Geometrical modeling for biaxial weft knitted fabrics based on rib structure

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Received 1 October 2015; revised received and accepted 23 December 2015

This study proposes a 3D geometrical model for biaxial weft knitted fabrics based on rib structure. Using modification of 3D plain loop to simplify the connection of back and face loops, unit-cell of a 1x1 rib structure is obtained. The warp and weft yarns can be drawn through this structure using Python scripting in Abaqus software. To verify the proposed geometrical model, the biaxial weft knitted fabric based on rib structure has been produced from polyester yarn on a modern flat knitting machine. Fabric is subjected to tensile loading in warp, weft and bias directions according to standard methods. Using a FEM software, tensile behavior of fabrics is numerically simulated. The results show that the geometrical model can successfully predict the tensile behavior of the biaxial weft knitted fabric.

Keywords: Biaxial weft knitted fabrics, Geometrical model, Numerical simulation, Tensile behaviour

1 Introduction

Technical textiles which exhibit desirable mechanical behavior during their end uses have attracted attention of engineers and designers. Use of technical textiles in various fields of application including cars and transportation industries, building constructions, aerospace equipment and even the clothing industries is being increased. Knitted structures due to their high formability could be used to form either simple or complex shapes for technical applications especially in composite manufacturing area¹⁻⁵. But from the other point of view, some researchers revealed that the knitted composites suffer from the lack of good mechanical performance as compared to other conventional reinforced textile composites⁶⁻⁹. Araújo et al.¹⁰ recognized the non-linear behavior of a knitted structure under tensile loading as the main reason for its low stiffness and strength properties. Researchers have offered various techniques in order to improve the mechanical behavior of weft knitted structures to be suitable for composite manufacturing¹¹⁻¹³. Considering the combination of weaving and knitting technologies, a new designed hybrid structure with improved mechanical behavior due to the constitutive straight fibres and yarns was proposed¹⁴⁻¹⁷. Along with this suggestion, it was concluded that by inserting the reinforcement yarns into the knitted structure directly either in warp (wale-wise) or weft (course-wise) directions, the anisotropy characteristics of the resulted composite would become suitable for particular application¹⁸.

Many researchers studied the application of the biaxial weft-knitted for reinforcing the thermoplastic and thermoset composites and the properties of the resulted structures¹⁶⁻²². Also, coated biaxial knitted fabrics are widely used for permanent works in various applications such as sport stadiums, transportation and commercial constructions. Indeed, these structures constitute an alternative to classical stiff roofs. Prior to the coating process, developing the analytical and numerical simulated models of reinforcing structure corresponding to their mechanical performance under different loading conditions, is considered as a key factor to achieve a desirable product ²³. So far, no geometrical model for biaxial weft knitted fabrics has been presented.

The present study has been divided into two sections. First, the 3D geometrical model for plain knitted loop proposed by Vassiliadis²³ has been modified to create a geometrical model for rib structure. In the second section, geometrical model for biaxial weft knitted fabrics comprising straight warp and weft yarns and rib structure is numerically simulated by a FEM method. This model will be helpful for predicting the mechanical and tearing behavior of these structures.
2 Modeling of Bi-axial Weft Knitted Fabrics

Bi-axial weft knitted fabrics can be produced based on different structures, namely rib, interlock and plain weft knitted fabrics. The knitted structure affects significantly on the mechanical and tearing behavior of bi-axial weft knitted fabric during the multi-axial loading. In this study, a rib knitted structure was chosen to produce the bi-axial weft knitted fabrics.

For better understanding of fabrics mechanical performance under tensile loading, developing a model of structural unit-cell will be necessary. In some literatures, researchers proposed different model of structural unit for the weft knitted fabrics. Since the biaxial weft knitted fabric is generally comprised of knit-loops and reinforcement yarns, some further modification must be applied to the models in order to do more accurate modeling of fabric structure. For this reason, the biaxial weft knitted structure must be divided into its constitutive components which are double-jersey knitted fabrics (plain rib) and a multi-layered structure of reinforcement yarns in warp and weft directions.

