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Current mode fractional order band pass and band reject filter using VDTAs

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In this proposed work, current mode fractional order band pass and band reject filters using voltage differencing trans-conductance amplifier (VDTAs) as an active building block have been proposed. The approximated transfer function of the band stop and band pass filters on the basis of the integer order transfer function has been shown and the similar form of the approximate transfer function has been implemented for the band stop and band pass filters by using VDTAs as an active building block. In this work, fractional order band stop and band pass filters of the different orders have been realized. The operation of the design has been tested using simulation results on PSPICE with TSMC CMOS 180 nm technology parameters.

Keywords: Voltage differencing trans-conductance amplifier, Fractional order, Band pass filter, Band reject filter

1 Introduction

Since last decade researchers are focusing on fractional calculus because of its various advantages over integer order system designing. The major advantage of the fractional order systems is that they fall in the category of infinite memory characteristics, while the integer order systems come under the category of finite memory characteristics. The application of the fractional calculus is in many fields of engineering and other streams like bioengineering[1], agriculture[2], electromagnetic[3], control system[4], etc. Many conceptual theories and stability analysis also discussed using fractional order calculus. Analog signal processing and generation circuits are the major point of attraction of the researchers[5,6].

The fractional derivative of order α defines by the Caputo can be written as:

$$\mathrm{a}\mathrm{D}_{\alpha}^{\mathrm{t}}\mathrm{f}(\mathrm{t}) = \left\{\left(\frac{1}{\Gamma(n-\alpha)}\right)\int_{a}^{t}\frac{f^{n}(\tau)}{(t-\tau)^{(\alpha-n+1)}}\right\}; (n-1) < \alpha < n$$

$$= \left\{\frac{d^{n}}{dx}\,f(t)\right\}; n = \alpha \qquad \ldots (1)$$

Where a the initial time and t is the time required for the process calculation. The presence of extra independent factor makes the above equation generalized form of the general integer order function. Assume zero initial conditions then by using Laplace transform to Eq. (1) results:

$$L\{0D_{\alpha}^{t}f(t)\} = S^{\alpha}F(t) \qquad \ldots (2)$$

Equation (2) represents the fractance device in the analog domain and the impedance function of fractance device is shown in Eq. (3):

$$\mathrm{Z(s)} = \mathrm{K_0 S^{\alpha}} = \mathrm{K_0\,(j\omega)^{\alpha}} \qquad \ldots \qquad (3)$$

Where α is the order of fraction and the constant is represented by K_0 . Then, the impedance Z in the polar form is represented as:

$$|\mathrm{z}| = \mathrm{K_0}\,\omega^{\alpha}\ \&\ \angle \mathrm{Z} = \alpha\frac{\pi}{2} \qquad \ldots \qquad (4)$$

From (3), it may be found that for different values of α, impedance Z will show different element characteristics like for α = 1, 0 and -1, impedance Z will show the inductor, resistor and capacitor characteristics, respectively. So a passive circuit detail that offers a regular section attitude with frequency can be referred to as a Fractional Order Element (FOE) which is the generalized detail of the already present electrical circuit factors[7].

In analog circuit designing there are a no of active building blocks are available like current conveyor, current feedback operational amplifier, current differencing buffered amplifier, voltage differencing buffered amplifier, etc. In the analog designing the current mode have its own advantage like low power, wide bandwidth, simple circuitry and high slew rate etc.

In this work, voltage differencing tans-conductance amplifier as an active block has been used to design

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*Corresponding author (E-mail: dprasad@jmi.ac.in) band reject filter and band pass filter. It can be used either in voltage mode or current mode. It works at high frequency operations and free from parasitic capacitance.

