Evaluation of submarine motions under irregular ocean waves by panel method

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This paper utilizes the Panel method to evaluate the effect of submerged depth in ocean waves on submarine motions. At the depth called the "wave base", the effects become so small that motions are almost negligible. The present research aims at recommending a safe depth for calm and stable motions of a submarine. Depth of $\lambda/2$ could be considered as an absolutely calm depth but a depth of 0.1$\lambda$ is recommended as an operationally safe and approximately calm depth for submarines. To achieve the objectives of the study, a naval submarine was analyzed at some depths accompanied by regular surface wave. By increasing the depth, the reductions in submarine motions are evaluated. The obtained results from the study might have beneficial outcomes for AUVs, and research in submersibles and naval submarines. As mentioned, the analysis is performed by Panel method and the results are compared with those of a CFD method.

[Keywords: Panel method; Irregular wave; Submarine; Motions; Maxsurf]

Introduction

Water wave is an orbital wave where particles move in orbital paths. These waves transmit energy along interfaces between two fluids of different densities. Below the surface, the circular orbital motion dies out quickly. At some depth below the surface, the circular orbits become so small that the motion becomes barely perceptible. This depth is called the "wave base", which size can be considered equaling half of a wave length ($\lambda/2$) measured from the still water level (Fig. 1). Since only wave length controls the depth of the wave base, so longer the wave, the deeper the wave base. A decrease in orbital motion as one goes deeper has many practical applications. For instance, submarines can avoid large ocean waves simply by submerging below the wave base. Even the largest storm waves become imperceptible if a submarine submerges to a depth of just 150 m\textsuperscript{1}. Floating bridges and floating oil rigs are constructed in such a way that most of their mass lies below the wave base, consequently they remain unaffected by wave motions. In fact, offshore floating airport runways have been designed drawing upon similar principles. Additionally, seasick scuba divers find relief when they submerge into the calm, motionless water below the wave base\textsuperscript{1}. Therefore, deep water is defined as depth more than $\lambda/2$. The hydrodynamic forces of ocean surface waves on submerged bodies have been studied in several different fields of engineering. Some example are as follows:

Offshore engineering

The wave impacts on such vertical and horizontal fixed cylinders as the structural members of a platform leg. Several extended studies have been conducted to analyze the diffraction around a submerged fixed cylinder. Thus Dean (1948)\textsuperscript{2} made use of a linearized potential theory to demonstrate the effects of reflection. Ursell (1949)\textsuperscript{3} and later Ogilvie

Fig. 1—Orbital motion in waves\textsuperscript{1}
(1963)\textsuperscript{4} presented the formulation of wave steepness up to the second order. Chaplin \textsuperscript{5} (1984) using experiment at method, measured the nonlinear force on a fixed horizontal cylinder beneath the waves. He analyzed the influence of the Keulegan-Carpenter number value on the harmonics of the applied force.

**Wave energy converter (WEC)**

Wave effects on the moored or prescribed motions of cylinders of energy converters just near the surface. This study is interesting if applied in offshore engineering for moored semi-submersibles\textsuperscript{6,11}. Wu \textsuperscript{(1993)} presented a formulation for calculating the forces exerted on a submerged cylinder undergoing large-amplitude motions. When the free surface condition linearized, the body surface condition is satisfied in its immediate position. The solution for the potential is stated as a multi-pole expansion. Wu obtained results for a circular cylinder in a purely vertical motion and clock-wise circular motion in a wave field (Wu, 1993).

**Submarine and submersible design**

Wave effects on the non-moored free submerged body near the free surface and at the snorkel depth. The present research study pursues the third category. In this work, we intend a safe depth for calm and stable motions of a submarine. This safe depth is not necessarily equal to wave base. In this study, a submarine design is analyzed at several depths accompanied by regular surface waves. By increasing the depth in degrees, the reduction in submarine motions is evaluated. Among the use of the results of study, one can refer to such cases as AUVs, research submersibles, and submarines. General discussion and specifications on submersible and submarine hydrodynamics are presented\textsuperscript{11,17}. As regards the submarine hydrodynamic field near the free surface effect or in snorkel depth (or periscope depth), three general considerations are presented.

**Resistance**

By focusing on the wave offering resistance to a submarine traveling below the free surface in still water (no ocean wave)\textsuperscript{18,25}.

