Effective surface properties of Sn isotopes

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We have studied the effective surface properties such as symmetry energy coefficient, pressure, and curvature coefficient of Sn isotopic series within the effective field theory motivated relativistic meanfield formalism (E-RMF). The densities of the nuclei, calculated within E-RMF, are used as the inputs to the coherent density fluctuation model (CDFM). We have used the recently predicted force parameter IOPB-I and compared the results with the calculated values of widely accepted and well-known NL3 parameter set. The small kinks, observed in the graph correspond to the magic number in the isotopic series.

Keywords: Symmetry energy coefficient, Pressure, Curvature coefficient, Skin thickness

1 Introduction

Nuclear symmetry energy has the key importance in locating the drip lines of an isotopic series, dynamics of heavy-ion reaction, and to determine the properties of a neutron star. Generally, it is defined as the energy required in converting isospin asymmetric nuclear matter to the symmetric one. The isospin asymmetry in nuclear matter arises due to the difference in the densities and masses of neutrons and protons. For interpreting neutron-rich nuclei and neutron star matter, the characterization of symmetry energy through experiment is a crucial step. But, symmetry energy is not a directly measurable experimental quantity. Hence, it is extracted from the quantities related to it. Theoretically, it is found that neutron skin thickness of \(^{208}\text{Pb}\) is correlated to nuclear symmetry energy, L-coefficient, and the radius of neutron star. The neutron skin thickness is not measured as precisely as that of proton radius yet it is a sensitive probe of symmetry energy.

The importance of symmetry energy and its sensitivity to the density have motivated us to study the same for the isotopic series of Sn. We have chosen Sn because its atomic number is one of the magic numbers and lies about in between the known nuclei. The study of properties of isotopic series of Sn nuclei enables one to locate the neutron and proton drip lines, to examine the difference between the properties of nuclei lie near to the beta stability line and those of drip line nuclei, and to look for the doubly magic isotopes of Sn. We have studied the effective surface properties like symmetry energy, pressure, curvature coefficient, and neutron skin thickness along with the bulk properties. These properties can be useful to synthesize drip line nuclei, to find the most stable isotope of Sn series, to constrain the equation of state (EOS) of neutron star, and to predict the nuclear synthesis through astrophysical processes.

2 Formalism

The bulk properties of nuclei like binding energy, and root mean square radius are calculated along with the densities within effective field theory motivated relativistic mean field model (E-RMF). The calculated densities are used as the inputs to the coherent density fluctuation model (CDFM) to study the symmetry energy, pressure and curvature coefficients. The E-RMF and CDFM are presented in this section briefly.

2.1 E-RMF

The relativistic mean field (RMF) theory is the relativistic generalization of non-relativistic Hartree or Hartree-Fock-Bogoliubov theory with the advantages that it accounts spin orbit interaction automatically and work well in high density region. It predicts the ground and excited state bulk properties of nuclei remarkably well over the nuclear landscape. In RMF, nucleons are supposed to interact with each
other through the exchange of mesons\textsuperscript{1,3}. The E-RMF has in principle, infinite number of mesonic terms with their self and cross coupling. But, the terms up to 4\textsuperscript{th} order of expansion in terms of ratios of mesonic fields and nucleonic mass give the converge solution. The detail formalism of E-RMF is given in literature\textsuperscript{1}. With the passage of time, several force parameters of RMF has been generated which predicts the properties of a nucleus under certain domain. In the series of parameters, IOPB-I\textsuperscript{1} is the latest which predicts the properties of nuclei over the nuclear chart and of neutron star. We have used the IOPB-I interaction within E-RMF and compared the calculated results with those of NL3 parameter; the well-known and widely accepted force parameter.

