Determining expedited time and cost of the end product with defective component parts using critical path method (CPM) and time-costing method

Singa Wang Chiu¹, Yuan-Shyi Peter Chiu²* and Chih-Chang Shih²
¹Department of Business Administration, ²Department of Industrial Engineering & Management, Chaoyang University of Technology, 168, Gifeng E. Rd., Wufeng, Taichung, Taiwan 413

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This paper extends an existing work that dealt with the issue of expediting completion time of the end product (ECTEP) and presents a revised algorithm for determining the expedited time and cost of end product in product structure diagram that takes defective component parts into consideration. The algorithm for ECTEP problem proposed in prior work assumes that all component-parts produced or purchased are of perfect quality. However, in real-life situations, production of defective items is inevitable. This study incorporates an adjusted unit procurement cost and adjusted quantity into the ECTEP solution procedure to cope with the existence of defective materials. Both stationary and random defective rates are examined. Critical path method and time-costing technique are employed for determining optimal component procuring alternatives when expediting end product’s completion time is desired.

Keywords: Component procurement decision, CPM, Expediting completion time, PSD, Time-costing method

Introduction

Classic product structure diagram (PSD) provides essential information for the consequent materials requirements planning (MRP). This includes the parent-child relationship between the end product and its components at each level; the number of components required at the child level in order to produce one unit at the parent level, and the number of periods required for production of each component.¹² Choosing an appropriate acquisition method (either a particular manufacturing process or a specific supplier) for each component part is one of the early tasks faced by product/process design team. This decision is critical since it will significantly affect the subsequent MRP. Studies³–⁸ have identified various factors that are important in materials procurement selection; key criteria among them are quality, on-time delivery, price, performance, and flexibility. Nowadays, in the shorter product life and time-based competition business environment, to be able to expedite completion time of end product with minimum cost becomes a very important task to any manufacturing firm, since it enables a firm to stay in competitive position facing various delivery time requirements from customers in the turbulent environment.

Chiu & Chiu⁹ in expediting completion time of the end product (ECTEP) problem incorporated expedited time and cost of end product into PSD. This study extends ECTEP problem, by examining parameters of adjusted unit procurement cost and adjusted procurement quantity to cope with the realistic issue of defective materials. Since little attention has been paid to ECTEP problem in PSD with imperfect quality components/materials; this paper proposes an extended ECTEP algorithm to serve this purpose.

Problem Statement and the Proposed Algorithm

Product structure tree refers to the relationship between the components at adjacent levels of the system containing quantity and lead-time information. In ECTEP problem⁹, there exist a number of procurement alternatives (several feasible in-house manufacture processes or multiple suppliers available in outsourcing case) for every component parts in PSD. In addition to unit production/purchase cost and individual lead-time requirement, each alternative also has its own yield rate. The following notations are used:

*TCT = total completion time needed to produce the end product.

\[ i = \text{ith level of PSD, where } i = 0, 1, 2, \ldots, l \]

\[ j = \text{jth component part (activity) of PSD, where } j = 1, 2, \ldots, m \]

*Author for correspondence
E-mail: ypchiu@mail.cyut.edu.tw
$A_{ij} = j^{th}$ component part in the $i^{th}$ level of PSD.

$k = k^{th}$ procurement alternative for $A_{ij},$ where $k=1, 2, \ldots, n.$

$S_{ijk} = k^{th}$ procurement alternative for $A_{ij}.$

$L_{ijk} = \text{lead time that is associated with the } k^{th} \ \text{procurement alternative for } A_{ij}.$

$N_{ij} = \text{quantities that are required for component part } A_{ij} \ \text{to form one unit of the end product in the original PSD.}$

$(dS)_{ijk} = \text{stationary defective rate of } k^{th} \ \text{procurement alternative for } A_{ij}.$

$(dR)_{ijk} = \text{random defective rate of } k^{th} \ \text{procurement alternative for } A_{ij}, \ \text{where } (dR)_{ijk} \ \text{follows a known probability density function.}$

$E[(dR)_{ijk}] = \text{expected value of random defective rate (dR)_{ijk} of } k^{th} \ \text{procurement alternative for activity } A_{ij}.$

$(YS)_{ijk} = \text{stationary yield rate } [1-(dS)_{ijk}] \ \text{of } k^{th} \ \text{procurement alternative for } A_{ij}.$

$(YR)_{ijk} = \text{random yield rate of the } k^{th} \ \text{procurement alternative for } A_{ij}, \ \text{where } (YR)_{ijk} \ \text{follows a known probability density function.}$

$E[(YR)_{ijk}] = \text{expected value of random yield rate (YR)_{ijk} of } k^{th} \ \text{procurement alternative for activity } A_{ij}.$