**Dynamic in still water**

By focusing on the submarine dynamic equations and coefficients affected by free surface of water. General dynamic equations of marine vehicles and submarines\textsuperscript{26,27} are considered the most prominent and comprehensive references in these fields. Revised standard submarine equations of motion are given\textsuperscript{15,28,30}. An interesting common study about submarine control is designing a control system for a submarine running near the free surface or snorkel depth. The controller design and maneuvering in still water are discussed in several studied\textsuperscript{11,35}.

**Dynamic under surface waves (seakeeping)**

By focusing on the submarine dynamic equations under ocean wave excitations, these cases are assessed\textsuperscript{36,44}. Collective helpful experimental results for wave forces on submerged bodies at several different wave conditions are presented\textsuperscript{45}.

Finally after a literature survey, one can state that approximately all references are based on a potential flow for inviscid fluid. For modeling the 3D objects and calculating their hydrodynamic coefficients, some such methods as Strip Theory and Conformal Mapping should be exploited which are basically incompatible with the submerged body (having no water plane area). Other measures for adjusting these potential flow solutions to the submerged bodies\textsuperscript{36} has clarified that, this latter manner could be used effectively only in the early stages of the design. In these early stages, some estimated and approximated values are sufficient. For executing the next stages and gaining better and careful results with exact models the 3D shape of submarine, numerical prediction of CFD method can serve as good option. Some technical explanation of numerical methods for modeling the submarine near the free surface are presented\textsuperscript{35}. The latter methods are more time consuming than analytical ones as they yield better results; however there are several CFD softwares capable of modeling the ocean waves (regular or irregular waves) like.Flow-3D\textsuperscript{46}, IOWA and OpenFOAM. Accordingly, the manner of study and our focus would be on Panel method by simulation in Maxsurf\textsuperscript{47}.

**Materials and Methods**

**Panel method applications**

There are two main methods in the numerical methods of the study based on the potential flow: Strip Theory and Panel methods. The Strip Theory method is well known and applicable for surface crafts and ships but it has no applicability for submerged bodies. The reason for this can be ascribed to a conformal mapping basis which requires a to water plane area. So to study the dynamics of submerged bodies like submarines by the potential flow, only the Panel method is applicable. The main
disadvantage of this method is an almost zero forward speed. Table 1 shows the main differences between the Strip Theory and the Panel methods. This study is accomplished via Maxsurf motions. To simulate the submerged submarine at viscous fluid and at non zero speed, only CFD methods based on solving RANS equations are utilized. This method is more accurate but more time consuming as regards solving and more complicated in terms of programming.

**Governing equations**

To apply the panel method, the wave height and the steepness are also assumed to be small such that use can be made of the linear wave theory. The fluid is considered to be inviscid and incompressible. The flow is assumed irrotational. Thus the flow field can be stated by a velocity potential gradient, which is governed by the Laplace equation and which simultaneously should satisfy the proper boundary conditions. The velocity potential in harmonic motions may be stated as follows:

$$\Phi(x,t) = R e[\phi(x)e^{-i\omega t}]$$

Governing equation:

$$\nabla^2 \phi(x) = 0 \quad \text{for} \quad x \in \Omega$$

Free surface boundary condition:

$$-\omega^2 \phi + \frac{\partial^2 \phi}{\partial z^2} = 0 \quad \text{on} \quad z = 0$$

Bottom boundary condition:

$$\frac{\partial \phi}{\partial z} = 0 \quad \text{on} \quad z = -h$$

Body boundary condition:

$$\frac{\partial \phi}{\partial n} = U_n \quad \text{on} \quad S$$

The velocity potential according to the linear theory is:

$$\phi = \phi_r + \sum_{j=1}^{6} \phi_j \phi_j$$

The body boundary conditions for diffraction and radiation velocity are as follows:

$$\frac{\partial \phi_r}{\partial n} = -i \omega \phi_r \quad \text{on} \quad S$$

$$\frac{\partial \phi_j}{\partial n} = -i \omega \phi_j \quad \text{on} \quad S$$

Drawing upon the Green theorem, the velocity potential turns out to be a solution of the following Fredholm integral equation of the second kind:

$$\alpha \phi(x) + \int_{S} \phi(\xi) \frac{\partial G(x,\xi)}{\partial n} dS = \int_{S} \frac{\partial \phi(\xi)}{\partial n} G(x,\xi) dS$$

