Steel fibers with non-circular cross-section used as reinforcement in self-compacting concrete

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The paper evaluates the methods of consideration of a non-circular shape of the cross-section of a steel fiber in determination of the aspect ratio of the fiber as well as the influence of these fibers on mechanical properties of the matrix. The analysis has been performed on the example of a composite made from self-compacting concrete and two types and three volume ratios (0.5%, 1.0% and 1.5%) of steel fibers with rectangular and part-of-a-circle cross-sections. Compressive and flexural tests have been performed. The influence of the above mentioned steel fibers on the mechanical properties of SCC has been analyzed in the view of the latest investigations performed on cement-based materials reinforced with fibers with non-circular cross-sections. Based on the research it is found that the compressive strength is slightly affected by the fibers. The flexural behavior of the SCC reinforced with non-circular fibers is comparable to that with the circular ones only in case of long corrugated fibers. In case of these fibers pronounced improvement of the flexural parameters has been observed proportional to the amount of fibers. The influence of short hooked fibers on the flexural behavior of SCC is less predictable and also different from the one observed for circular fibers. It has been shown that the differences in definition of the aspect ratio of the fiber without consideration of its cross-sectional shape can lead to underestimation of the aspect ratio. The real aspect ratio was used in the formula to describe the flexural tensile strength of SFR-SCC, however, it seems not to be crucial as other factors are unknown.

Keywords: Steel fibers, Aspect ratio, Flexural tensile strength, Toughness

Randomly distributed short steel fibers are a popular addition to brittle concrete matrix which enhances mechanical properties, ductility and energy absorption capacity under impact of the matrix. Steel fiber-reinforced concrete (SFRC) is commonly used in structural engineering, mainly in road pavements, lining or tunnel segments and slabs

The market of steel fibers has developed widely in the recent years. Currently, about 30 major global producers offer steel fibers with a great variety of geometrical parameters. The efficiency of steel fibers in the concrete matrix along with the mechanical and physical/chemical properties depend strongly on their geometrical parameters like diameter, length, longitudinal profile and cross-sectional shape. To improve the mechanical anchorage of the fiber in the matrix the shape of the fiber can be modified along its length, e.g., the fiber can be crimped, hooked or flattened at the ends. According to Katzer and Domski, hooked types are the most popular next to the deformed wire and crimped fibers. As far as the cross-sectional shape is concerned, the most popular is circular. However, the researchers found the advantages that may be achieved by changing the shape of the cross-section of the fiber from a circle into a more sophisticated figure. The greatest enhancement was obtained by creating the Torex fibers, which joint the advantages of non-circular cross-section with a twist longitudinal shape. Nowadays, fibers with rectangular, section-of-a-circle or triangular cross-sections are eagerly investigated.

The most important factor which characterizes geometrical parameters of steel fibers is an aspect ratio. The aspect ratio is a main parameter used by researchers, also by the authors, to compare the effectiveness of the fibers in the concrete matrix. According to the newest statistics, the aspect ratio of the fibers available in the market can be in a wide range from 20.4 to 152. However, 50% of all types offered by the producers have an aspect ratio ranging from 45 to 63.5. It directly influences the workability and spacing of fibers in fresh concrete mix.

The paper deals with an issue of how the cross-section of the fiber should be taken into account in the calculations of its aspect ratio. The paper proves that
calculations made according to the current recommendations lead to overestimation of this parameter and, as a result, lowering the real value of the aspect ratio of the fibers with non-circular cross-sectional shape. The authors believe that the use of the proposed method for calculation of the aspect ratio will lead to some further conclusions.

Fibers with rectangular and a part-of-a-circle cross-section were chosen to investigate the above mentioned problem. These fibers are available in the market but are not well recognized. Thanks to the rectangular cross-section a hook shape was attained in relatively short fibers (20 mm), which makes them untypical also considering the length. Thus, analyzing these fibers can be valuable taking into account the limited knowledge about their behavior in SCC matrix. The compressive strength and flexural tensile strength of such hardened SCC were studied and analyzed in the view of the state of the art presented elsewhere11-20.