1x1 rib structure is produced by two sets of needles being alternately set or gated between each other. In this structure, the face and back loops on each side are connected together using a linking yarn. Ray and Banerjee simulated the 1x1 rib loop formation process on a dial and cylinder machine. Their proposed model could predict loop length and yarn tension profile inside knitting zone for any combination of input parameters. Jeddi and Zarinian attempted to present an ideal model loop for 1x1 rib knitted fabric taking into account yarn swelling. Kurbak also presented an ideal model for these knitted fabrics for full relaxation state. The both models cannot be practically used for modeling the biaxial weft knitted fabrics, because they are not a precise model and a different geometry is needed while the warp and weft yarns are inserted through the rib structure. Therefore, it will be essential to propose a new geometrical model for this structure.

The yarn is assumed to behave as a cylindrical elastic rod and follows the linear elastic property. For a rib structure, the shape of structural knitted cell is divided into two segments, one segment being the needle loop and two arms of two plain-type face loops of the cell, which follows the improved mathematics-based equation of Vassiliadis. Another segment is the linking portions between face and back loops. Then, an equation is taken as the linking portion between face and back loops. The analysis is introduced for three-dimensional models.

Figures 1(a) & (b) show the central axis and 3D unit-cell of loop structure proposed by Vassiliadis et al. The model is generated by using the main structural parameters of a single jersey fabric, the course spacing \(c\), the wale spacing \(w\) and the diameter of the yarn \(D\).

Vassiliadis et al. stated that the total loop is generated from a quarter of it using the symmetry of the ideal loop shape. Thus, only the central axis of a quarter of the loop (part EMKA) was considered. According to Fig. 1(b), the part EMKA was divided into three parts of known mathematical curves. The part EM was considered as an elliptic arc in the three-dimensional space that can be represented as the following equations:

Part EM \(0 < y < c/2\):

\[
x(y) = -\frac{D}{c} \cdot y
\]

\[
z(y) = \sqrt{(r + D/2)^2 - y^2} - (r + D/2)
\]

Considering condition \(c/2 < y < c/2 + R\) for part MK, this part can be defined using following equations:

\[
x(y) = h - a \cdot \sqrt{1 - \left(\frac{y-c/2}{b}\right)^2}
\]

\[
z(y) = \sqrt{(r + D/2)^2 - y^2} - (r + D/2)
\]

Also, part KA is defined using following equations:

Part KA \((y = c/2 + R) < x < w/4\):

\[
z(x) = OZ - \sqrt{A^2 - (x-OX)^2}
\]

\[
y(z) = \sqrt{(r + D/2)^2 - (z + r + D/2)^2}
\]

\[
OZ = \frac{x_2 - OX)^2 - (x_1 - OX)^2 + z_1^2 - z_2^2}{2z_1 - 2z_2}
\]

\[
OX = w/4
\]
To prepare a rib structure, point A' at the end of the central axis of face loop structure should be connected using a linking yarn to a corresponding loop placed in the fabric technical back. Spatial coordinate of this point cannot provide a correct linking to the back loop. On the other word the plain loop drawn according to the above equations has been just developed for a plain single jersey structure and it can not be used for a rib structure. In order to utilize this loop model for creating the 1x1 rib structure, it should be modified.

It is assumed that the face loop is connected to the back loop at point N. According to the Fig. 1(a), this point is considered as the middle point of the curve K₁M₁. Calculating the coordinates of point N is possible using Eqs (3) and (4), as shown below:

$$A = \sqrt{(x_i-OX)^2 + (z_i-OZ)^2} \quad \ldots (9)$$

$$y_N = \frac{-c-R}{2} \quad \ldots (10)$$

$$x(y_N) = h - a\sqrt{1 - \left(\frac{y_N - c/2}{b}\right)^2} + \frac{w}{2} \quad \ldots (11)$$

$$z(y_N) = \sqrt{(r + D/2)^2 - y_N^2} - (r + D/2) \quad \ldots (12)$$

Therefore, the curve NN' will be the path of linking yarn which connects this loop to its corresponding back loop at the technical back of weft knitted fabric. Fig. 1(c) depicts the geometrical model of linking yarn between points N and N'.

It is assumed that the linking yarn consists of three different parts, namely NP, PQ and QN'. To achieve the equation of central axis of linking yarn, the equations of each part should be determined.