Here, it is assumed that when in calm water, that the body is rigid and in a state of stable equilibrium. Taking into account the hydrodynamic forces, the motion equations are obtained thus:

$$\sum_{j=1}^{6} \left[ -\omega^2 (M_{ij} + A_{ij}) - i \omega (B_{ij} + \dot{B}_{ij}) + (C_{ij} + K_{ij}) \right] = F_i$$

The right hand side is drift forces. The mean drift forces and moments are evaluated based on the direct pressure integration method. Pinkster and Oortmerssen (1997) derived the second order drift force and moments acting on the floating body as follows:

$$F^{(2)} = \int_{\Omega} \rho g \left( \frac{\partial \phi}{\partial t} \right)^2 n d\Omega$$

$$- \int_{S} \frac{1}{2} \rho \nabla \Phi^{(1)} \cdot \nabla \Phi^{(1)} n dS$$

$$- \int_{S} \rho \chi^{(1)} \cdot \nabla \Phi^{(1)} n dS$$

$$+ \alpha^{(1)} \times M \Theta^{(2)}$$

$$- \int_{S} \rho \nabla \Phi^{(2)} n dS$$

$$M^{(2)} = \int_{\Omega} \frac{1}{2} \rho g \left( \frac{\partial \phi}{\partial t} \right)^2 (x \times n) d\Omega$$

$$- \int_{S} \frac{1}{2} \rho \nabla \Phi^{(3)} \cdot \nabla \Phi^{(3)} (x \times n) dS$$

$$- \int_{S} \rho \chi^{(3)} \cdot \nabla \Phi^{(3)} (x \times n) dS$$

$$+ \alpha^{(1)} \times \dot{M}^{(2)}$$

$$- \int_{S} \rho \nabla \Phi^{(2)} (x \times n) dS$$
The model specifications

The general shape of the submarine is provided in Figures 2 and 3. It has the general shape of a naval submarine with a sailing mast on the top of the hull and a snorkel mast for snorting depth. The model submarine has a weight of 134.5 tons and a length of 29 m. It is a small-sized naval submarine. The main advantage of the present research is that it addresses small and medium submarines because they can't submerge to very high depths, equalling "wave base". Therefore our focus is on finding a real accessible calm depth for submarines of this type. To explain more, such submarines have a maximum dive depth of 100 m. In a wave length of 300 m, "the wave base" is 150 m which is a lot more than the maximum dive depth of a submarine. At this stage, we try to determine the minimum logical, calm and safe depth for small and medium submarines.

The mass distribution for dynamic modelling is presented in Tab.2.

The LCG=LCB is considered from a mid-ship section. The vertical center of gravity (VCG) is considered from base line at the bottom of the cylindrical hull. The longitudinal radius of gyration (Rxx) is considered 40%BOA and Ryy=Rzz=25%LOA. The hydrostatic properties of the model are listed in Table 3.

Irregular wave specifications and wave spectrum

This study uses JONSWAP energy spectrum as a base for nonlinear wave. After analyzing the data collected during the Joint North Sea Wave Observation Project (JONSWAP), Hasselmann et al. (1973), found that the wave spectrum is never fully developed. It continues to develop through non-linear, wave-wave interactions even for very long durations and distances. Hence, an extra and somewhat artificial

<table>
<thead>
<tr>
<th>Total mass (t)</th>
<th>LCG (m)</th>
<th>VCG (m)</th>
<th>Rxx (m)</th>
<th>Ryy (m)</th>
<th>Rzz (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>134.5</td>
<td>3</td>
<td>1.237</td>
<td>1.1</td>
<td>7.25</td>
<td>7.25</td>
</tr>
</tbody>
</table>

Table 2 — Mass distribution of the simulated model

Table 3 — Hydrostatic properties of the model
factor was added to the Pierson-Moskowitz spectrum to improve the fit to their measurements. The JONSWAP spectrum is thus a Pierson-Moskowitz spectrum multiplied by an extra peak enhancement factor $\gamma$. 

$$S_j(\omega) = \frac{\alpha g^2}{\omega^5} \exp \left[ -\frac{5}{4} \frac{\omega_p^4}{\omega^4} \right] \gamma^r$$

$$\gamma = \exp \left[ -\frac{(\omega - \omega_p)^2}{2\sigma^2\omega_p^2} \right]$$

Wave data collected during the JONSWAP experiment were used to determine the values for the constants in the above equations:

$$\alpha = 0.076 \left( \frac{U_{10}^2}{Fg} \right)^{0.22}$$

$$\omega_p = 22 \left( \frac{g^2}{U_{10}F} \right)^{1/3}$$

$$\gamma = 3.3$$

$$\sigma = \begin{cases} 0.07 & \omega \leq \omega_p \\ 0.09 & \omega > \omega_p \end{cases}$$

where $F$ is the distance from a lee shore, called the fetch, or the distance over which the wind blows with constant velocity. Therefore, based on JONSWAP, the characteristics for irregular waves are shown in Table 4.

