Influence study of Weibull distribution parameter on double stress down-step accelerated life test

Qingwei Liang* & Dongdong Wang

College of Marine Science and Technology, Northwestern Polytechnical University, Xi’an Shaanxi, 710072, P. R. China

* [E-mail: liangqingwei@nwpu.edu.cn]

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Based on the study of time and efficiency of double step-down-stress accelerated life test, the influence of the change of the shape parameters of Weibull distribution on the time and efficiency of the double step-down-stress accelerated life test is studied. Combined with Matlab software, Monte Carlo method is used. The simulation of different shape parameter of Weibull distribution is carried out. Results show that with the increase of sample size, the failure time curve of Weibull distribution stably intersects in 60%–70% of the sample size. When shape parameter m is relatively small, small failure sample number of double step-down-stress accelerated life test can be chosen to carry out truncation test, which will help to shorten test time. When shape parameter m is relatively large, the large failure sample number of double step-down-stress accelerated life test can be chosen to carry out truncation test. This conclusion can also be applied to all double stress accelerated life test.

[Key words: Accelerated Life Test, Monte Carlo, Weibull distribution, Shape parameter m, Sample size]

Introduction

Accelerated life test is used as a technical approach to solve the high reliability, long life product quantitative assessment. It is also one of the issues of the reliability engineering. At present, life assessment of electronic equipment is the forefront in the field of reliability engineering, and accelerated life test is the most effective method in this topic. However, using accelerated life testing, the evaluation of some high reliable and long-life equipment still requires very long time. Therefore, seeking more economical and efficient accelerated life test method becomes one of the key areas of reliability engineering. In order to improve the efficiency and reduce the cost of accelerated test, a step-down-stress accelerated life test is proposed in literature, which can improve the test efficiency in the reliability evaluation of loss type-long life products. In literature, the double cross-step-down-stress and cross-step-up-stress methods are simulated under different life distributions, and the conclusion is drawn that the cross step-down test is more effective than the cross step-up test. In literature, the double synchronous-step-down-stress accelerated life test under Weibull distribution is simulated and the conclusion is drawn that the synchronous-step-down test is more effective than the cross-step-down test.

The conclusion of the above literature on the accelerated life test of Weibull distribution is based on the analysis of the shape parameter m, the failure sample number and the samples size is determined. But for the Weibull distribution accelerated life test, what effect will be for different shape parameter m, failure sample number and sample size on test time and efficiency? In this paper, Monte Carlo numerical simulation, combining with Matlab software, is used to study the influence of shape parameter of Weibull distribution on accelerated life test. At the same time, according to the failure sample number and the sample size, the analysis of accelerated life test under Weibull distribution is carried out.

Materials and Methods

Double stress accelerated life test

In this paper, the double synchronous-step-down-stress accelerated life test is analyzed. The specific steps are as follows:
Monte Carlo Simulation of Double Stress Test

1. Hypothesis of simulation test condition

Monte Carlo method is a probabilistic simulation method, which get the approximate solution of physics and engineering problem by stochastic simulation. Exponential distribution, Gamma distribution, Weibull distribution and Lognormal distribution are commonly used distributions in product reliability research. In this paper, Monte Carlo simulation is carried out by using the double synchronous-step-down-stress accelerated life test with Weibull distribution as the example.

(1) When the product life is Weibull distribution, the product life in the normal stress level combination \((0,0)\) and accelerated stress level combination \((i,j)\) are all Weibull distribution. Then the life distribution function is:

\[
F_y(t) = 1 - \exp\{-(t/\eta_0)^m\} \quad t > 0
\]

(2) Under accelerated stress, the failure mechanism of product remains same. That is, the shape parameter \(m_0\) of Weibull distribution which express the failure mechanism remains same. then:

\[
m_{00} = m_y = m
\]

(3) The characteristic life \(\eta_0\) of the sample and two accelerating stresses satisfies the acceleration model

\[
\eta_y = \varphi(S^1, S^2)
\]

