Development of closed form design formulae for aperture coupled microstrip antenna

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Closed form equations are presented for designing aperture coupled microstrip antennas to ensure impedance matching with the feed network with a low return loss over a wide frequency band. The features like compactness, integrability with printed circuits and shielding of the radiating patch from the radiation emanating from the feed structure etc. make these antennas attractive for present day scientific and industrial applications in fields like mobile computing and communication.

Keywords: Microstrip, Antenna, Impedance matching

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Introduction

An aperture coupled microstrip antenna (Fig. 1) consists of a rectangular patch of dimensions $a \times b$ printed on a substrate of thickness $h$ and dielectric constant $\varepsilon_{ra}$. A microstrip line feeds the microstrip patch through an aperture or slot in the common ground plane. The aperture is of dimensions $l_a \times l_w$ and centered at $(X_o, Y_o)$. The width of the microstrip line is $W$ and it is printed on a substrate described by thickness $t$ and dielectric constant $\varepsilon_{rf}$. The characteristic impedance of the microstrip line is denoted by $Z_{om}$ and that of the slot line corresponding to the coupling slot by $Z_{os}$. Coupling of the slot to the dominant mode of the patch and the microstrip line occurs because the slot interrupts the longitudinal current flow in them.

The coupling slot is nearly centered with respect to the patch where the magnetic field of the patch is maximum. This is done to enhance coupling between the magnetic field of the patch and the equivalent magnetic current near the slot. The coupling amplitude can be determined from the following expression:

\[
\text{Coupling} \approx \iiint_{V} (M \cdot H) dv \approx \sin \left( \frac{\pi x_0}{l_a} \right) \quad \cdots (1)
\]

where $x_0$ is the offset of the slot from the patch edge.

The biggest problem encountered by engineers, while working with aperture coupled microstrip antennas, is that no closed form design formula is available. The paper presents closed form design expressions for optimal design of the feed when the patch is designed according to the relations available readily.

Theory for Structural Analysis

In a simplified equivalent circuit of an aperture-coupled microstrip antenna (Figs 2 & 3), the patch is characterized by admittance, $Y_{patch}$, and the aperture by an admittance, $Y_{ap}$. The patch admittance is determined at the centre of the slot.

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Fig. 1—Expanded view of an aperture-coupled microstrip antenna
In feed configuration, the patch antenna appears in series with the feed because of slot coupling. The nonresonant slot is represented as an inductor in series with the $R-L-C$ network representing the patch resonator. The open circuited microstrip stub of length $L_s$ can be replaced by a shunt capacitor $C_s$ such that
\[
\frac{1}{\omega C_s} = Z_0 \tan(\beta L_s)
\]
where $Z_0$ is the characteristic impedance and $\beta$ is the propagation constant of the microstrip feed line.

The coupling of the patch to the aperture is described by an impedance transformer of turns ratio $n_1 = L_o/b$. If $Z_1$ and $Z_2$ are the impedances looking toward the left and right of the aperture (Fig. 3), then
\[
Z_{patch} = Z_1 + Z_2 = \frac{1}{Y_1} + \frac{1}{Y_2}
\]

where
\begin{align*}
Y_1 &= Y_o\left\{\frac{(G_r+jB_{open}) + jY_o \tan(\beta L_1)}{(G_r+jB_{open}) + jY_o \tan(\beta L_1)}\right\} \\
L_1 &= X_o \\
Y_2 &= Y_o\left\{\frac{(G_r+jB_{open}) + jY_o \tan(\beta L_2)}{(G_r+jB_{open}) + jY_o \tan(\beta L_2)}\right\} \\
L_2 &= a - L_1
\end{align*}

Here $(Y_o, \beta)$ characterize the rectangular patch antenna as a microstrip line of width $b$, and $(G_r+jB_{open})$ is the edge admittance of the patch. The value of $Y_{ap}$ can be obtained from transmission line model of a slot and is given by
\[
Y_{ap} = -j2Y_o \cot(\beta s l_o/2)
\]