The submergence depth should be stated as wave length ($\lambda$). For deep water the formula $\lambda = \frac{2}{\pi} \sigma^2$ could be applied where the wave length equals 100 m. The headings of 0, 45, 90, 135 and 180 degrees are considered in the encounter frequencies of 0.2~2 (rad/s) for 10 frequencies. The speeds of 1,3,5,7,9 knots are considered for calculating the encounter frequency but generally the Panel method is applicable for very small speeds and Froud numbers of 0~0.1.

**Results and Discussion**

**Modeling by panel method and results**

The simulation is performed for 11 different drafts and depths. The depth is considered between the top side of the cylindrical part of the hull and the still water surface. The descriptions for each depth are presented in Table 5.

The general form of meshing the body in Panel method at surfaced and submerged conditions is shown in Figure 4. At surface conditions, the body is meshed up to the surface draft.

The visualized results of simulations for submarine motions and irregular wave surface are shown in Figure 5. As can be seen, by increasing the depth of submergence, a decrease in motion amplitude occurs.

Table 6 provides the sample result at snorkel depth and encountered wave angle of 180 degrees at JONSWAP spectrum with a significant wave height of 2 meters, a time period of 10 second and a wave length of 100 meters.

<table>
<thead>
<tr>
<th>Table 5 — Descriptions of surfaced or submerged depth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Depth (m)</td>
</tr>
<tr>
<td>-----------</td>
</tr>
<tr>
<td>4.2</td>
</tr>
<tr>
<td>3.9</td>
</tr>
<tr>
<td>0</td>
</tr>
<tr>
<td>-1</td>
</tr>
<tr>
<td>-3</td>
</tr>
<tr>
<td>-5</td>
</tr>
<tr>
<td>-8</td>
</tr>
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<td>-12</td>
</tr>
<tr>
<td>-16</td>
</tr>
<tr>
<td>-25</td>
</tr>
<tr>
<td>-50</td>
</tr>
</tbody>
</table>

**Table 4 — Characteristics of JONSWAP irregular wave**

<table>
<thead>
<tr>
<th>Significant wave height (m)</th>
<th>Modal period (s)</th>
<th>Average period (s)</th>
<th>Zero-up crossing period (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>9.95</td>
<td>8.37</td>
<td>7.87</td>
</tr>
</tbody>
</table>
As is usual in marine applications, the results of seakeeping modelling are shown in the form of polar diagrams. The polar diagrams are easy to understand for any headings. In this diagram, the heading angle is shown from 0 to 180 degrees and the RMS values for every seakeeping parameter (e.g. heave) are given in several radiuses. The polar diagram for each depth of submergence of submarine is shown in Figure 6.

The total results for the main headings of 0, 90 and 180 degrees are provided in Table 7.

For instance, two diagrams for two conditions are presented in Figure 7: 1) RMS pitch angle at the heading of 180 degree and different depths and 2) RMS heave at the heading of 180 degree and different depths. These diagrams illustrate a descending trend when increasing the depth. But there are some distortions and inconsistencies at the depths near the water surface. The reason can be attributed to two factors: At surface conditions or near surface depths, there are some huge forces and moments bringing about large values of heave and pitch motions; in large motions, panel method is not valid. However, the meshing of the submarine body is executed up until the waterline level, as is shown in Figure 4-a.
Therefore, at large motion amplitudes, the main body can jump out of water or dive in water while there is no meshing inside the water for the non-meshed area of the body. These two parameters indicate that we can ignore the results of surface and near surface depths (first three depths). By studying other cases, it becomes clear that by increasing the depth, a fast decrease in RMS values occur. This decreasing trend shows that at a depth of 8 m (\(\lambda/12.5\)), RMS pitch is only 30% of a 1 m depth (\(\lambda/100\)). Also, at the depth of 8 m (\(\lambda/12.5\)), RMS heave is only 20% of a 1-meter depth (\(\lambda/100\)). This is one main result of the present study which shows the depth of about 0.1\(\lambda\) can be recommended as an operationally calm, stable, and safe for naval or research submarines. Depth of 50 meters (\(\lambda/2\) equal wave base depth) is absolutely calm and depth; however, it may be inaccessible for small and medium submarines. A logical and accessibly recommended depth for all submarine types is 0.1\(\lambda\).