2 Circuit Configuration

The symbolic representation of the circuit of active cell, VDTA is shown in Fig. 1. It has five terminals, out of five, *P* and *N* are input and *Z*, *X*+ and *X*- are output terminals with all input output terminals are at high impedance level. The mathematical relation between input and output terminals of an ideal VDTA is represented by hybrid matrix given below:

$$\begin{bmatrix} I_Z \\ I_{X+} \\ I_{X-} \end{bmatrix} = \begin{bmatrix} g_{m1} & -g_{m1} & 0 \\ 0 & 0 & g_{m2} \\ 0 & 0 & -g_{m2} \end{bmatrix} \begin{bmatrix} V_P \\ V_N \\ V_Z \end{bmatrix} \qquad \ldots(5)$$

As discuss above, the symbolic representation of circuit of VDTA is shown in Fig. 1 where input terminals *P* and *N* and output terminals are *X*+, *X*- and *Z*. The CMOS implementation of VDTA is shown in Fig. 2. g_{m1} and g_{m2}, the trans-conductance gain of the CMOS circuit of VDTA shown in Fig. 2 are given as:

$$gm_1 = \frac{(g_3+g_4)}{2} \qquad \ldots(6)$$

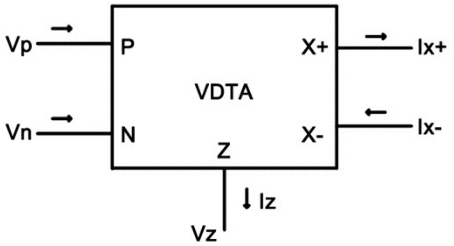

Fig. 1 – VDTA symbol.

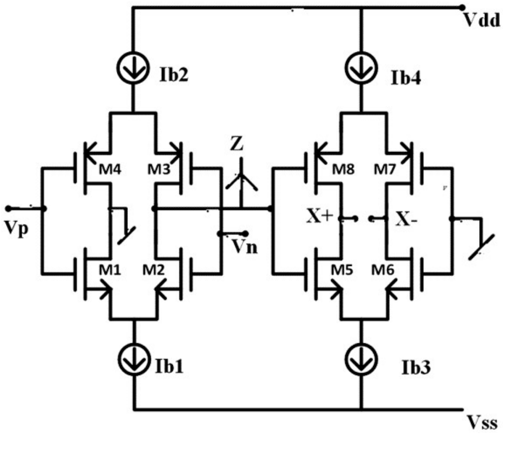

Fig. 2 – CMOS realization of VDTA block.

$$gm_1 = \frac{(g_5+g_8)}{2} \qquad \ldots(7)$$

or

$$gm_2 = \frac{(g_6+g_7)}{2}$$

The trans-conductance of *n*th CMOS is given as:

$$g_{mn} = \sqrt{I_{Bn}\,\mu_n\,C_{ox}\left(\frac{w}{l}\right)_n} \qquad \ldots \qquad (8)$$

Where the μ_n is the mobility of the charge carrier, C_{ox} is the gate oxide capacitance per unit area; *w* and l are used to denote the channel width and length, respectively[8].

VDTA found application in analog signal processing and signal generation circuits but we didn't find its application in fractional order signal processing and signal generation application[9-14]. So this work has tried to fill this vacancy.

The condition $(x+\alpha)<2$ must be satisfy to implement the fractional order filters directly for the stabile design, Where x is an integer number. The general transfer functions of the band pass and band reject filters[15] of fractional order 1+α are given in Eqs (9) and (10), respectively:

$$T_{FBPF} = \frac{K_1 K_3 (\tau s)^{\alpha}}{(\tau s)^{1+\alpha} + K_3 (\tau s)^{\alpha} + K_2} \qquad \ldots \qquad (9)$$

$$T_{FBRF} = K_1 \frac{\tau s^{1+\alpha} + K_2}{(\tau s)^{1+\alpha} + K_3 (\tau s)^{\alpha} + K_2} \qquad \ldots(10)$$

