

## Mathematical modeling of process parameters on hard turning of AISI 316 SS by WC insert

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This paper presents a newly developed mathematical modeling for process parameters on hard turning of AISI 316 stainless steel by tungsten carbide inserts (WC). Regression analysis and ANOVA theory was used to predict surface roughness (Ra) and tool wear ( $V_B$ ). WC tool inserts performed machining of AISI 316 SS to study main and interaction effects of process parameters [cutting speed (Vs), feed rate (fs) and depth of cut ( $a_p$ )]. Adequacies of developed model were verified by calculation of correlation coefficient (r). These models can be effectively used to predict Ra of work piece and  $V_B$  of WC insert.

Keywords: ANOVA, Factorial design, Hard turning, Predictive model, Surface roughness, Tool wear

### Introduction

AISI 316 stainless steel (SS) is used in aerospace and nuclear industries. Studies<sup>1</sup> are available on machined part quality and tool wear of cutting tool inserts (WC) by various statistical methods. Optimum surface roughness in hard turning using Taquchi method is reported<sup>2</sup>. However, modeling of machining parameters [cutting speed (Vs), feed rate (fs) and depth of cut ( $a_p$ )] on surface roughness (Ra) and tool wear ( $V_B$ ) has not been carried out. In present work, mathematical model was developed to determine Ra and  $V_B$ <sup>3</sup> and main and interaction effects of process parameters (Vs, fs and  $a_p$ ) were studied.

### Experimental

#### Identification of Process Parameters and their Limits

While hard turning, important process parameters during experimentation that affect Ra and  $V_B$  are Vs, fs and  $a_p$ <sup>1</sup>. Upper and lower limits for process parameters were decided based on earlier reports<sup>4</sup> (Table 1). For recoding and processing experimental data, upper (+1) and lower levels (-1) of parameters were coded. Coded values of any intermediate levels were calculated as<sup>5</sup>

Table 1—Control parameters and their levels

Sl No	Parameter	Levels			
		Actual		Code	
		Low	High	Low	High
1	Cutting speed (Vs), m/min	24.75	94.24	-1	+1
2	Feed rate (fs), mm/rev	0.25	0.381	-1	+1
3	Depth of cut ( $a_p$ ), mm	0.4	1.0	-1	+1

$$X_i = X - \frac{(X_{\max} - X_{\min}) / 2}{(X_{\max} + X_{\min}) / 2} \dots(1)$$

where,  $X_i$ , required coded value of parameter for any value X from  $X_{\min}$  to  $X_{\max}$ ;  $X_{\min}$ , lower limits of parameter;  $X_{\max}$ , upper level of parameter. In present study,  $2^n$  ( $n$ , number of factors) trails are required to include all possible combination of levels<sup>6</sup>. Hence, number of combination of settings or treatment combinations of variables, in which experiments to be conducted, are 8.

#### Design Matrix

Using Montgomery method<sup>5</sup>, two levels full factorial design ( $2^3$  factorial designs) was used. In present study

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Table 2—Experimental design matrix and observed values of surface roughness (Ra) and tool wear ( $V_b$ )

Standard order	Coded value			Ra, $\mu\text{m}$		$V_b$ , mm	
	Vs	fs	$a_p$	Batch1	Batch 2	Batch1	Batch 2
1	-1	-1	-1	7.5	7.2	0.15	0.17
2	+1	-1	-1	5.6	5.0	0.12	0.14
3	-1	+1	-1	8.4	8.6	0.24	0.22
4	+1	+1	-1	6.2	6.0	0.10	0.12
5	-1	-1	+1	7.9	7.3	0.20	0.18
6	+1	-1	+1	5.8	5.2	0.07	0.10
7	-1	+1	+1	7.9	7.3	0.34	0.30
8	+1	+1	+1	5.9	5.0	0.08	0.10

Table 3—Yate algorithm to calculate sum of squares of surface roughness (Ra)