**Comparison with CFD Results**

The authors of the present work have published a similar study via CFD method\(^{48}\). The simulation was
executed utilizing FLOW-3D software based on solving the RANS equations, as shown in Figure 8.

The regular wave is defined input boundary condition. Here the following parameters are defined in Flow-3D: Wave amplitude of 0.18 m, wave period of 1 s, a mean fluid depth (according to the domain depth) of 3.5 m, and a current velocity regarded at zero. Based on these defined parameters, deep water condition is compatible because \(d/\lambda>0.5\). For deep waters according to formula \(\lambda=1.56T^2\), the wave length is 1.56 m. Wave speed according to \(C=1.25\sqrt{\lambda}\) is 1.56 m/s.

To study the wave effects on the submarine, several depths for the submarine situation \(h\) are considered according to Figure 8-a and Table 8.

The time history of pitch angle in 12 conditions is analyzed. Table 9 provides the results for each depth. The percentage of decrease in the last column is based upon the comparison where \(h=0\) and hence the

<table>
<thead>
<tr>
<th>h (m)</th>
<th>Pitch deg</th>
<th>Heave m</th>
<th>Pitch deg</th>
<th>Heave m</th>
<th>Pitch deg</th>
<th>Heave m</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.2</td>
<td>18.5</td>
<td>2.7</td>
<td>18.9</td>
<td>3.3</td>
<td>16.5</td>
<td>3.17</td>
</tr>
<tr>
<td>3.9</td>
<td>6.7</td>
<td>1</td>
<td>8.42</td>
<td>1.1</td>
<td>6.6</td>
<td>0.82</td>
</tr>
<tr>
<td>0</td>
<td>8.25</td>
<td>0.41</td>
<td>5.72</td>
<td>0.43</td>
<td>6.14</td>
<td>0.41</td>
</tr>
<tr>
<td>-1</td>
<td>11.6</td>
<td>0.76</td>
<td>11.55</td>
<td>0.71</td>
<td>12.2</td>
<td>0.82</td>
</tr>
<tr>
<td>-3</td>
<td>6.9</td>
<td>0.36</td>
<td>6.9</td>
<td>0.31</td>
<td>7.1</td>
<td>0.37</td>
</tr>
<tr>
<td>-5</td>
<td>6.5</td>
<td>0.31</td>
<td>6.17</td>
<td>0.23</td>
<td>6.3</td>
<td>0.29</td>
</tr>
<tr>
<td>-8</td>
<td>3.8</td>
<td>0.17</td>
<td>3.7</td>
<td>0.13</td>
<td>3.7</td>
<td>0.16</td>
</tr>
<tr>
<td>-12</td>
<td>2.24</td>
<td>0.13</td>
<td>2.17</td>
<td>0.13</td>
<td>2.21</td>
<td>0.13</td>
</tr>
<tr>
<td>-16</td>
<td>1.27</td>
<td>0.125</td>
<td>1.24</td>
<td>0.135</td>
<td>1.26</td>
<td>0.13</td>
</tr>
<tr>
<td>-25</td>
<td>0.43</td>
<td>0.12</td>
<td>0.41</td>
<td>0.13</td>
<td>0.42</td>
<td>0.12</td>
</tr>
<tr>
<td>-50</td>
<td>0.06</td>
<td>0.06</td>
<td>0.05</td>
<td>0.07</td>
<td>0.05</td>
<td>0.06</td>
</tr>
</tbody>
</table>
average=((h₀-hᵢ)/h₀)*100. It should be noted that the static pitch angle of this submarine is taken at 0.34 degree.