Where Eqs (11) and (12) shows the -3 dB frequency of the fractional order band pass and band reject filters, respectively:

$$\left(\frac{\omega_p}{\omega_0}\right)^{2(1+\alpha)} - (1-\alpha)K_2\left(\frac{\omega_p}{\omega_0}\right)^{(1+\alpha)}\sin\left(\frac{\alpha\pi}{2}\right) - \alpha K_2 K_3\left(\frac{\omega_p}{\omega_0}\right)^{\alpha}\cos\left(\frac{\alpha\pi}{2}\right) - \alpha K_2^2 = 0 \qquad \ldots(11)$$

$$\begin{aligned} &\frac{2(1+\alpha)\omega^{(3+4\alpha)}}{\omega_0^{(4+4\alpha)}}\left(\omega_0^{(1+2\alpha)} - 1\right) \\ &\quad - \frac{4K_2(1+\alpha)\omega^{(2+3\alpha)}\sin\left(\frac{\alpha\pi}{2}\right)}{\omega_0^{(3+3\alpha)}}\left(1 - \omega_0^{(1+\alpha)}\right) \\ &\quad + \frac{2K_3^2\omega^{(1+4\alpha)}}{\omega_0^{(2+4\alpha)}} + \frac{2K_2K_3(1+2\alpha)\omega^{(1+3\alpha)}\cos\left(\frac{\alpha\pi}{2}\right)}{\omega_0^{(2+3\alpha)}} \\ &\quad - 2K_2K_3^2\frac{\omega^{3\alpha}}{\omega_0^{(1+3\alpha)}}(1-\alpha)\sin\left(\frac{\alpha\pi}{2}\right) \\ &\quad - 4\alpha K_3K_2^2\frac{\omega^{2\alpha}}{\omega_0^{(1+2\alpha)}}\cos\left(\frac{\alpha\pi}{2}\right)\sin\left(\frac{\alpha\pi}{2}\right) \\ &\quad - 2\alpha K_3K_2^2\frac{\omega^{(\alpha-1)}}{\omega_0^{\alpha}}\cos\left(\frac{\alpha\pi}{2}\right) = 0 \end{aligned} \qquad \ldots(12)$$

The quality factor for the band pass and band reject filter can be calculate by using Eq. (13) given as:

$$Q = \frac{\omega_p}{\omega_{h2} - \omega_{h1}} \qquad \ldots (13)$$

Where the factors k_i in the above equation which are derived by the proper algorithm to minimize the errors in frequency response are given below[15] in Eq. (14):

$$K_1 = 1 \qquad \ldots (14a)$$

$$K_2 = 0.2937\alpha + 0.71216 \qquad \ldots (14b)$$

$$K_3 = 1.068\alpha^2 + 0.161\alpha + 0.3324 \qquad \ldots (14c)$$

An approximate transfer function of the fraction order band pass and band stop filters in Eq. (15) and (16), respectively have been realized by using the following leader feedback topology as given below[15]:

$$T_{FBPF}(s) = \frac{\frac{A_1}{\tau_1}S^2 + \frac{A_2}{\tau_1\tau_2}S + \frac{A_3}{\tau_1\tau_2\tau_3}}{S^3 + \frac{1}{\tau_1}S^2 + \frac{1}{\tau_1\tau_2}S + \frac{1}{\tau_1\tau_2\tau_3}} \qquad \ldots (15)$$

$$T_{FBRF}(s) = \frac{A_0S^3 + \frac{A_1}{\tau_1}S^2 + \frac{A_2}{\tau_1\tau_2}S^1 + \frac{A_3}{\tau_1\tau_2\tau_3}}{S^3 + \frac{1}{\tau_1}S^2 + \frac{1}{\tau_1\tau_2}S + \frac{1}{\tau_1\tau_2\tau_3}} \qquad \ldots (16)$$

Where, τ is the time constant and *A* is the gain factor.

The proposed circuit of fractional order band passes and band stop filter using VDTA is shown in Figs 3 and 4, respectively. The transfer function for circuit shown in Figs 3 and 4 are given in Eq. (17) and (18), respectively:

$$\frac{I_{out}}{I_{in}} = \frac{S^2\frac{gm\ gm R}{C_1}\left(\frac{R_1}{R_1+R}\right) + S\frac{gm\ gm\ gm\ R}{C_1C_2}\left(\frac{R_2}{R_2+R}\right) + \frac{gm\ gm\ gm\ gm R}{C_1C_2C_3}\left(\frac{R_3}{R_3+R}\right)}{S^3 + S^2\frac{gm\ gm\ R}{C_1} + S\frac{gm\ gm\ gm\ R}{C_1C_2} + \frac{gm\ gm\ gm\ gm\ R}{C_1C_2C_3}} \qquad \ldots (17)$$