Order	Response, Ra		Total response	1	2	3	Sum of squares
	Batch 1	Batch 2					
1	7.5	7.2	14.7	25.3	54.5	105.8	699.60
Vs	5.6	5.0	10.6	29.2	51.3	-17.2	18.4
Fs	8.4	8.6	17.0	26.2	-8.9	-3.0	0.5625
Vs fs	6.2	6.0	12.2	25.1	-8.3	-0.6	0.0225
$a_p$	7.9	7.3	15.2	-4.1	3.9	-3.2	0.64
Vs $a_p$	5.8	5.2	11.0	-4.8	-0.9	0.6	0.0225
fs $a_p$	7.9	7.3	15.2	-4.2	-0.7	-4.8	1.4
Vs fs $a_p$	5.9	5.0	10.9	-4.1	0.1	0.8	0.04

Table 4—Yate algorithm to calculate sum of squares of tool wear ( $V_b$ )

Order	Response, $V_b$		Total response	1	2	3	Sum of squares
	Batch 1	Batch 2					
1	0.15	0.17	0.32	0.58	1.25	2.62	0.429
Vs	0.12	0.14	0.26	0.67	1.37	-0.94	0.0552
fs	0.24	0.21	0.45	0.55	-0.29	0.36	0.0081
Vs fs	0.10	0.12	0.22	0.82	-0.67	0.42	0.0110
$a_p$	0.20	0.18	0.38	-0.06	0.09	0.12	0.0009
Vs $a_p$	0.07	0.10	0.17	-0.23	0.27	-0.40	0.01
fs $a_p$	0.34	0.30	0.64	-0.21	-0.17	0.18	0.0020
Vs fs $a_p$	0.08	0.10	0.18	-0.46	-0.25	-0.08	0.0004

Table 5—ANOVA for surface roughness (Ra)

Source of variation	Degree of freedom (d.f)	Sum of squares (SS)	Mean squares (MS = SS/d.f)	F <sub>cal</sub> = MS/error	%Contribution
Replicates	1	0.86	0.86	14.8532	3.837
<b>Main effects</b>					
Vs	1	18.4	18.4	317.7892	82.103
fs	1	0.5625	0.5625	9.715	2.509
a <sub>p</sub>	1	0.64	0.64	11.053	2.856
<b>Factor interaction</b>					
Vs fs	1	0.0225	0.0225	0.3886	0.101
Vs a <sub>p</sub>	1	0.0225	0.0225	0.3886	0.101
fs a <sub>p</sub>	1	41.4	41.4	24.179	6.247
<b>Three factor interaction</b>					
Vs fs a <sub>p</sub>	1	0.04	0.04	0.6908	0.1784
Error	8	0.4632	0.0579		2.067
Total	15	22.41			

(Table 2), standard order is shown, however, experiments were performed in a random order. Work piece material (316 SS) was turned to different cutting speed levels. Ra of work piece and V<sub>B</sub> of WC were performed at different process parameters. Experiments were conducted as per conditions in design matrix to avoid systematic errors infiltrating into the system<sup>4</sup>.

**Development of Mathematical Models**

Let Y be Ra or V<sub>B</sub>, then response function/ can be given as

$$Y = f(A, B, C) \dots(2)$$

Chosen model includes effects of main variables and first order interactions of all variables. It is a portion of power series-polynomial expressed as

$$Y = \beta_0 + \beta_1 A + \beta_2 B + \beta_3 C + \beta_4 AB + \beta_5 AC + \beta_6 BC + \beta_7 ABC \dots(3)$$

**Determining Significant Factors**

Yates method<sup>5</sup> has been used to find sum of squares for main and interaction effects of Ra (Table 3) and V<sub>B</sub> (Table 4). Higher order interactions are practically insignificant and hence not considered. Significant factors and their interaction on the process were determined using ANOVA (Tables 5, 6).

**F-Distribution**

Comparison of F<sub>calculated</sub> with F<sub>critical</sub> gives insight into the significance of process parameters and their interaction with the process. This analysis was done using F-distribution at 5% and 1% significant levels and F<sub>critical</sub> values (from F-table) were obtained as 5.31 and 11.26, respectively. Based on comparison made in Table 5 and 6, following conclusions were made: 1) Replication of experiments does not affect results; 2) Except factors Vs, fs and Vs a<sub>p</sub>, all other factors have real effect on response function Ra; and 3) Except factors a<sub>p</sub>, fs a<sub>p</sub>, and Vs fs a<sub>p</sub>, all other factors have real effect on response function V<sub>B</sub>.