According to Fig. 1(c), central angle $\angle NOP$ or ($\theta$) alters in range of $\theta_N$ to 90°. In order to achieve the equation of arc $PN$, curvature radius of arc (R), angle $\theta_N$ and the coordinate of point O ($\Delta_0,y_0$) are four
unknown parameters which should be calculated. The coordinates of each point placed on central axis of arc NP can be defined as follows:

\[ \Delta = \Delta_0 - R \cos \theta \]  \hspace{1cm} (13)
\[ y = y_0 - R \sin \theta \]  \hspace{1cm} (14)

In Fig. 1(c), \( \bar{n}_1 \) and \( \bar{n}_2 \) are the vertical and horizontal components of the tangent to the curve at point N respectively. By taking the derivative of Eqs (13) and (14), the amount of angle \( \theta_N \) can be determined using following equations:

\[ \tan \theta = \frac{\bar{n}_1/\bar{n}_2}{(\frac{dy}{d\theta})} = (R \sin \theta)/(-R \cos \theta) = -\tan \theta \]  \hspace{1cm} (15)

Considering the triangle ONG, curvature radius of part NP (\( R \)) can be determined using the following equation:

\[ R \sin \theta_N = y_0 - y_N = (y_p + R) - y_N \]  \hspace{1cm} (16)

As the coordinates of points N and P are already specified from Eqs (10) and (11), parameter \( R \) is calculated as follows:

\[ R = (y_p - y_N)/ \left( \sin \theta_N - 1 \right) \]  \hspace{1cm} (17)

Considering the triangle ONF, the coordinates of point O (\( \Delta_0, y_0 \)) are defined using following equations:

\[ \Delta_0 = \Delta_N + R \cos \theta_N \]  \hspace{1cm} (18)
\[ y_0 = y_N + R \sin \theta_N \]  \hspace{1cm} (19)

Therefore, the coordinates of each point which is placed on arc NP can be specified. Considering the conditions for arc NP (\( \theta_N \leq \theta \leq \pi/2 \)), the coordination for each point placed on this arc is resulted as:

\[ x = x_N + (\Delta_0 - R \cos \theta) \cos \alpha \]  \hspace{1cm} (20)
\[ y = y_0 - R \sin \theta \]  \hspace{1cm} (21)
\[ z = z_N + (\Delta_0 - R \cos \theta) \sin \alpha \]  \hspace{1cm} (22)

Considering the Fig. 1(c), part PS is assumed to be half of straight line PQ. For part PS, it can be concluded:

\[ (2L + 2R \cos \theta_N) \sin \alpha = t - D - 2(Z_N - Z_{E1}) \]  \hspace{1cm} (23)

\[ (2L + 2R \cos \theta_N) \sin \alpha = t - D - 2 \left( \sqrt{(r + D/2)^2 - y^2} - (r + D/2) \right) \]  \hspace{1cm} (24)

\[ L = \frac{t - D - 2 \left( \sqrt{(r + D/2)^2 - y^2} - (r + D/2) \right)}{2 \sin \alpha} \]  \hspace{1cm} (25)

where \( t, D \) and \( \alpha \) are the fabric thickness, yarn diameter and angle of portion PS with \( x \) axis respectively. With taking into account the conditions of the points placed on the portion PS (\( \Delta_0 \leq \Delta \leq \Delta_0 + L \) and \( x_p \leq x \leq x_p + L \cos \alpha \)), each point of this portion possesses the following coordinate.

\[ y = y_p; \]  \hspace{1cm} (26)
\[ z = \tan \alpha \cdot x + z_p - \tan \alpha \cdot x_p \]  \hspace{1cm} (27)

The only unknown parameter which should be calculated is \( \alpha \). Figure 1(d) shows the top view of face and back loops which are connected together by line NN’. The value of angle \( \alpha \) can be calculated by considering the triangle NN’F as follows:

\[ \alpha = \arctan \left( \frac{t - 2(L - 2Z_{E1})}{W - 2x_N} \right) \]  \hspace{1cm} (28)
\[ \alpha = \arctan \left( \frac{t - D - \left( \sqrt{(r + D/2)^2 - y^2} - (r + D/2) \right)}{W - 2x_N} \right) \]  \hspace{1cm} (29)

The coordinate of the point N’ will be determined as follows:

\[ x_{N'} = x_N + (R \cos \theta_N) \cos \alpha + (2L + R \cos \theta_N) \cos \alpha \]  \hspace{1cm} (30)
\[ y_{N'} = y_N \]  \hspace{1cm} (31)
\[ z_{N'} = z_N + (R \cos \theta_N) \sin \alpha + (2L + R \cos \theta_N) \sin \alpha \]  \hspace{1cm} (32)
Using modification of the plain loop to connect the back and face loops, unit-cell of a 1x1 rib structure is obtained [Fig. 2(a)]. The warp and weft yarns can be drawn through this structure as shown in Fig. 2(b).