$Y_{ijk} = \text{yield rate } \{\text{a general form stands for either } (YS)_{ijk} \ \text{or } E[(YR)_{ijk}]\} \ \text{of the } k^{th} \ \text{procurement alternative for activity } A_{ij}.$

$\sigma_{ijk} = \text{standard deviation of random defective rate of } k^{th} \ \text{procurement alternative for activity } A_{ij}.$

$(CU)_{ijk} = \text{unit procurement cost when } k^{th} \ \text{procurement alternative is selected for activity } A_{ij}.$

$C_{ijk} = \text{adjusted unit procurement cost (including cost for defective components/materials) when } k^{th} \ \text{procurement alternative is selected for activity } A_{ij}.$

$N_{ijk} = \text{adjusted quantities required (including extra quantities to compensate defective items) when } k^{th} \ \text{procurement alternative is selected for component part } A_{ij}, \ \text{to form one unit of the end product.}$

$(CI)_{ijk} = \text{increases in cost per unit time shortened, when } k^{th} \ \text{procurement alternative for } A_{ij} \ \text{is chosen as part of the solution.} \ \ \ \text{((CI)_{ijk}}=(N_{ijk})[(C_{ijk}-\text{C}_{ijk*})/(L_{ijk} - L_{ijk})], \ \text{where } C_{ijk} \text{ is adjusted unit cost associated with } S_{ijk} \text{ of the current procurement solution for } A_{ij}.$

TPC = total production cost for producing one unit of end product. i.e. TPC= $\Sigma\Sigma\Sigma_{k} \{N_{ijk} C_{ijk}\}, \ \text{where only } k \ \text{belongs to the solution set.}$

In this study, $C_{ijk}$ and $N_{ijk}$ are incorporated into revised ECTEP algorithm to cope with the existence of defective component parts. Both stationary and random yield rates of materials are examined and practical cases regarding reimbursement (in case of purchase) and rework (when items are produced in-house) are considered as follows$^{10-13}.$

Case 1

If defective items are purchased from outside suppliers and they will not be reimbursed. In this case, $C_{ijk}$ and $N_{ijk}$ must be calculated, where $Y_{ijk}=(YS)_{ijk}, \ \text{C}_{ijk}=(CU)_{ijk}/Y_{ijk}, \ \text{and } N_{ijk}=N_{ijk}/Y_{ijk}.$

Case 2

Suppose that defective items are received from outsider suppliers and they will be reimbursed. In this case, $C_{ijk}=(CU)_{ijk}, \ \text{and to avoid shortage } N_{ijk}=N_{ijk}/Y_{ijk} \ \text{where } Y_{ijk}=(YS)_{ijk}.$

Case 3

If defective component parts are produced in-house and they are all scrap items. In this case, $C_{ijk}=(CU)_{ijk}/Y_{ijk} \ \text{and } N_{ijk}=N_{ijk}/Y_{ijk} \ \text{where } Y_{ijk}=(YS)_{ijk}.$

Case 4

Suppose that the defective rate $(dR)_{ijk}$ is a random variable with a known probability density function. Expected values of defective rate $E[(dR)_{ijk}]$ and yield rate $E[(YR)_{ijk}]$ must be computed, and let $Y_{ijk}=E[(YR)_{ijk}].$ For sensitivity analyses on total cost for producing one unit of end product, one reasonable suggestion among others will be to use $E[d_{ijk}] \pm 3\sigma_{ijk}$ to calculate the optimistic and the pessimistic costs.

It is noticed that if defective items can be reworked and repaired, an accurate estimation of $C_{ijk}$ may depend on various assumptions made in realistic production models$^{10,11,14-17}.$

Revised ECTEP Algorithm

Revised algorithm starts with using the least $C_{ijk}$ to find the initial solution set $S_{ijk}$ for every $A_{ij}$ in PSD.
Then, critical path method (CPM) is used to find critical path (CP) based on parent-child relationship among activities in initial solution set \( S_{ijk} \). Following by the repeated use of time-costing method to choose (and to replace with) the alternative that has the minimal increases in cost per unit time shortened \( \min\{ (CI)_{ijk} \} \) until no further reductions on completion time of end product are possible. Revised ECTEP algorithm is summarized as follows:

**Step 1**
Identify the practical case of defective components in PSD and calculate all \( C_{ijk} \) for each \( S_{ijk} \), for \( i=0, 1, 2, \ldots, l; j=1, 2, \ldots, m; \) and \( k=1, 2, \ldots, n \).

**Step 2**
Obtain the initial solution set \( S_{ijk} \) by the least cost \( (C_{ijk}) \) method. For alternatives in every \( A_{ij} \) in PSD, choose least \( C_{ijk} \) and obtain the initial solution set \( \{ S_{ijk} \} \). Total cost for producing one unit of end product using the initial procurement solution; \( \text{TPC} = \sum_i \sum_j \sum_k \{ N_{ij} C_{ijk} \} \).