$$\frac{I_{out}}{I_{in}} = \frac{S^3 g_m\ R\left(\frac{R_1}{R_1+R}\right) + S^2\frac{gm\ gm R}{C_1}\left(\frac{R_2}{R_2+R}\right) + S\frac{gm\ gm\ gm\ R}{C_1C_2}\left(\frac{R_3}{R_3+R}\right) + \frac{gm\ gm\ gm\ gm R}{C_1C_2C_3}\left(\frac{R_4}{R_4+R}\right)}{S^3 + S^2\frac{gm\ gm R}{C_1} + S\frac{gm\ gm\ gm R}{C_1C_2} + \frac{gm\ gm\ gm\ gm R}{C_1C_2C_3}} \qquad \ldots (18)$$

In the designed filter circuit the time constant as well as the gain is given by R_i C_i and $A_i = R_i/(R_i+R)$, respectively. By using the formula and expression given in Eqs (15) and (16), the fractional order band pass and band reject filter is designed. In this work three grounded capacitor are used. The trans-conductance of each VDTA block is considered in the deriving the expression of both the filters.

3 Simulation Results

Simulations have been performed by using PSPICE program with TSMC CMOS 180 nm technology parameters. The aspect ratio of the transistors is given in the Table 1. In the given VDTA the supply voltages are taken[8] V_{DD}= $-V_{SS}$= 0.9 V and the biasing currents I_{B1}=I_{B2}=I_{B3}=I_{B4}= 150 μA are used. According to these supply voltage and biasing current values it can be observed from the simulation results of VDTA, that the value of trans-conductance is g_{m1} = g_{m2} =

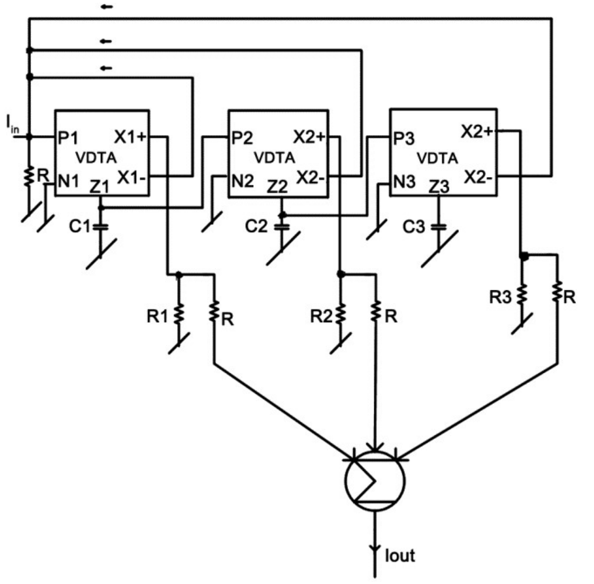

Fig. 3 – Proposed design of fractional order Band Pass filter using VDTAs.

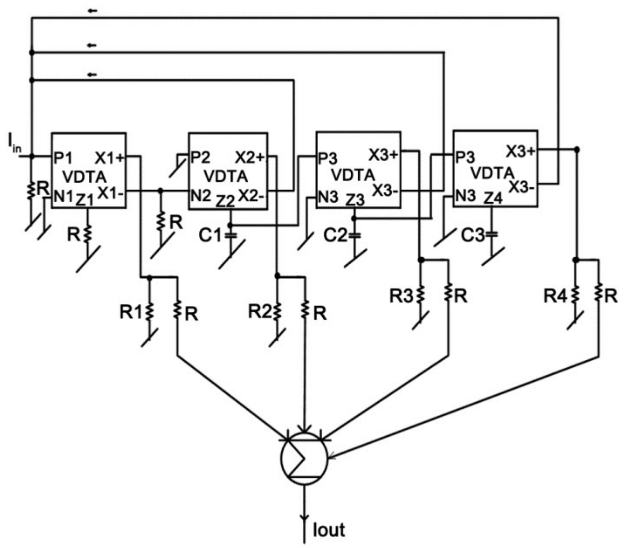

Fig. 4 – Proposed design of fractional order Band Reject filter using VDTAs.

