are computed for simulated and observed hydrographs of daily discharges for each year for the entire periods. In some parts of hydrographs, different daily discharges are determined, because of different parameters in a day. For 21, 24 and 28 April 1987, different daily discharges were obtained by EBM. Hourly changing of radiation and temperatures of these days are plotted to evaluate the difference (Fig 4 a, b). Changing of radiation is normal when it compared with other days and also changing temperature is normal. The reasons for different calculations of snowmelt may be the effect of topographic parameters (elevation, aspect, slope) and meteorological variables (prevailing wind direction, temperature).

In addition to these, daily mean values of input parameters for March 1987 are as follows: temperature, -5.9 ºC; radiation, 705 kJ/m²; velocity of wind, 1.997 km/h; humidity, 0.753; and, rainfall, 0.001056 m/day. All these values are too small to produce snowmelt for this month. After March, beginning of the first half of the April 1987, the calculated snowmelt begins to increase. During April and May (respectively), daily mean values of input parameters are as follows: temperature, 2.83, 12.16ºC; radiation, 782.83, 962.98 kJ/m²; velocity of wind, 2.53, 2.88 km/h; humidity, 70, 51 %; and, rainfall, 0.0016, 0.0014 m/day.

To judge accuracy of simulated and observed streamflows, linear scale plots of simulated and observed hydrographs of daily discharges for each year for entire calibration and verification periods were made (Fig 5). The high coefficients of correlations of nine seasons were calculated (Table 2).

### Table 1—Values of intervals of the $K_s$, $K_{sat}$, $L_k$, $\alpha_v$, $\alpha_{dvi}$ constants

<table>
<thead>
<tr>
<th>Values</th>
<th>$K_s$</th>
<th>$K_{sat}$</th>
<th>$L_k$</th>
<th>$\alpha_v$</th>
<th>$\alpha_{dvi}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reported in literature$^j$</td>
<td>0.005–0.5</td>
<td>10–50</td>
<td>0.01–0.1</td>
<td>0.5–0.95</td>
<td>0.4–0.9</td>
</tr>
<tr>
<td>Calculated for EBM$^{15}$</td>
<td>0.005–0.5</td>
<td>10–30</td>
<td>0.03–0.05</td>
<td>0.75–0.85</td>
<td>0.50–0.60</td>
</tr>
</tbody>
</table>

$^j$ Values of intervals of the $K_s$, $K_{sat}$, $L_k$, $\alpha_v$, $\alpha_{dvi}$, are calibrated with daily mean flow data of Kirkgöze discharge gauging station of State Hydraulic Works (DSI) for all periods (Table 1).
Fig 2 — Simulated and observed daily discharges

Fig 3 — Using EBM comparison of computed and observed streamflow-water (1987)
Results of the Regression Model

A multiple LRM is established only for 1987 season using 552 data points. The criterion variable is the observed daily discharge ($Q_c$). Five predictor variables are mean daily temperature ($T$), wind velocity ($V$), relative humidity ($RH$), radiation ($Rd$) and precipitation ($Y$).

The correlation matrix (Table 3) is a 6x6 symmetric matrix. The determinant is computed to be 0.0552; such a low value indicates that intercorrelation is high. Standard error is 4.06 and degree of freedom is 86. Regression equation is obtained using the least square method and is as follows:

$$Q_c = 16.117 + 0.5667T + 0.109V - 18.388RH + 0.001593Rd + 220.291Y$$

Observed values and predicted values from LRM are plotted with time (Fig 6). The computed coefficient of correlation ($R$) is 0.87 ($R^2=0.757$), which means that the equation involving five predictor variables explains 75.7 percent of the variation in the snowmelt discharges ($Q_c$). $T$ is the most effective variable in the LRM (Table 3). The

<table>
<thead>
<tr>
<th>Years</th>
<th>Total simulated discharges</th>
<th>Total observed discharges</th>
<th>Coefficient of Determination $R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1987</td>
<td>699.150</td>
<td>675.827</td>
<td>0.9913</td>
</tr>
<tr>
<td>1988</td>
<td>508.500</td>
<td>508.214</td>
<td>0.9423</td>
</tr>
<tr>
<td>1989</td>
<td>283.700</td>
<td>310.274</td>
<td>0.8288</td>
</tr>
<tr>
<td>1990</td>
<td>576.670</td>
<td>517.253</td>
<td>0.9738</td>
</tr>
<tr>
<td>1991</td>
<td>529.340</td>
<td>562.412</td>
<td>0.9918</td>
</tr>
<tr>
<td>1992</td>
<td>211.220</td>
<td>212.990</td>
<td>0.9450</td>
</tr>
<tr>
<td>1993</td>
<td>557.330</td>
<td>521.330</td>
<td>0.8988</td>
</tr>
<tr>
<td>1994</td>
<td>318.140</td>
<td>289.379</td>
<td>0.9442</td>
</tr>
<tr>
<td>1995</td>
<td>312.420</td>
<td>327.833</td>
<td>0.9907</td>
</tr>
</tbody>
</table>

Table 2—Simulated and observed discharges and coefficient of determinations for 1987-1995

Fig 5 — Relationship of simulated total daily discharges and observed total daily discharges

Fig 4 — Variations in melt factors on 21, 24, 28 April 1987: a) Temperature; b) Radiation

Fig 6 — Plot of observed and predicted discharge values.
coefficient of correlation for temperature is 0.834. It means that $T$ explains 69.6 percent of the variation in $Q_c$ by itself. Thus, $V$, $RH$, $Rd$ and $Y$ on snowmelt results an increase of only 6.1 percent in $Q_c$ of the total variation (75.7%).

**Conclusions**

EBM and LRM for prediction of daily discharges of snowmelt were applied in eastern part of Turkey (Karasu-Kirköge Basin). Time delay in runoff generation is determined as three days due to catchment routing, water retention and movement hydraulic conductivity, refrozen snow and soil conditions, which changes from point to point in the area. Values of intervals of the model constants ($K_S$, $K_{Sat}$, $L_k$, $\alpha_{vvo}$, $\alpha_{vvo}$) were determined for all periods for EBM. High correlation coefficients ($R=0.88-0.99$ $R^2=0.77-0.98$) were computed for simulated and observed hydrographs of daily discharges for each year. In some parts of hydrographs, different daily discharges were calculated due to the effect of topographic parameters (elevation, aspect and slope) and meteorological variables (prevailing wind direction, temperature). The changing of daily temperature and radiation is discussed. While temperature is under 0°C in the middle of the night to early noon, especially between 22 and 10 h, there is no snowmelt and the radiation does not affect to produce snowmelt between 18 and 5 h. Relative humidity is inverse effective parameter to produce snowmelt. Wind velocity and direction of wind are also effective to produce snowmelt, especially from southwester.

The computed coefficient of correlation for regression model is 0.87 ($R^2=0.757$), which means that the equation involving five predictor variables, which explains 75.7 percent of total variation in the snowmelt discharges ($Q_c$). The order of importance of
the predictor variables is obtained as 0.84, -0.75, 0.47, 0.39 and 0.02 for $T$, $RH$, $Rd$, $V$ and $Y$, respectively for the regression model. Consequently, EBM is more accurate than LRM for prediction of snowmelt in eastern part of Turkey due to higher correlation coefficient.

References
1. Tarboton D G & Luce C H, Utah energy balance snow accumulation and melt model (Utah Water Research Laboratory, Utah State Univ, Utah) 1996.