2.2 CDFM

The main assumption of CDFM is that the surface properties of nuclei can be found by folding the corresponding quantities of nuclear matter with the weight function. In the CDFM, the symmetry energy and related quantities for a nucleus are supposed to be the infinite superstitions of corresponding nuclear matter quantities weighted by weight function\textsuperscript{(8,9)} |f(x)|\textsuperscript{2}:

\[ s = \int_{0}^{\infty} |f(x)|^2 S_{NM}(x) dx \]  \quad \text{... (1)}

\[ p = \int_{0}^{\infty} |f(x)|^2 p_{NM}(x) dx \]  \quad \text{... (2)}

\[ K_{sym} = \int_{0}^{\infty} |f(x)|^2 K_{sym}^{NM}(x) dx \]  \quad \text{... (3)}

Where, s, p and \( K_{sym} \) are the symmetry energy, pressure and curvature coefficient of a nucleus. The expression for the corresponding quantities of nuclear matter (see Eqs 1-3) is given explicitly in the earlier studies\textsuperscript{8}. The weight function is obtained by using the density of a nucleus calculated within E-RMF model as:

\[ |f(x)|^2 = -\frac{1}{\rho_0(x)} \frac{d\rho(r)}{dr} \]  \quad \text{... (4)}

provided \( r = x \), where \( \rho(r) \) is the density of a nucleus and \( \rho_0(x) = 3A/4 \pi x^3 \) is the flucton density.

3 Results and Discussion

The binding energies of the nuclei as a function of neutron number of Sn isotopic series corresponding to IOPB-I and NL3 parameters are shown in Figure 1. It can be seen in the figure that both the parameter sets predicts almost same binding energy with a little difference in neutron rich region.

In the binding energy versus neutron number (N) curve (Fig. 1), two kinks are observed corresponding to N= 50 and 82 which are the magic numbers. These kinks exhibit that the nuclei are more stable at this N number as compared to the surrounding isotopes. In the isotopic series of Sn, we have two doubly magic nuclei with N=50 and 82 exhibiting more stability. The calculated results agree well with the available experimental data. After verifying the calculated results (BE) with the available experimental data, we have proceeded to calculate the surface properties of the nuclei. As stated, the densities of the nuclei calculated within E-RMF are used as the input to CDFM. The densities of \(^{116}\text{Sn} \) (dashed lines) and \(^{132}\text{Sn} \) (bold lines) as the representative case are shown in Fig. 2. The color code corresponds to the parameter

\[ \begin{align*}
\text{Fig. 1 – Binding energy (BE) versus neutron number of Sn isotopic series.} \\
\text{Fig. 2 – The density profile of } ^{116}\text{Sn and } ^{132}\text{Sn corresponding to IOPB-I and NL3 parameter sets.}
\end{align*} \]
sets. It is clear from the figure that the central and surface density of $^{132}\text{Sn}$ is larger than that of $^{116}\text{Sn}$ due to the more number of nucleons (neutron). $^{132}\text{Sn}$ is the doubly magic nuclei whose density changes monotonously, while $^{116}\text{Sn}$ which lies near the beta stability valley has the oscillatory behaviour from 2-4 fm. By using the densities of the nuclei in Eq. 4, the weight functions are calculated, which are shown in Fig. 3 for the representative cases. The $^{116}\text{Sn}$ have slightly larger magnitude of weight function as compared to the doubly magic $^{132}\text{Sn}$.

The symmetry energy, pressure, and curvature coefficient of nuclear matter are calculated by using the Bruckner potential. These calculated values are folded with the weight function to find the corresponding quantities for finite nuclei. Figure 4 shows these properties as the function of neutron number. The values corresponding to both the parameter sets almost follow the same nature. The behaviour of the surface properties with $N$ is monotonous in nature. Yet, there are kinks in the curve which correspond to the magic number $N=50$ and 82 of the nuclei. It is clear from the figure that in going far from the magic number, the surface properties of the nuclei are decreased. The same properties are further plotted with the neutron skin thickness which is shown in Fig. 5. Here, again the kinks in the curves are at magic neutron numbers. The NL3 parameter predicts larger skin thickness than IOPB-I set (which is clear from the figure) and hence large neutron star mass and radius. Thus, NL3 equation of state (EOS) is recalled as the stiffer in nature than IOPB-I.

4 Conclusions

In summary, we have studied the bulk as well as effective surface properties like symmetry energy, pressure, and curvature coefficient for Sn isotopic series. We have used E-RMF formalism to calculate the ground state bulk properties of the nuclei by using the recently predicted IOPB-I parameter and compared with the results of NL3. The nature of doubly magic isotopes of Sn is observed showing the more binding energy as well as effective surface properties that the rest of the nuclei. The calculated results can be used for theoretical study and experimental use to synthesize the drip lines nuclei.

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References

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