A transformer of turns ratio $n_2$, used to describe the coupling of the patch to the microstrip feed line is modeled from the discontinuity $\Delta V$ in modal voltage of the feed microstrip line, that is $n_2 = \Delta V/V_o$ where $V_o$ is the slot voltage. Thus
\[
n_2 = \frac{(J_0(\beta s w/2) J_0(\beta m w/2))}{(\beta^2_m - \beta^2_s \beta_m^2)} \left[\frac{\beta_m^2 k_2 e_{rf}}{(k_2 e_{rf} \cos(k_2 h) - k_1 \sin(k_2 h)) + (\beta_m^2 k_1 / k_2 \cos(k_2 h) + k_2 \sin(k_2 h))}\right]
\]

where, $J_0(.)$ is the zeroth order Bessel function; $k_1 = k_{o\beta} \sqrt{\epsilon_{res} - \epsilon_{rem}} / \sqrt{\epsilon_{res} + \epsilon_{rem} - 1}$; $k_2 = k_{o\beta} \sqrt{\epsilon_{res} + \epsilon_{rem} - 1} / \sqrt{\epsilon_{res} + \epsilon_{rem} - 1}$; and $\beta_s = k_{o\beta} \sqrt{\epsilon_{res}}$ and $\beta_m = k_{o\beta} \sqrt{\epsilon_{rem}}$.

Here $(w, C_{rem}, \beta_m, Z_{om})$ are microstrip line parameters and $(l_o, C_{res}, \beta_o, Y_{om})$ are the corresponding slot line parameters. Taking into account the impedance of the microstrip open circuited stub of length $l_o$, the input impedance of the antenna at the centre of the slot is given by
\[
Z_{in} = \left[n_2 + (n_1^2 Y_{patch} + Y_{ap}) - jZ_{om} \cot(\beta ml_o)\right]
\]

Setting $n_2^2 B_{patch} + B_{ap} = 0$ for resonance and using Eqs (3) and (4) yields the following condition for resonance:
\[
B_{patch} = -B_{ap} / n_1^2 \approx 4b^2 / (Z_{om} \beta s l_o^3)
\]

Therefore increasing $l_o$ results in a decrease in $B_{patch}$ and consequently the resonant frequency decreases.

**Development of Closed Form Design Formulae**

In the analysis, series expansion of Bessel function of first kind was taken as
\[ J_n(x) = \frac{1}{2^n \sqrt{n+1}} \sum_{r=0}^{n} (-1)^r \frac{x^{n+2r}}{r! (n+1)(n+2) \ldots \ldots \ldots (n+r)} \]  
\[ \cdots \) (10) \]

where \( r = 0, 1, 2, 3, 4, \ldots \), and terms involving third or higher degrees of the argument were neglected.

Further for inverse cotangent function, first six terms of corresponding Maclaurin Series are considered as

\[ \cot^{-1} Z = \left( \prod \frac{1}{Z} - Z + \frac{Z^3}{3} - \frac{Z^5}{5} + \frac{Z^7}{7} - \frac{Z^9}{9} + \ldots \ldots \right) \]  
\[ \cdots \) (11) \]

This, according to Eqs (2)-(7), gives

\[ \left( \frac{l_a}{b} \right)^2 \frac{Y_{\text{patch}} - j(2 \cot(B \beta_s l_a / 2))/A}{(1 - \frac{\beta_s^2 w^2}{16} - \frac{\beta_m^2 l_w^2}{16})^2 (X_1 + X_2)^2} / Z_{\text{in}} \]  
\[ \cdots \) (15a) \]

where, \( X_1 = \beta_m^2 k_3 C_{\text{ef}} \left[ \frac{k_2 C_e \cos(k_1 h) - k_3 \sin(k_1 h)}{k_1 \cos(k_1 h) + k_3 \sin(k_1 h)} \right] \), and

\[ X_2 = \beta_m^2 k_1 / \left( k_1 \cos(k_1 h) + k_3 \sin(k_1 h) \right) \].

The expressions for the characteristic impedance and guide wavelength of a slot line on a substrate of low \( \varepsilon_r \) have been obtained by curve fitting of the numerical results obtained using Galerkin’s method in the Fourier transform domain. These expressions are for \( 0.075 \leq w/\lambda_0 \leq 1.0 \) and \( 2.22 \leq \varepsilon_r \leq 3.8 \).

\[ \lambda_s/\lambda_o = 1.194 - 0.24 \ln \varepsilon_r - \left[ (0.62 \varepsilon_r^{0.835} (w/\lambda_o)^{0.48}) / (1.344 + w/\lambda_o - 0.0617 \ln(h/\lambda_o)) \right] \]  
\[ \cdots \) (12) \]

average error = 0.69%, max error = -2.6% (evaluated at two points, for \( w/\lambda_o > 0.8 \)) and