**Step 3**
Construct a project network based on the PSD. The end product is the last activity in project network and it is considered to be finished only if all preceding activities are completed. Therefore, minimum time to complete the project (end product) is equivalent to the length of time of longest path in the project network.

**Step 4**
Search for the critical path (CP). Using parameters of current solution set \( \{ S_{ijk} \} \) and critical path method to find CP in the PSD project network. Computations of earliest starting and finishing times and latest starting and finishing times for all \( A_{ij} \) are required when performing the forward and backward passes in the CP analysis. Total completion time of project TCT, is the time needed to produce the end product. It is noticed that expediting completion time of the end product is equivalent to reducing time of CP activities.

**Step 5**
Determine step-by-step the expedited time and cost of the end product.

**Step 5-A**
Seek and gather all possible procurement alternatives \( \{ S_{ijk} \} \) from the current CP activities that can further reduce TCT. Compute their corresponding \( (CI)_{ijk} \) values, where \( (CI)_{ijk} = (N_{ij})[(C_{ijk} - C_{ijk}*)/(L_{ijk} - L_{ijk})] \). Here, the general rule is to successively reduce the project time by one unit of time, until no further reductions are possible. If none exist, then go to Step 6.

**Step 5-B**
Select \( \min\{ (CI)_{ijk} \} \) and its corresponding \( S_{ijk} \) from the resulting set of all possible procurement alternatives found in Step 5-A. Perform the following: i) Replace current procurement solution of \( A_{ij} \) with the new selected \( S_{ijk} \); ii) Update TCT, i.e. \( \text{TCT} = \text{TCT} - 1 \); and iii) Update TPC, i.e. \( \text{TPC} = \text{TPC} + (CI)_{ijk} \). Repeat on Step 4.

**Step 6**
Compute adjusted quantities \( N_{ijk} \) for all \( S_{ijk} \) in the optimal solution sets. Compute \( N_{ijk} = N_{ij}(1/Y_{ijk}) \) for all procurement alternatives in final solution sets of \( \{ S_{ijk} \} = 1 \), where \( i=0, 1, 2, \ldots, l; j=1, 2, \ldots, m; \) and \( k=1, 2, \ldots, n \).

**Step 7**
Stop. Optimal expedited time and cost of the end product in PSD are obtained.

**Results and Discussion**
To demonstrate extended ECTEP algorithm (Fig.1), a production of trumpet problem is used. For demonstration purpose (Table 1), following additional assumptions are made for each component parts (activity): 1) Activity number; 2) Number of procurement alternatives (either different manufacturing methods or suppliers); 3) Lead time of each alternative; 4) Unit acquisition cost (price) of each alternative; 5) Defective component parts are received from outside supplier and they will not be
reimbursed; and 6) Dummy activities will be shown in dash line with lead time zero in project network. Proposed ECTEP algorithm is applied as follows:

**Step 1**
Identify defective components and calculate \((YS)_{ijk}\) and \(C_{ijk}\) for each \(S_{ijk}\) (Table 1).

**Step 2**
Find initial solution set of \(S_{ijk}\) by the least cost method. The resulting solution is \(\{S_{011}, S_{112}, S_{212}, S_{211}, S_{221}\}\), where \(C_{ijk} = \min \{C_{ijk}'s\} \) for \(k = 1, 2, \ldots, n\).

\[
TPC = \sum_{i} \sum_{j} \sum_{k} \{N_{ij} C_{ijk}\} = \$77.73.
\]

**Step 3**
Construct a project network (Fig. 2).

**Step 4**
Search for critical path (CP). Applying CPM, one obtains CP = \(\{S_{221}, S_{222}, S_{011}\}\) (Fig. 2, CP: in thicker line). Total completion time needed to produce Trumpet (end product), TCT= 8 weeks.

**Step 5**
**Step 5-A**
Seek and gather all possible procurement alternatives \(\{S_{ijk}'s\}\) from current CP activities that can further reduce TCT. Compute their corresponding \((CI)_{ijk}\) values. Table 2 presents the resulting alternatives \(\{S_{ijk}'s\}\), where the linear relationship is assumed between the increases in cost and the unit time shortened.

**Step 5-B**
Select \(\min \{(CI)_{ijk}\}\) and its corresponding \(S_{ijk}\) from \(\{S_{ijk}'s\}\). One finds (Table 2) \(\min \{(CI)_{ijk}\} = $1.83\) and hence \(S_{223}\) is selected to replace \(S_{221}\) for activity \(A_{22}\). New solution = \(\{S_{011}, S_{112}, S_{212}, S_{211}, S_{223}\}\). TCT=8-1=7 weeks. TPC = TPC+\((CI)_{ijk}\) = \$77.73+$1.83 = \$79.56.