Evaluation of an Aspect Ratio of Fibers with Non-circular Cross-section

As the aspect ratio (the ratio of the length \(l\) to the diameter \(d\) of the fiber) is an important parameter in analysis of the effectiveness of the fibers in the matrix, it should be properly determined for every cross-sectional shape of the fiber. The EN 14889-1:200623 proposes to replace the non-round cross-sectional shape of a fiber with an equivalent circular shape, with a diameter calculated from the cross-section with the same area. According to ref. 23 the equivalent diameter of a fiber with non-circular cross-section can be determined:

For a rectangular cross-section as:

\[
\frac{\pi w d}{4} = \phi \tag{1}
\]

where \(w\) and \(t\) are the width and depth of the fiber;

For non-circular cross-section (also section-of-a-circle) as:

\[
\frac{\pi m \cdot 10^6}{l_d \cdot \pi \rho} = \phi \tag{2}
\]

where \(m\) is mass of the fiber, \(l_d\) is developed length of the fiber, \(\rho\) is steel density.

The developed length is a length of the deformed fiber after straightening the fiber without deforming the cross-section.

Thus, according to ref.23 the equivalent diameter, here designated as \(d_{EN}\), is dependent only on the cross-sectional area \(A\) and is equal:

\[
d_{EN} = 1.13\sqrt{A} \tag{3}
\]

Changing the shape of the cross-section from a circle into any other cross section increases the effectiveness of the fibers in the composite by 13%. The equivalent diameter \((d_{EN})\) is independent from the cross-sectional shape of the fibers, which can be seen in Fig. 1 (green line).

The approach that permits to consider the cross-sectional shape of the fibers in the aspect ratio was proposed by Naamanin1 by using the fiber intrinsic efficiency ratio (FIER):

\[
FIER = \frac{\psi - l}{A} \tag{4}
\]

where \(\psi\) is perimeter of the fiber cross-section, \(l\) is length of the fiber, \(A\) is cross-sectional area.

To avoid introduction of another parameter (FIER) into the analysis, the calculation of the FIER parameter for the circular fiber is given as:

\[
\frac{\psi - l}{A} = \frac{4 l}{d} \tag{5}
\]

The equivalent diameter can be derived from Eq. (5) to obtain a simple method for calculation of the equivalent diameter:

\[
d_{FIER} = \frac{4 A}{\psi} \tag{6}
\]

Fig. 1 — Variation of the equivalent diameter according to Eqs (5) and (8)
Using $d_{\text{FIER}}$ ensures more accurate determination of the aspect ratio for all kinds of cross-sectional shapes in contrary to application of $d_{\text{EN}}$, which is discussed in this paper. The problem has been presented on the example of two types of steel fibers: hooked with rectangular cross-section (KE) and corrugated with a section-of-a-circle cross-section (SW). As mentioned above, the effectiveness of the fibers with the same cross-sectional area depends on the shape of their cross-section. Changing the cross-section from a circle into a rectangle or a part-of-a-circle causes an increase in FIER, which reduces the equivalent diameter (Fig. 1). This reduction is related to the height-to-radius ($h/r$) or width-to-depth ($a/b$) ratio. For the studied fibers the $h/r$ was equal to 0.44 and $a/b$ achieved the value of 0.29. As the fibers with different cross-sectional area were considered, the final value of $d_{\text{FIER}}$ was equal to 0.77 and 0.93 for KE and SW fibers, respectively (Table 1).

The comparison of the $d_{\text{FIER}}$ with $d_{\text{EN}}$, made with an assumption of the unified area of the cross-sections of different types, implies that the diameter according to EN23 is overestimated (Fig. 1). This leads to lowering of the real aspect ratio of the fibers with non-circular cross-sections. The values of the aspect ratio calculated with the use of $d_{\text{FIER}}$ were equal to 26 and 39 for KE and SW steel fibers, respectively (Table 1). Comparing the values with the conventionally determined aspect ratios, which were 19 and 27 for KE and SW fibers, respectively, a non-negligible increase (more than 30%) can be noted. Thus, to compare other fibers parameters which influence their effectiveness in concrete matrix (longitudinal shape, material, etc.) firstly the aspect ratio of the fibers should be properly determined taking into account the actual cross-sectional shape.