2. Monte Carlo simulation

Monte Carlo method is used to simulate the synchronous-step-down test of the Weibull distribution. The simulation steps are as follows:

(1) Random samples of Weibull distribution is generated

If the distribution function \(F(x)\) of the random variable \(X\) is continuous, then \(r = F(x)\) is uniform distribution random variable on interval \((0,1)\). It can be gotten frome formula (5), if \(p = F(x)\), then \(p = 1 - \exp\{-(t/\eta)^m\}\). \(p\) is the uniform distribution random variable on interval \((0,1)\). The sampling formula of Weibull distribution can be generated by using inverse function method\(^{15,16}\).

\[
t = \eta(-\ln(1 - p))^{1/m}
\]

So that random number \(p_1, p_2, p_3, \cdots, p_n\) of \(n\)
uniform distribution is produced on interval (0,1) by computer, which is treated as the reliability corresponding to the individual failure time of sampling. They are substituted into formula (8).

(2) The highest combination level sampling formula

The synchronous-step-down test is started at the highest level combination \((l, k)\). So \(\theta = \theta_k\). Therefore, the highest combination level sampling formula is:

\[
t_{\theta_k} = \eta_k \left( -\ln (1 - p_\lambda) \right)^{1/m} \quad k = 1, 2, 3, \cdots, n \quad (9)
\]

(3) The conversion of the failure time of each level combination

(a) the sample data generated by formula (9) arranged from small to large, the previous \(r = r_k + r_{(l-1,k-1)} + \cdots + r_k + r_{(i-1,k-1)}\) data is taken as failure sample. The remaining \(n - r\) is taken as truncated sample of fixed number truncated test. And then the time \(t_i\) \((i = 1, 2, \cdots, r_k)\) of previous \(r_k\) data is taken as the failure time of highest level combination.

(b) \(t_i\) of \(r_{(l-1,k-1)}\) data is taken. At this time, \(i = (r_k + 1, r_k + 2, \cdots, r_k + r_{(l-1,k-1)})\) minus the cumulative failure time \(t_{r,k}\) respectively, and multiply the acceleration factor \(m>1\), then the failure time of combination level \((l-1,k-1)\) can be obtained.

(c) And so on, until failure time of the lowest level combination \((1,1)\) is reached. At this time, all the failure times under all step-down-stress levels in the synchronous-step-down test are obtained.

**Results and Discussion**

**Total time of double-stress synchronous step-down test**

A set of data of the double-stress accelerated life test in literature is taken as an example, Weibull distribution is used to simulate the double synchronous-step-down stress test. A double-stress accelerated life test, which take temperature \(T\) (unit K) and voltage \(U\) (unit V) as accelerated stresses, is carried out. \(T\) and \(U\) stress levels in normal and accelerated states are as follows:

- \(T_0 = 353, T_1 = 373, T_2 = 388, T_3 = 403\)
- \(U_0 = 100, U_1 = 200, U_2 = 300, U_3 = 400, U_4 = 500\)

Sample size is \(n=100\). Sample life obeys exponential distribution accelerated model equations

\[
\ln \theta_j = -20 + \frac{20000}{T} - 4.5 \ln U
\]

Change trajectory of acceleration stress level is:

\((3, 4) \rightarrow (2, 3) \rightarrow (1, 2) \rightarrow (1, 1)\)

Total number of tests is \(L = (3 + 4 + 1)/2\). The mean value of each truncated sample is:

\[
r_{34} = 23, r_{23} = 17, r_{12} = 6, r_{11} = 5
\]

According to the simulation process method described in Section 2, Monte Carlo simulation is carried out by Matlab. Simulation carried out for 1000 times. Failure time of the double synchronous-step-down-stress life test under Weibull distribution is shown in Table 1.