3 Materials and Methods
The biaxial weft knitted fabric based on rib structure was fabricated on a modern flat knitting machine (Stoll CMS 400, E5). In these flat knitting machines which are equipped with close-type carriage, some modification is required for warp yarns insertion. A new kind of yarn guide (Fig. 3) which can be fixed on the yarn feeder guide rails must be designed for this condition. The yarns as the warp reinforcements are threaded from both sides of the flat knitting machine and are fed to this warp yarn guide. The hole gauge of this yarn guide is equal to the needle gauge. Since the knitting quality and the ultimate mechanical behavior of the resulted biaxial knitted structure is strongly affected by the yarn tension, employing a yarn creel unit equipped with suitable tensioners would be essential for providing a constant tension to the whole yarns.

Two kinds of polyester yarns were used for stitch and straight yarns. Linear density of stitch yarn and straight yarn (warp and weft yarns) are 850 and 2780 denier respectively. The thickness of the fabrics was measured as 2.5 mm. Three weft and four warp yarns per centimeter were inserted in the knit structure. Also the knit density of rib structure was 15.4 loop/cm². The view of produced biaxial weft knitted fabric based on rib structure is depicted in Fig. 2(d).

In order to specify the tensile properties including stiffness (E), tenacity and breaking elongation of the stitch and straight yarns, they were tested by Zwick tensile tester according to ASTM D2256.

Five specimens (5 x 15 cm) of biaxial weft knitted fabric were cut and tested by Zwick tensile tester in course, wale and bias directions according to ASTM D 5034-95. All samples were conditioned at 65%±2% relative humidity and 21±1 °C temperature prior to each test.

3 Results and Discussion
3.1 Verification of Proposed Model Results
A program was created in Python software based on the mentioned model and governing equations. Entering fabric thickness, wales and courses distance and yarn diameter as input data, the program rendered three dimensional dots coordinates. Modeling the rib structure is feasible through solid elements. The
modeling and simulation of the mechanical tests of fabric unit-cell is performed by the Abaqus software.

Linear elastic behavior has been assumed for each yarn. Elastic constants (E) assigned to weft, warp and ground yarns are 113, 113 and 176 MPa respectively; and $\nu$ (Poisson’s ratio) is 0.3 for all the yarns.

Contacts between all yarns pairs are considered in the unit-cell model in Abaqus software. Penalty method is used as the contact enforcement method to apply contacts in calculations. After the geometrical design a meshing phase is followed. The generation of the mesh, i.e. the segmentation of the virtual bodies into small parts, is a stage of great significance for the accuracy of the simulation since it presides over the success of the load transfer, the stress and strain distribution. The meshed unit-cell shown in Fig. 2(b) consists of 8417 reduced integration linear hexahedral elements (C3D8R).

To solve the task, the explicit numerical method of Abaqus software is used, characterized in that the calculation results are obtained by explicit integration method. Loading on unit-cell is carried out in course, wale and bias directions.

Since the unit-cell represents the full thickness of the fabric, the upper and lower surfaces are unconstrained. Periodic boundary conditions are applied to the other surfaces of the model, based on the coordinate system shown in Fig. 2(d), as shown below:

- Displacement in $z$ direction (normal to the fabric plane) is identical for points with similar $x$ on bottom and top surfaces.
- Displacement in $z$ direction is identical for points with similar $y$ on left and right surfaces.
- The $y$ distance of all points with similar $x$ on bottom and top surfaces should be equal.
- The $x$ distance of all points with similar $y$ on left and right surfaces should be equal.