Repeat on Step 4 (second iteration) Applying CPM again, two critical paths, CP\(_1\)=\(\{S_{221}, S_{212}, S_{211}, S_{011}\}\) and CP\(_2\)=\(\{S_{223}, S_{222}, S_{011}\}\), are obtained. **Step 5-A**: Result is shown in Table 3. **Step 5-B**: Applying multiple CP analysis and time-costing method, reduces TCT by one unit of time. One can either choose \(S_{124}\) or \(\{S_{213} and S_{225}\}\) together; whichever is less expensive. Since

<table>
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<th>Type of component part</th>
<th>(A_{ij})</th>
<th>Procurement alternatives, (S_{ijk})</th>
<th>(N_{ij})</th>
<th>(L_{ijk}) weeks</th>
<th>((dS)_{ijk})</th>
<th>((CU)_{ijk})</th>
<th>((YS)_{ijk})</th>
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min\{C_{Ijk}\}$ = $4.42$, one chooses $S_{124}$ to replace $S_{122}$ for activity $A_{12}$. New solution becomes $\{S_{211}, S_{112}, S_{124}, S_{211}, S_{223}\}$. TCT = $7-1=6$ (weeks). 

TPC = $79.56 + 4.42 = $83.98.

**Repeat on Step 4 (third iteration)**

Two critical paths $C_{P1}\{S_{211}, S_{124}, S_{011}\}$ and $C_{P2}\{S_{223}, S_{124}, S_{011}\}$ are found. **Step 5-A**: One can either choose $S_{124}$ or $\{S_{213}$ and $S_{225}\}$ together. **Step 5-B**: Select $S_{124}$ again, for the reason that $\min\{C_{I124}\}$ = $4.42$. In this iteration, no replacement of current solution is needed, since $S_{124}$ is already in the solution. TCT = $6-1=5$ (weeks). TPC = $83.98 + 4.42 = $88.40.

**Repeat on Step 4 (fourth iteration)**

Applying CPM again, one obtains two critical paths: $C_{P1}\{S_{213}, S_{124}, S_{011}\}$ and $C_{P2}\{S_{223}, S_{124}, S_{011}\}$. **Step 5-A**: One must choose $S_{213}$ and $S_{225}$ together. **Step 5-B**: Choose $S_{211}$ and $S_{223}$ to replace $S_{211}$ and $S_{223}$, respectively. New solution becomes $\{S_{011}, S_{112}, S_{124}, S_{211}, S_{223}\}$. TCT = $5-1=4$ (weeks). TPC = $88.40 + 6.15 = $94.55. No further reductions are possible.

The step-by-step optimal solutions of component procurement alternatives for expediting Trumpet (end product) and their related parameters (Table 4) are obtained.

**Step 6**

Compute $N_{ijk}$ for all $S_{ijk}$ in the optimal solution sets (Table 5). It is noticed that when preparing the bill of materials, required quantity must be adjusted ($N_{ijk}=N_{ij} \cdot (1/Y_{ijk})$) to compensate for the various defective rates. Fig. 3 depicts defective component effects on optimal TPC when ECTEP algorithm is applied.

**Case of Random Defective Components**

Suppose random defective rate is assumed and procurement alternatives and their parameters are
given (Table 6). As stated earlier, it is suggested that the practitioners employ expected values of defective rate $E[d_{ijk}]$ and yield rate $E[Y_{ijk}]$ in the proposed ECTEP algorithm. For sensitivity analysis on total production cost for producing one unit of end product, a wider 99% confidence interval of $E[d_{ijk}]+3\sigma_{ijk}$ are employed to obtain the optimistic and pessimistic costs accordingly (Fig. 4). For the case of random defective rate, when preparing the bill of materials, required quantity should be adjusted in accordance with the worst-case scenario. That is to use $N_{ijk}=N_{ij}/[1-(E[d_{ijk}]+3\sigma_{ijk})]$ to compensate for random defective items and to prevent materials shortage from happening.
Conclusions

In the shorter product life and time-based competition business environment, expediting completion time of end product becomes important to manufacturers, since it enables the firms to stay in competitive position. This study extends a prior work on ECTEP problem and proposed a revised algorithm for determining ECTEP in PSD taking defective component parts into consideration. This paper incorporates adjusted unit procurement cost and adjusted quantity into a revised solution procedure for ECTEP problem to address the issue of defective components/materials. Both stationary and random defective rates are examined. The critical path method and time-costing method are employed for determining the step-by-step optimal component procuring alternatives when expediting production time of end product is desired. Although a noticeable amount of articles pointed out many important factors and methods in procurement alternative selection, little attention has been paid to the issue of ECTEP in PSD with consideration of defective components/materials. This paper intends to serve this purpose and presents a revised algorithm for addressing this realistic problem.

References