**Experimental Procedure**

**Materials, mix design and specimen preparation**

The materials used in the composition of the self-compacting concrete were as follows: Portland cement CEM I 42.5R, natural sand (0-2 mm), two fractions of coarse aggregate (2-16 mm), silica fume, stabilizer and superplasticizer. The composition of the

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**Table 1 — Properties of steel fibers**

<table>
<thead>
<tr>
<th>Cross-sectional shape</th>
<th>Designation</th>
<th>Length [mm]</th>
<th>Width [mm]</th>
<th>Depth [mm]</th>
<th>Effective diameter ($d_{\text{EN}}$)</th>
<th>Effective diameter ($d_{\text{FIER}}$)</th>
<th>Aspect ratio [23] (length/$d_{\text{EN}}$)</th>
<th>Aspect ratio [1] (length/$d_{\text{FIER}}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>KE</td>
<td>20 ± 1</td>
<td>1.7 ± 0.017</td>
<td>0.5 ± 0.05</td>
<td>1.04</td>
<td>0.77</td>
<td>19</td>
<td>26</td>
</tr>
<tr>
<td></td>
<td>SW</td>
<td>35 ± 3,5</td>
<td>2.30 ± 2.95</td>
<td>0.7 ± 0.07*</td>
<td>1.28</td>
<td>0.93</td>
<td>27</td>
<td>39</td>
</tr>
</tbody>
</table>

Fibershape flat with hooked end corrugated

Tensile strength [MPa] 770 ± 115 800 ± 120
e (GPa) 210 201

*—measured by author

**Table 2 — Composition of SFR-SCC mix**

<table>
<thead>
<tr>
<th>Cement composition (%)</th>
<th>Natural sand (0-2mm) (kg/m³)</th>
<th>Coarse aggregate (2-8mm) / (8-16 mm) (kg/m³)</th>
<th>Water (kg/m³)</th>
<th>Silica fume (kg/m³)</th>
<th>Steel fibers (%) by volume</th>
<th>Superplasticizer (kg/m³)</th>
<th>Stabilizer (kg/m³)</th>
<th>W/B</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>S</td>
<td>CA</td>
<td>W</td>
<td>SF</td>
<td>F</td>
<td>SP</td>
<td>ST</td>
<td></td>
</tr>
<tr>
<td>485</td>
<td>749</td>
<td>467.7 / 467.7</td>
<td>203</td>
<td>48.5</td>
<td>0.5-1.0-1.5</td>
<td>17.2</td>
<td>1.6</td>
<td>0.38</td>
</tr>
</tbody>
</table>

**Table 3 — Chemical and physical properties of cement CEM I 42.5R**

<table>
<thead>
<tr>
<th>Physical properties</th>
<th>Setting time (h:min)</th>
<th>Compressive strength (MPa) after days</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Initial setting</td>
<td>Final setting</td>
</tr>
<tr>
<td></td>
<td>Initial setting</td>
<td>Final setting</td>
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<td></td>
<td>Initial setting</td>
<td>Final setting</td>
</tr>
</tbody>
</table>
SCC mix is presented in Table 2. High content (485 kg/m$^3$) of cement (Table 3) was applied, which has been also investigated by other researchers$^{25}$. Silica fume in the amount of the 0.6% of cement mass, which chemical properties are presented in Table 4, was used in the mix. The Glenium SKY 592 Superplasticizer and Stabilizer RheoMATRIX were applied to the mix in the amounts of 3.5% and 0.3% of the mass of cement, respectively (Table 5). The water/binder ratio was equal to 0.38.

Two types of commercially available steel fibers, which geometrical parameters were discussed above, were used as reinforcement. Each type of fibers was added to the SCC by 0.5%, 1.0% and 1.5% of concrete volume. Geometrical and mechanical properties of the fibers are given in Table 1.

The mix proportioning system, used by Okamura and Ozawa$^{26}$, assumes general supply from ready-mixed concrete plants. In the mixing procedure the sand and coarse aggregate are firstly mixed for one minute and then the cement is added to the mixer (Fig. 2). After one minute, 70% of water with fibers is applied to be mixed for the next minute. Then, 20% of water with superplasticizer is added and mixed for the next 2 min. Last part of the water and stabilizer are the final components which are mixed till the whole process of mixing is finished after 7 min from the beginning$^{27}$.