Figures 4(a) and (b) show the stress distribution under course-wise and wale-wise loading. It can be seen in Figs 4(a) and (b) that the weft yarn bears the maximum stress in coarse-wise extension while the warp yarn bears the maximum stress in wale-wise extension, as expected.

To verify the accuracy of the geometrical model, it is numerically simulated using a FEM software. Then, the stress-strain curve resulted from the experimental method is compared with the corresponding curve extracted from FEM software.

For a particular testing direction, the tensile behaviour of the rib structure is non-linear. A careful examination of the rib structure behaviour during testing can reveal a two-stage deformation process. In the first stage, the deformation of the knitted fabric is due mainly to the straightening of the curved yarns. This process continues up to the jamming state, in which these straightened yarns start to elongate\(^{10}\).

It should be stated that inserting the warp and weft yarns through the rib knitted structure will change the mechanical behavior of the structure in wale, course and bias directions. When the biaxial weft knitted fabric is subjected to coarse-wise or wale-wise tensile loading, a linear stress-stain curve is resulted. Although both straight yarns and knit structure are subjected to tensile loading, but due to different extensibility of faric components, warp or weft yarns are elongated to breaking point before straightening of the curved yarns and association of knit structure. The curve continues up to the peak point in which the warp or weft yarns break. The applied tensile force at peak point is so great that is able to break the straightened yarns of the knitted structure. Consequently, the biaxial weft knitted fabric breaks
completely. Indeed, the warp or weft yarns are the only elements which tolerate the wale-wise or course-wise loading. This can be observed in simulated stress distribution of fabric unit-cell [Figs 4(a) and (b)].

Figure 5 shows the stress-strain curves resulted from experimental and modeling methods for the state in which the biaxial weft knitted fabric is subjected to course-wise and wale-wise loading. The comparison of both curves shows that the model can predict successfully the tensile behavior of the structure in weft or warp directions.

Due to anisotropic properties of the biaxial knitted fabrics, their mechanical properties in different directions are not the same. For verification of the proposed model, the tensile properties of the structure in the bias direction (45°), where the knitted structure associates in load bearing, has been investigated.

Figure 4(c) depicts the stress distribution of fabric unit-cell under bias loading. The figure shows that loops are tolerating the applied force. The stress-strain curves resulted from experimental and numerical modeling in bias direction are shown in Fig. 6.

Opposite to linear behavior of the biaxial knitted fabric in weft or warp directions, a non-linear behavior can be observed in the stress-strain curve obtained from bias direction. At the first stage of this curve, applied force increases slightly until jamming state of the knit loops. In this stage, applying a small amount of the tensile force creates high fabric elongation that refers to a low stiffness. After jamming state, the curved yarns are straightened and the slope of the curve will increase. Presence of weft and warp yarns through the knit structure cause to increase the inter-yarn frictions, which results in an increase in resistance against the tensile force. Also, the results reveal that biaxial knitted fabric shows lower breaking force in bias direction than in the weft and warp directions, while the breaking elongation is significantly higher.

Generally, the stress-strain curves extracted from FEM software and experimental method show good conformity in Bias direction. This means that the proposed geometrical model can be successfully used for predicting the mechanical properties of biaxial weft knitted fabric.

Considering Figs 5 and 6, it can be found that stress values obtained from numerical modeling method is slightly higher than that obtained from experimental method. It might be due to the following experimental issues, which are not included in the simulations:

- Yarns tensile properties such as yarn tensile strength and elongation are reduced during knitting process.
- The fabric gripped between tensile tester jaws is not probably stretched adequately and the weft and warp yarns may be in relaxed states before test initiation.

4 Conclusion

A geometrical model has been proposed for biaxial weft knitted structure. This structure is numerically simulated using FEM software. Tensile behavior of the fabric in weft, warp and bias direction is studied. When the biaxial weft knitted fabric is subjected to tensile loading in coarse-wise or wale-wise directions, warp or weft yarns are the only elements which tolerate the wale-wise or course-wise loading. Therefore, the fabric shows a linear behavior during tensile loading. Opposite to linear behavior of the biaxial knitted fabric in weft or warp directions, a non-linear behavior can be observed in the force-elongation curve. This shows the association of knit
structure in load bearing. Comparison between experimental and modeling results show that the geometrical model can successfully predict the tensile behavior of the biaxial weft knitted fabrics.

References