**Evaluation of Coefficients Models**

General model is given as

$$Y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_5 + \beta_6 x_6 + \beta_7 x_7 \dots(4)$$

Taking cognizance of findings from the comparison of F<sub>calculated</sub> and F<sub>critical</sub> values and their interactions, Eq. (4) was modified by deleting factors having no effect on response function. Since Vs fs and Vs a<sub>p</sub> factors have no real effect on Ra, Eq. (4) is rewritten as

$$Y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_6 x_6 \dots(5)$$

where β is average of response (Ra or V<sub>B</sub>) and β<sub>1</sub>, β<sub>2</sub>, ....., β<sub>i</sub> are calculated as

Table 6--ANOVA for tool wear (VB)

Source of Variation	Degree of freedom (d.f)	Sum of squares (SS)	Mean squares (MS = SS/d.f)	F cal = MS/error	% Contribution
Replicates	1	0.00005	0.00005	0.9689	0.054
<b>Main effects</b>					
Vs	1	0.0552	0.0552	106.976	60.13
fs	1	0.0081	0.0081	15.69	8.823
ap	1	0.0009	0.0009	1.744	1
<b>Factor interaction</b>					
Vs fs	1	0.0110	0.0110	21.317	11.982
Vs ap	1	0.01	0.01	19.3798	10.89
fs ap	1	0.002	0.002	3.9147	2.2
<b>Three factor interaction</b>					
Vs fs ap	1	0.0004	0.0004	0.7751	0.435
Error	8	0.00413	0.00052		4.5
Total	15	0.0918			

**SPEED Vs SURFACE ROUGHNESS RESIDUAL PLOT**

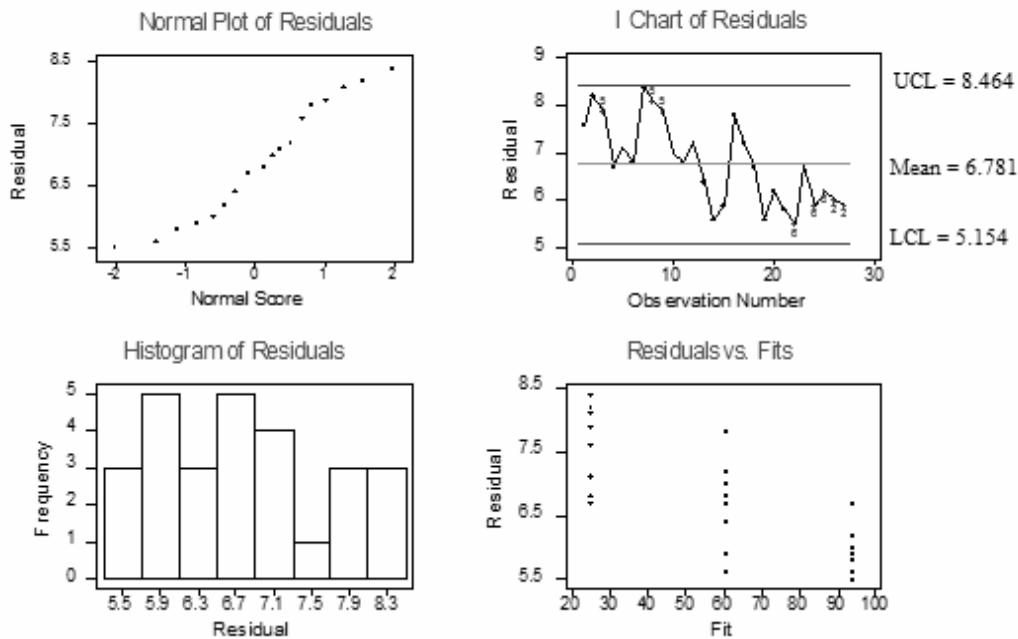


Fig. 1—Surface roughness residual plot

$$\beta_i = \frac{\sum X_i \times Y_i}{N}$$

where  $i$  vary from 1 to  $N$ ;  $X_i$ , corresponding coded value of process parameters;  $Y_i$ , corresponding response output

variable;  $N$ , total number of treatment combinations.

