Study of $B \to \pi \pi$ puzzle in the standard model

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Decays of B-mesons is one of the most promising research areas to obtain information about the flavour structure of the standard model (SM) as well as to get a track of new physics (NP) path. The discrepancies among the measurements of branching ratios and CP asymmetries of $B \to \pi \pi$ decays ($B^+ \to \pi^+ \pi^0$, $B^0 \to \pi^+ \pi^-$ and $B^0 \to \pi^0 \pi^0$) such as large direct CP asymmetry for $B^0 \to \pi^+ \pi^-$ mode and large branching ratio for $B^0 \to \pi^0 \pi^0$ mode, originate the $B \to \pi \pi$ puzzle. According to diagrammatic approach of this $b \to d$ transition the ratio of color-suppressed amplitude ($C$) to color-allowed one ($T$) demands small ratio which is in contradiction with the SM approach. To make out the ambiguity we need to scrutinize the decays with different topological amplitudes. In this paper, the decays are studied in the SM by taking different values of $C_T$ as constraints. We find that larger ratio of $C_T$ is explained successfully in the SM but the lower ratio is not for which NP is needed.

Keywords: Standard model, $B \to \pi \pi$ puzzle, Color-suppressed amplitude, Color-allowed amplitude

1 Introduction

In recent years, B-meson decays play an important role for theorists as well as experimentalists to study some disagreements between the standard model predictions and experimental observations. Different pioneering measurements of B-factories with BaBar & Belle experiments as well as Tevatron provide latest experimental state of B-decays governed by the dedicated B-decays experiment of the large hadron collider LHCb. There are three decays of $B \to \pi \pi$ system: $B_d^+ \to \pi^+ \pi^0$, $B_d^0 \to \pi^+ \pi^-$ and $B_d^0 \to \pi^0 \pi^0$. Different observables of above three decays have been measured: three branching ratios and three direct CP asymmetries. After the measurements, some discrepancies are observed between theoretical and experimental predictions, this is known as “$B \to \pi \pi$ puzzle”. In literature, it is observed that the inconsistency with the SM predictions can be explained by performing a full fit with the obtained data as constraints. But every time the fit provides poor value. Then the value of CKM phase angle $\gamma$ is added as a constraint and it recovers the picture of SM explanation upto some extent.

In this work, we have used the updated experimental results to perform the fitting. While observing the fitted results we get a specific region where SM is able to explain the puzzle and in other region the SM results become terrible. It demands the entry of new physics (NP) for proper explanation.

2 Amplitude Structure and Observables of $B \to \pi \pi$ Decays

To present the $B \to \pi \pi$ decays in terms of topological amplitudes, we include three types of amplitudes – a “tree” contribution $t$, a “color-suppressed” contribution $c$ and a “penguin” contribution $p$. These amplitudes contain both leading order and penguin contributions. Taking the gluonic amplitude very small in compare to QCD penguin contribution, the amplitudes can be parameterized as:

$$t = T + P^C_{EW}, \quad c = C + P^C_{EW} \quad \text{and} \quad p = P - \frac{1}{3} P^C_{EW}, \quad \ldots \ (1)$$

where, $T$, $C$, $P^C_{EW}$ and $P^C_{EW}$ are both color-allowed, color-suppressed tree and electroweak penguin amplitudes, respectively. The diagrammatic approach of $B \to \pi \pi$ decay amplitudes can be represented as:

$$A^{+0} = t + c, \quad A^{-} = c + p \quad \text{and} \quad A^{00} = c - p \quad \ldots \ (2)$$

Considering the SM approach, we have neglected the annihilation, exchange, penguin-annihilation
diagrams and used $SU(3)$ flavour symmetry\textsuperscript{7-9} to relate $P_{EW}$ and $P_{EW}^C$ to $T$ and $C$ as:

$$P_{EW} = \frac{3 C_9}{2 C_1} R_T \quad \text{and} \quad P_{EW}^C = \frac{3 C_9}{2 C_1} R_C,$$

where, $R = \left| \frac{V_{tb}^* V_{td}}{V_{ub}^* V_{ud}} \right| = 2.19$ and $C_9$ and $C_1$ are Wilson coefficients\textsuperscript{10}.