Table 1 shows the accumulated test time from zero failure number to a specified failure number when the double synchronous-step-down-stress accelerated life test of the Weibull distribution \((m=1)\) is in each of the accelerating stress level combination. All the accumulated times are added as the total time required for the entire test. It is compared with the total time of the double synchronous-step-down-stress accelerated life test of the Weibull distribution \((m=1.5)\) which is carried out in literature (shown in Table 2).

From Table 2, the total time of the double synchronous-step-down-stress test of \(m=1\) is 73.78% of \(m=1.5\). This raises a question: how does shape parameter of Weibull distribution affect the step-down acceleration test? The question is studied in the following. When the double synchronous-step-down-stress test meets the truncated number, the level of two stresses are reduced at the same time. Therefore, \(T = 403, U = 500\) and \(T = 383, U = 400\) is taken (sample capacity \(n=100\)) to carry out synchronous-step-down analysis. The simulation results are shown in Fig. 1 (a) and Fig. 1 (b).
Effects of shape parameters $m$ of Weibull distribution on failure time of step-down accelerated life test

In the literature\(^\text{(1,10)}\), $m = 1.5$ of Weibull distribution is taken when Monte Carlo simulation is carried out. When shape parameter $m$ is taken different values, how it works on the failure time of double synchronous-step-down-stress accelerated life test? This paper will focus on this problem.

For Weibull distribution, the value of different shape parameter $m$ will affect the probability density of Weibull distribution. Shape parameter $m$ has a very important influence on the Weibull distribution. The stability and consistency of the production process is reflected by shape parameter $m$. The greater shape parameter $m$ is, the better the stability and the consistency is\(^\text{(19)}\).

In general, the values of shape parameter $m$ can not be too large (aggressive) and too small (conservative) in order to prevent aggressive and overly conservative. Therefore, according to the actual engineering experience, for many products and materials\(^\text{(17)}\), shape parameter $m$ is located between 0.5 and 5. In this section, analysis is carried out when shape parameter is $0.5 \leq m \leq 3$, sample size is $n = 100$ and double-stress is $T = 403$, $U = 500$. Results are shown in Fig. 2:

**Table 2**—Comparison of test data

<table>
<thead>
<tr>
<th>Stress level</th>
<th>$T_i$</th>
<th>$V_j$</th>
<th>$\eta_{ij}$</th>
<th>$r_{ij}$</th>
<th>Failure time/h</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(3,4)$</td>
<td>403</td>
<td>500</td>
<td>5.27</td>
<td>23</td>
<td>0.0018 0.1342 0.1870 0.1953 0.2783 0.3667 0.5239</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.5316 0.7070 0.8189 0.8865 0.8921 0.8970 0.9623</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.0981 1.1329 1.2272 1.3046 1.4025 1.4057 1.4103</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.5477 1.6080 1.3217 2.5196 4.2780 4.4729 4.9378 4.9598 5.1447</td>
</tr>
<tr>
<td>$(2,3)$</td>
<td>388</td>
<td>400</td>
<td>97.98</td>
<td>17</td>
<td>6.3204 8.7106 9.1017 10.2114 11.6524 15.7632 16.3254</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>19.1368 21.7098 29.0166 103.6796 105.4421 130.3430 162.0538</td>
</tr>
<tr>
<td>$(1,2)$</td>
<td>373</td>
<td>300</td>
<td>2842</td>
<td>6</td>
<td>10.4175 29.0166 103.6796 105.4421 130.3430 162.0538</td>
</tr>
<tr>
<td>$(1,1)$</td>
<td>373</td>
<td>200</td>
<td>17621</td>
<td>5</td>
<td>370.0030 372.7039 374.6515 541.0847 711.9556</td>
</tr>
</tbody>
</table>

**Table 1**—Simulation failure time of double synchronous-step-down-stress life test ($m = 1$)

From Fig. 1 (a) and 1 (b), it can be seen that the failure time trend with shape parameter $m$ of Weibull distribution is consistent when the double synchronous-step-stress $(403, 500) \rightarrow (388, 400)$ is reduced. The difference is that when the double-stress reduced synchronously, the failure time increased. Therefore, when the double-stress reduced synchronously, such as $(403, 500) \rightarrow (388, 400) \rightarrow (373, 300) \rightarrow (373, 200)$, the changes of failure time are the same with parameter $m$. Following will analyze the effect of the shape parameter $m$ of Weibull distribution on failure time of step-down accelerated life test.