Thus coefficients  $\beta_0$ ,  $\beta_1$ ,  $\beta_2$ , and  $\beta_6$  were calculated. Similar procedures were adopted to find coefficients for modeling VB as a response function.

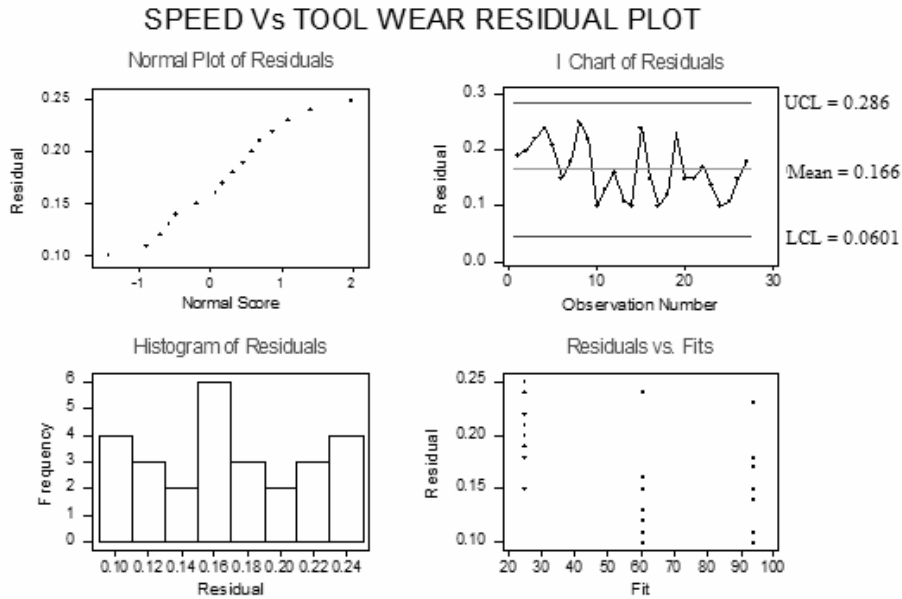


Fig .2—Tool wear residual plot

### Final Models

Mathematical models developed from ANOVA (Analysis of variance) are given as

$$R_a = 6.7 - 0.825 V_s + 0.25 f_s - 0.225 f_s a_p \quad \dots(6)$$

$$V_b = 0.1638 - 0.06 V_s + 0.0113 f_s - 0.0188 V_s f_s + 0.015 f_s a_p \quad \dots(7)$$

### Checking Adequacy of Developed Models

Coefficient of correlation ( $r$ ) is calculated as<sup>6</sup>

$$R = \frac{\sqrt{\sum (y_e - \bar{y}_0)^2}}{\sqrt{\sum (y_0 - \bar{y}_0)^2}}$$

where  $Y_e$ , estimated values of  $Y$  using final model;  $Y_0$ , observed values of  $Y$ ;  $\bar{Y}_0$ , mean of observed values of  $Y$ .

Calculated  $r$  values for  $Y_1$  model and  $Y_2$  model [Eqs (6) and (7)] are 0.8584 and 0.76 respectively. These values show that correlation values of developed models are high.

### Results and Discussion

Residual plots for response parameter  $R_a$  (Fig. 1) reveal any model inadequacy or unusual problem with normality assumptions<sup>4</sup>. Moreover, all residual values fall within control limits as well. Similar trend is observed in residual plots drawn for  $V_b$  (Fig. 2).

### Conclusions

Mathematical model developed from ANOVA efficiently predicts main effects and interaction effects of different influential combinations of machining parameters on hard turning to study work piece surface roughness and tool wear of cutting tool insert. Developed models can be used to predict values of surface roughness of work piece and tool wear of cutting tool insert from any combinations within the ranges of variable studied. Among different process parameters during hard turning, effect of cutting speed is more and combination of feed rate and depth of cut influences only lesser extent.

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