### Tests
The rheological properties were tested with a slump-flow test and L-box test 10 min after the end of mixing (Table 6). The air content was determined with the pressure method and stability of the mix was determined with the VSI test. Between measurements the mix was kept covered in a mixer. The tests were conducted in the temperature of 20°C.

Mechanical properties in compression and flexure were tested on 28-daySFR-SCC specimens. Compressive tests were carried out in 3000 kN hydraulic compression testing machine on cubes with dimensions of 150×150×150 mm according to PN-EN 12390-3$^{28}$. For every mix 6 specimens were tested with a constant strain rate.

The laboratory investigations were performed according to ASTM C1609$^{29}$ to obtain the flexural
Results and Discussion

Compressive tests

The cube compressive strength of SFR-SCC mixes is presented in Table 6, where the coefficient of variation is given in parentheses. The matrix compressive strength was equal to 85.7 MPa. The test results indicate that steel fibers have a minor effect on $f_c$ (Fig. 4). The addition of steel fibers caused variation of the compressive strength of the SCC matrix from -2% to 9%. The observed failure mode of the specimens confirms this fact. The specimens reinforced with fibers did not break apart but behaved in a ductile manner. The degree of the specimen destruction decreased with an increase of the amount of fibers.

Steel fibers similar to these presented in this paper were tested elsewhere where a pronounced increase or a decrease of $f_c$ was noted. Hooked fibers (20 mm long) with rectangular cross-section (1.8×0.5 mm) and $l/d = 20$ were analyzed by Khaloo et al. The tests were performed on medium- (40 MPa) and high- (60 MPa) strength self-compacting concrete. The adverse to be presented in this paper compressive response was reported. In general, the compressive strength was decreased in the range of 0.5%–2.0% of the amount of fibers, where the reduction of $f_c$ was more pronounced in medium-strength SCC. A decrease in $f_c$ from 4% to even 18% with an increase of fibers volume ratio in medium-strength self-compacting concrete was observed. The compressive strength of the high-strength SCC was less sensitive to the addition of fibers where only 7% decrease was noted.

The highest amounts of the KE fibers comparable to these presented in this paper were examined in SCC matrix by Ponikiewski et al. The amounts of fibers equal to 1.2%, 1.78% and 2.0% resulted in a pronounced increase of $f_c$ equal to 38%, 30% and 27%, respectively. The compressive strength of the SCC matrix was 73.4 MPa.

The fibers with non-circular cross-section but corrugated shape were analyzed. An enhancement in the compressive strength was noted while in ref. a decrease of this property was reported.

Khaloo et al. performed a research of two types of concrete matrices with strengths equal to 30 MPa and 45 MPa. Three volume ratios equal to 0.5%, 1.0% and 1.5% of two types of fibers with cross-sectional dimensions: 0.35×0.8 mm and 0.35×1.0 mm, lengths of 25 mm and 35 mm and $l/d$ equal to 42 and 52, respectively, were applied. The longer fibers increased the compressive strength in case of both concrete matrices. However, the concrete with lower $f_c$ was more sensitive to the addition of longer fibers for all investigated amounts of fibers. 10% and 7% of increase in $f_c$ was noted for the matrix compressive strength equal to 30 MPa and 45 MPa, respectively. Presence of shorter fibers resulted in smaller increase of $f_c$. The compressive strength did not increase proportionally to the fibers content.

Mohammadi et al. also investigated two lengths (25 mm, 50 mm) of fibers with cross-section 0.6×2.0 mm with aspect ratio equal to 27 and 54, respectively. Volume ratios of steel fibers applied to the concrete matrix with $f_c$ equal to 57.8 MPa were 1.0%, 1.5% and 2.0%. The enhancement of $f_c$, in contrary to ref., decreased with the length of fibers. The maximum increase in the compressive strength was equal to 11% and 26% for 50 mm and 25 mm long fibers, respectively.