**Effects of shape parameters $m$ of Weibull distribution on failure time of step-down accelerated life test**

In the literature\(^\text{(1,10)}\), $m = 1.5$ of Weibull distribution is taken when Monte Carlo simulation
It can be seen from Fig. 2 that failure time increases with the increase of shape parameter $m$. At the same time, when shape parameter grows, failure time curve of small $m$ is lower than big $m$ at the beginning. With the increase of sample size, failure time curve of small $m$ become higher than big $m$. The failure time curve of different $m$ intersects in 60%-70% of sample size.

And then, when shape parameter $m$ is different, whether the failure time curve of Weibull distribution will intersect within a certain sample range? In this paper, simulation results are shown in Fig. 3 (the upper limit of the shape parameter is infinite and the lower limit is 0.5, and the double stress is the $T = 403$, $U = 500$).

From the results it can be seen that when the upper limit $m$ is infinite and the lower limit is 0.5, failure time curve of Weibull distribution intersects 60%-70% of sample size.

The above study was carried out in the case of a sample size $n=100$. Will the different sample sizes have an impact on this conclusion? The effects of different sample sizes on the failure time of step-down accelerated life test were studied.

Influence of the different sample size $n$ on failure time of step-down accelerated life test

Corresponding to the actual project, sample sizes are set as follows: $n=50$, $n=100$, $n=150$, $n=200$, and a set of double stress $T = 403$, $U = 500$ is selected for simulation. The results are shown in Fig 4.

It can be seen from Fig. 4 that when the different sample sizes are taken, failure time curves of Weibull distribution are intersected stably in 60%-70% of sample size as the sample size increases.

Therefore, when the selected failure sample number is less than 60%-70% of sample size, the smaller the shape parameter $m$, the shorter the test time of the double step-down-stress accelerated life test will be. When the selected failure sample number is more than 60%-70% of sample size, the bigger the shape parameter $m$, the shorter the test time of the double step-down-stress accelerated life test will be.
stress accelerated life test will be. That is, when shape parameter \( m \) is relatively small, small failure sample number of double step-down-stress accelerated life test can be chosen to carry out truncation test, which will help to shorten test time. When shape parameter \( m \) is relatively large, large failure sample number of double step-down-stress accelerated life test can be chosen to carry out truncation test.

**Influence of shape parameter \( m \) on the double stress step-down accelerated life test**

It can be seen that, the change trends of shape parameter \( m \) is consistent for different products when the stress of each group step-down synchronously for double synchronous-step-down-stress accelerated life test. At the same time, above results shows failure time trend when the different shape parameter \( m \) of Weibull distribution and different sample size are simulated. These conclusions can also be applied to other double stress accelerated life test.

**Conclusion**

In this paper, the computer simulation test is carried out on the double synchronous-step-down-stress accelerated life test when the product is Weibull distribution \( m = 1 \). Failure time and the average time of the statistical test are obtained. Compared with \( m = 1.5 \), the results show that the test time of double stress accelerated life test of \( m = 1 \) is shorter than the test time of \( m = 1.5 \). The failure time of Weibull distribution of different shape parameter \( m \) is discussed. At the same time, the effect on the average time of the test of Weibull distribution is carried out when the sample size is different. Results showed that failure time curve intersected steadily in 60%-70% of sample size as the sample size increases. When shape parameter \( m \) is relatively small, small failure sample number of double step-down-stress accelerated life test can be choosen to carry out truncation test, which will help to shorten test time. When shape parameter \( m \) is relatively large, large failure sample number of double step-down-stress accelerated life test can be choosen to carry out truncation test. This conclusion can also be applied to all double stress accelerated life test.

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