It should be obvious by now that by increasing the depth, the wave effect decreases and the pitch angle approaches the static trim angle. The last column of Table 9 can smoothly describe the pitch angle reductions in percentages. In a depth of 0.03λ there is a 33% reduction and in a depth of 0.06λ, there is a 51% reduction. Intense gradient of the pitch angle will continue until a depth of 0.09λ which causes the submarine to experience a 59% reduction in the pitch angle. After this depth, there is a gentle variation. Values of RMS at the depths of λ, 1.5λ, and 2λ are equal to static trim angles that is to say, no wave effects on the submarine are observed. At almost around the depth of λ/2, the wave effect becomes negligible. The reason for this phenomenon can be explained by resorting to the principle 'wave base' described in the

**Conclusion**

In conclusion, the results obtained from simulations in Panel method and CFD method might be abstracted in Figure 9 which appropriately illustrates the gradient of motions versus the depth of submergence. A depth of λ/2 could be considered absolutely calm; a depth of 0.1λ however could be recommended as operationally safe and approximately calm depth for all types of submarines.

**Nomenclature**

- \( \lambda \) Wave length (m)
- \( \alpha \) Pitch angle (degree)
- A Wave amplitude (m)
- CFD Computational Fluid Dynamics
- d Depth of water (m)
- Ds Diameter of submarine body
- DOF Degree Of Freedom
- GMO General Moving Object
- h Distance from top of the object (submarine) to the water surface (m)
- IHSS Iranian Hydrodynamic Series of Submarines
- L Length of object (submarine)
- R orbital radius of wave articles path (m)
- Rs Radius of submarine body
- RMS Root Mean Square

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Table 8 — Considered conditions for analyses

<table>
<thead>
<tr>
<th>Submarine depth (m)</th>
<th>Description (equivalent to)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Body tangent to free surface</td>
</tr>
<tr>
<td>2</td>
<td>Rs (or) 0.03( \lambda )</td>
</tr>
<tr>
<td>3</td>
<td>Ds (or) 0.06( \lambda )</td>
</tr>
<tr>
<td>4</td>
<td>1.5Ds (or) 0.09( \lambda )</td>
</tr>
<tr>
<td>5</td>
<td>2.5Ds (or) 0.16( \lambda )</td>
</tr>
<tr>
<td>6</td>
<td>3.5Ds (or) 0.22( \lambda )</td>
</tr>
<tr>
<td>7</td>
<td>5.5Ds (or) 0.35( \lambda )</td>
</tr>
<tr>
<td>8</td>
<td>7.5Ds (or) 0.48( \lambda )</td>
</tr>
<tr>
<td>9</td>
<td>9.5Ds (or) 0.61( \lambda )</td>
</tr>
<tr>
<td>10</td>
<td>≅( \lambda )</td>
</tr>
<tr>
<td>11</td>
<td>≅ 1.5( \lambda )</td>
</tr>
<tr>
<td>12</td>
<td>≅ 3( \lambda )</td>
</tr>
</tbody>
</table>

Table 9 — RMS values for considered conditions

<table>
<thead>
<tr>
<th>depth (m)</th>
<th>depth (( \lambda ))</th>
<th>RMS (degree)</th>
<th>Percentage of Decrease (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>3.43</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0.05</td>
<td>2.29</td>
<td>33</td>
</tr>
<tr>
<td>3</td>
<td>0.1</td>
<td>1.67</td>
<td>51</td>
</tr>
<tr>
<td>4</td>
<td>0.15</td>
<td>1.42</td>
<td>59</td>
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<tr>
<td>5</td>
<td>0.25</td>
<td>1.38</td>
<td>60</td>
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<tr>
<td>6</td>
<td>0.35</td>
<td>1.22</td>
<td>64</td>
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<td>1</td>
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<td>0.75</td>
<td>0.82</td>
<td>76</td>
</tr>
<tr>
<td>9</td>
<td>0.95</td>
<td>0.44</td>
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</tbody>
</table>

Fig. 9 — Gradient of RMS pitch versus submergence depth of submarine via Panel method and CFD method.

Introduction, i.e., if a submarine dives to the depth more than \( \lambda /2 \), it doesn't experience wave effects. For long swell waves, the value of \( \lambda /2 \) may be more than the collapse depth of the submarine, which is an impossible thing to happen. In this condition, if the submarine dives to a depth of about 0.1\( \lambda \), it can avoid 60% of motions and shakes. For instance, in swell waves (being very similar to regular waves) with a time period of 15 s, the wave length is 351 m. The half wave length is about 175 m a dangerous depth which can have catastrophic consequences for a submarine. This being so, if a submarine dives to a depth of 0.1\( \lambda \) equaling about 35 m, it can navigate in much more calm and stable conditions.
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