636.6 μA/V. The DC transfer characteristics of *Ix*+ and *Ix*- *V*s V_Z is shown in Fig. 5.

The behavior of the proposed design is evaluated experimentally by using VDTA. The value of the passive components is calculated for band pass and band reject filter of different orders with Butterworth characteristics and cutoff frequency 10 kHz is given in the Table 2. Figures 6 and 7 show the response of the fractional order band pass and band reject filters of different orders using VDTAs, respectively, where the peak frequency of the band pass filter of order 1.1, 1.5 and 1.8 is 8.46 kHz, 8.09 kHz and 10.51 kHz, respectively and the peak frequency of the band reject filter of order 1.1, 1.5 and 1.8 is 9.57 kHz, 9.45 kHz and 9.52 kHz, respectively.

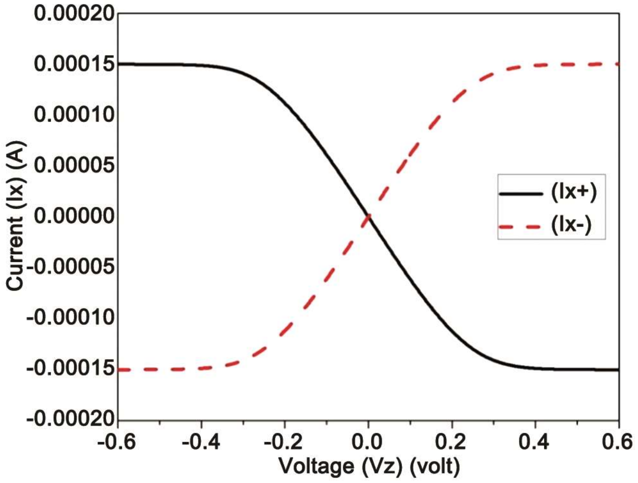

Fig. 5 – DC transfer characteristics of VDTA.

Table 1 – Transistor aspect ratio for VDTA.

Transistors	Width (μm)	Length (μm)
M1	3.6	0.36
M2	3.6	0.36
M3	16.64	0.36
M4	16.64	0.36
M5	3.6	0.36
M6	3.6	0.36
M7	16.64	0.36
M8	16.64	0.36

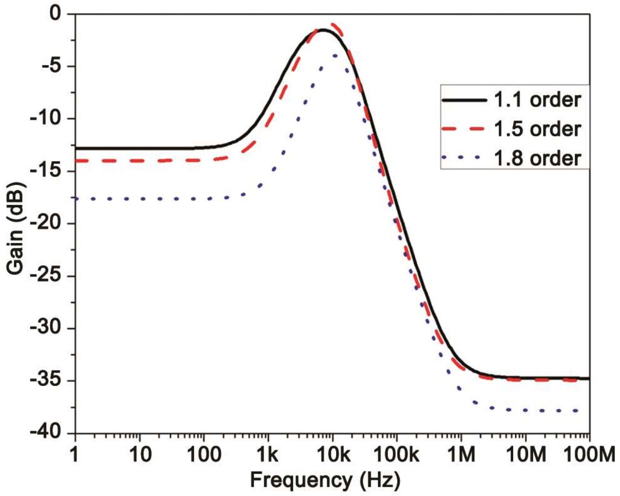

Fig. 6 – Frequency response of the band pass filter of different orders.