Fixing the phase\textsuperscript{2} we consider:

$$\exp(i \delta_T + \gamma) \quad \text{and} \quad P = -\bar{P}. \quad \text{This negative sign comes for the weak phase} \pi \text{ associated with } P.$$

Taking the above consideration into account branching ratios and CP asymmetries $A_{CP}$ are represented essentially in terms of diagrams and phases. Experimental values of these observables\textsuperscript{3,11} are given in Table 1.

### 3 Naive Explanation of $B \rightarrow \pi \pi$ Puzzle

The main discrepancies observed in $B \rightarrow \pi \pi$ decays are as follows\textsuperscript{1}:

i) the direct CP asymmetry for $B^0 \rightarrow \pi^+ \pi^-$ is very large in comparison to the SM value and

ii) the branching ratio has larger value for $B^0 \rightarrow \pi^0 \pi^0$ than theoretical expectations.

This is the naive $B \rightarrow \pi \pi$ puzzle which is the centre of interest for decades.

### 4 Statistical Point of View to Analyse

To solve the puzzle, we need to change its usual pattern of representation. In this regard, we will study the decays in a statistical view by performing $\chi^2$ fit including some theoretical inputs. The value of $\chi^2$ is used to find the deviation of experimental values from the expected values\textsuperscript{5}. It can be defined as:

$$\chi^2 = \sum_i \left( \frac{f_i^{th} - f_i^{exp}}{\Delta f_i} \right)^2, \quad ... (3)$$

where, $f_i^{th}, f_i^{exp}$ and $\Delta f_i$ represent theoretical expressions of various observables, experimental values and the experimental errors of corresponding observables, respectively. By minimising this $\chi^2$ value we obtain several best fit values. The probability distribution for $\chi^2$ can be calculated\textsuperscript{2,5} by:

$$P(\chi^2) = \int \frac{1}{\sqrt{2 \pi n}} \left( \chi^2 \right)^{n/2 - 1/2} e^{-\chi^2/2} d(\chi^2) \quad : ... (4)$$

This probability depends on the only parameter 'n', i.e., degrees of freedom.

### 5 The SM Fits

In order to calculate the value of $\chi^2$ as given in Eq. 3, we need $f_i^{th}$. For this, we use Eq. (1 and 2) in terms of diagrams and phases. So we have 6 observables in terms of amplitudes. Hence, we get the $\chi^2$ expression as a function of diagrams and phases. By minimising this $\chi^2$ expression we get different set of best fit values for several theoretical inputs. Here, we have 3 branching ratios, 3 CP asymmetries and $\gamma$ as known parameters. And the three magnitudes ($C$, $P$, $T$) and two strong phases are unknown parameters. Thus, we have more known parameters than unknown parameters, so it is possible to fit the parameters. To study the puzzle in terms of color-suppressed and color-allowed amplitudes, we will observe the contribution of the amplitudes by introducing an additional theoretical input in terms of $C_T$. We have performed fits for several values of $C_T$. The fitting is started in descending order of $C_T$ values. The obtained best fit values are recorded in the Tables 2–5.

From the p-value of Table 2 it can be said that it is an acceptable fit. In similar manner with same
constraints we have performed other fits which are recorded in Tables 3-5 gradually increasing the contribution of colour-allowed amplitude ($T$) in $B \to \pi \pi$ decays.

### 6 Conclusions

For the lower value of $\frac{\gamma}{T}$, the SM cannot explain $B \to \pi \pi$ puzzle properly as it has low probability whereas for the higher value of $\frac{\gamma}{T}$ the puzzle is explained successfully. So we can say that SM cannot explain the $B \to \pi \pi$ puzzle fully. And we realise to introduce NP in terms of topological amplitudes to get an acceptable solution of the puzzle. On the other hand, the quality of fits can be improved with enhancement of number of constraints by including more observables.

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### References