\[ Z_{\text{as}} = 133 + 10.34(Cr_{1.8})^2 + 2.87(2.96 + (Cr_{1.582})^2) \left[ (l_w/\lambda_o + 2.32)(19.1 - (Cr_{1.5})^2) \right]^{1/2} \]  
\[ \left( 684.45h/\lambda_o \right)(Cr_{1.35})^{2.3} + 13.23((C\varepsilon_{1.722}) l_w/\lambda_o)^2 \]  
\[ \cdots \) (13) \]

average error = 1.9 %, \( \left| \text{max error} \right| = 5.4 \% \) (evaluated at three points, for \( w/\lambda_o > 0.8 \)). Therefore

\[ Z_{\text{in}} = \left( \frac{1 - \beta_m^2 w^2}{16} - \frac{\beta_m^2 l_w^2}{16} \right)^2 (X_1 + X_2)^2 / \left( l_a/l_b \right)^2 Y_{\text{patch}} - j^2 \cot(B \beta_s l_a / 2)/Z_{\text{as}} \]  
\[ \cdots \) (14) \]

Equating real and imaginary parts of \( Z_{\text{in}} \) as given by Eq. (14) to those of the characteristic impedance of the feed line, required design equations are obtained as

\[ l_a = (2n + 1) \prod / \beta_s \]  
\[ l_w = 4/\beta_m ((1 - (\beta_w / 4)^2) / B)^2 \]  
\[ \cdots \) (15b) \]

Here \( B = \left( \frac{l_a}{b} \right)^2 Y_{\text{patch}} Z_{\text{in}} \) \( (X_1 + X_2)^2 \)

and \( l_s = 1 / \beta_m \left( \cot(Y/Z_{\text{om}}) \right) \) \[ \cdots \) (15c) \]

where \( Y \) and \( Z_{\text{om}} \) are the values of the real and imaginary parts of the microstrip feed line characteristic impedance and the other parameters are as defined earlier.

Thus a set of closed form design equations is obtained for the aperture length \((l_a)\), width \((l_w)\) and corresponding stub length \((l_s)\) of the aperture coupled microstrip patch antenna.

Validation of Design Procedure

In order to check the validity of the design equations developed, an aperture coupled rectangular microstrip patch antenna was designed for operation at \( K_s \) band with the following parameters: for antenna substrate, thickness=1.5875 mm, \( \varepsilon_r = 2.4 \); and, for feed line substrate, thickness=0.79375 mm, \( \varepsilon_r = 2.4 \). The corresponding resonant patch length and width are 3.70 mm and 5.60 mm respectively. For a 50 Ohm feed line, the aperture parameters, according to the design equations developed in this paper, are obtained as \( l_a = 5.234 \text{ mm}, \ l_s = 2.79 \text{ mm} \) and \( l_w = 2.98 \text{ mm} \).

Computer simulation using IE3D software package was run for the designed antenna, which yielded a \(- 9.5 \text{ dB (VSWR<2:1)}\) bandwidth of 1.853 GHz and undistorted radiation pattern providing half power beam width of 79.284 degrees. The simulated plots for return loss vs frequency and radiation pattern at resonance are shown in Figs 4 and 5 respectively.

To extend the validation, further investigations were carried out in array environment. A \( 4 \times 4 \) uniform array was investigated at a frequency band centered around 18.5 GHz with inter-element spacing of 12 mm along both orthogonal directions (Fig. 6). The individual antennas had configuration as mentioned earlier, obtained by using the design procedure developed. Since the memory requirement for computer simulation of such a large structure is prohibitively large, the entire array was fabricated on a single substrate and tested experimentally. For
Fig. 4—Return loss vs. frequency plot for single aperture coupled patch (simulated)

Fig. 5—Radiation pattern for single aperture coupled patch (simulated)

Fig. 6—4×4 array of aperture coupled microstrip patches

Fig. 7—Return loss vs. frequency plot for aperture-coupled microstrip array (experimental)

Fig. 8—Radiation pattern for aperture-coupled microstrip array (experimental)
return loss (Fig. 7), measured values show very good impedance matching with the feed indicated by a less than –10 dB return loss over 17-20 GHz. The measured radiation pattern at resonance (Fig. 8) expectedly provides half power beam width of about 25 degrees and side lobe level of – 8.5 dB.

Conclusions

A set of closed form design formulae is presented for the feed to an aperture coupled microstrip antenna. Its validity is also checked through simulation of a single patch as well as fabrication and measurement in an array environment. It is established that usage of the relations developed results in wideband performance with very low return loss, thereby indicating a good design from practical point of view. However, this work was carried out for rectangular coupling slots only. Closed form design equations for coupling slots of other shapes can be formulated in a similar fashion.

References

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