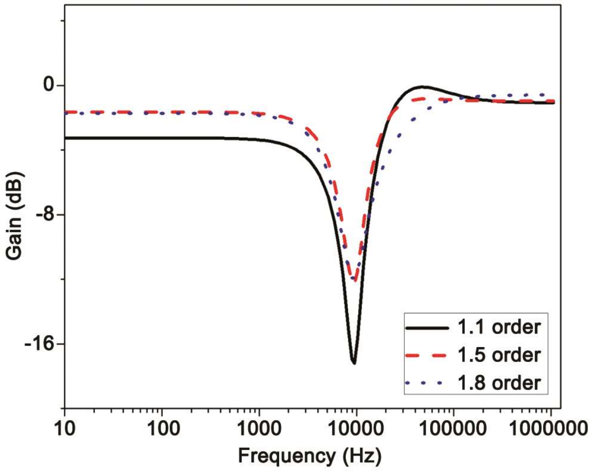

Fig. 7 – Frequency response of the band reject filter of different orders.

Table 2 – Values of the passive component used in proposed design.

Element	Band pass filter			Band reject filter		
Order	1.1	1.5	1.8	1.1	1.5	1.8
R (kΩ)	1.6	1.6	1.6	1.6	1.6	1.6
*R*1 (kΩ)	1	1	1	12	10	14.13
*R*2 (kΩ)	5.23	5.25	1.268	13	4	1.5
*R*3 (kΩ)	0.45	0.378	0.231	0.12	0.12	0.12
*R*4(k)	-	-	-	5	10	10
*C*1 (nF)	3.1	3.5	3.01	2.33	3.5	3.01
*C*2 (nF)	8.01	8.7	9.0	6.2	8.01	9.0
*C*3 (nF)	45.12	33.3	26.2	16.5	17.11	22.02

The quality factor for the band pass filter of order 1.1, 1.5 and 1.8 is 0.485, 0.52 and 0.801, respectively and for the band reject filter of order 1.1, 1.5 and 1.8 is 1.13, 0.80 and 0.348, respectively. The predicted value of peak frequency was 10 Hz. The correct operation of the proposed filters of different orders shown in Figs 6 and 7 is verified. The deviation in the response of the filter is due to the parasitic of the VDTA and the passive components, which are used in the proposed circuit. These effects can be minimized by good layout techniques.

4 Conclusions

In this work band pass and band reject filters of different orders are designed by using VDTA as an active building block. The advantage of this work is that a less number of active block and passive elements are used, which makes this design area efficient and less noisy. The VDTA active block has its own advantage in synthesis and designing. The simulation results and the mathematical expression of band pass and band reject filters of different orders; verify the functionality of the proposed work in terms of peak frequency. The design is considered as efficient design in terms of area and noise.

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References

1 Magin R L, *Fractional calculus in bioengineering bellgel house*, (Begell House: USA), 2006.

2 Jesus I S, Machado T J A & Cunha B J, *J Vib Control*, 14 (2008) 1389.

3 Faryad M & Naqvi Q A, *Prog Electromagn Res*, 75 (2007) 384.

4 Li H, Luo Y & Chen Y Q, *IEEE Trans Control Syst Tech*, 8 (2010) 516.

5 Ali A S, Radwan A G & Soliman A M, *IEEE Trans Emerg Sel Topics Circuits Syst*, 3 (2013) 346.

6 Comedang T & Intani P, *Circuits Syst*, 7 (2016) 4201.

7 Soltan A, Radwan A G & Soliman A M, *J Adv Res*, 5 (2014) 157.

8 Yesil A, Kacar F & Kuntman H, *Radioengineering*, 20 (2011) 632.

9 Prasad D, Bhaskar D R & Srivastava M, *Circuits Syst*, 4 (2013) 32.

10 Prasad D, Bhaskar D R & Srivastava M, *Indian J Pure Appl Phys*, 51 (2013) 864.

11 Prasad D, Srivastava M & Bhaskar D R, *Int Sch Res Network*, 2014 (2014) 762845.

12 Maundy B, Elwakil A & Freeborn T, *Signal Process*, 91 (2011) 484.

13 Krishna B T & Reddy K V V S, *Active Passive Electron Component*, 2008 (2008) 369421.

14 Prasad D, Srivastava M & Bhaskar D R, *Circuits Syst*, 4 (2013) 169.

15 Tsirimokou G, Psychalinos C & Elwakil A, *Design of CMOS analog integrated circuit*, (Springer International Publishing), 2017.