Chen et al. noted a decrease in $f_c$ determined on specimens prepared from steel fiber-reinforced recycled aggregate concrete, where 32 mm long fibers were used. A decrease in the range of the amount of fibers from 0.5% to 1.5% was non-negligible and equal to up to 18%.
Kaïkea et al.\textsuperscript{18} investigated 55 mm long fibers in high performance concrete with the compressive strength of the matrix equal to 75 MPa. For fibers volume ratio equal to 1\% and 2\% the authors noted a pronounced increase of $f_c$ equal to 16\% and 29\%, respectively. The same length of a fiber was analyzed by Ponikiewski\textsuperscript{19}, where for $V_f = 1\%$ the increase of $f_c$ was also significant and equal to 20\%\text{÷}26\%, depending on the tested element. The main purpose was to determine the distribution of the fibers in self-compacting concrete, so the results of the compressive strength were strongly dependent on the element tested. For shorter fibers (35 mm) of the same type also an increase in $f_c$ was noted.

El-Dieb\textsuperscript{20} investigated 25 mm long twisted steel fibers with triangular cross-section (0.5 mm). The amounts of fibers of 0.08\%, 0.12\%, 0.52\% were applied in self-compacting concrete to perform compressive and splitting tensile tests. Analyzing compressive strength, a pronounced increase was noted as the percentage of fibers increased (from 12\% to 23\%).

Most of the referred works indicate an improvement of the compressive strength of the matrix reinforced with steel fibers with non-circular cross-sections, reaching even 30\%\textsuperscript{12-15,17-20}. Only some researchers noted a decrease of $f_c$\textsuperscript{11,16}. The ambiguity of increase or decrease of the compressive strength due to addition of steel fibers noted by researchers can be explained. It is well-known that the fibers affect mostly the post-cracking behavior of the matrix. Generally, the fibers delay the appearance of cracks and prevent from their farther development due to fiber bridging effect\textsuperscript{2}. The negative effect connected with inclusion of the fibers is the fact that the fibers are a kind of perturbation in the matrix, which causes rather a decrease of $f_c$. Further, based on the results presented in refs\textsuperscript{11,13}, the concrete of higher strength was less sensitive to the application of the fibers. Considering the above, there are no obvious dependencies between the value of the compressive strength of SFRC and the length or aspect ratio of the fibers.

**Flexural tests**

The average load–deflection curves obtained in flexural tests taken from three specimens are presented in Fig. 5a and 5b for KE and SW fibers, respectively. The paper covers the first-crack, peak and post-peak parameters from third-point bending tests. The first-crack strength, flexural tensile strength and residual strengths were calculated according to the equations:

Equivalent bending strength:

$$f = \frac{PL}{bh^2}$$

where $L$, $b$ and $h$ are the span, width and depth of the beam.

Flexural toughness per unit volume under flexure was determined as the area under the load–deflection ($F$–$\delta$) curve to the deflection equal to $L/150=2$ mm:

$$T_u = \int_0^s F(\delta)d\delta$$

The calculated mean values of mechanical properties with the coefficient of variation were summarized in Table 7.

Referring to Fig. 5, randomly distributed short steel fibers improved the peak and the post-peak mechanical properties of the reference mix for both types of fibers. However, the SW fibers showed better performance than the KE fibers in case of all flexural parameters.

The flexural tensile strength of plain SCC was equal to 4.96 MPa. The addition of SW steel fibers caused a pronounced increase in $f_{ts}$ (Fig. 5b). For the highest investigated amounts of fibers the increase was equal to 56\% of $f_{ts}$ of the reference mix. Flexural tensile strength and other investigated flexural parameters (toughness, residual strengths) increased proportionally to $V_f$ of SW fibers.

Flexural behavior of SCC reinforced with KE steel fibers was less predictable than in case of SW fibers. An increase of flexural tensile strength was low and disproportional to the amount of fibers. Further, the bending response changed from brittle into ductile when the 1.5\% amount of fibers was applied. In that case, the deflection corresponding to the maximum load was much larger than for other tested mixes with KE fibers. That fact has its reflection in the values of toughness. For mixes reinforced with 0.5\% and 1.0\% it was almost the same and for 1.5\% a pronounced increase was noted (Fig. 6b).

Similar randomly distributed fibers, were investigated in refs\textsuperscript{11,12} Comparable changes in the shape of the load–deflection curve obtained in flexural tests on SCC reinforced with KE steel fibers were also noted in ref.\textsuperscript{12}
The fibers dosage for which a pronounced increase in the deflection corresponding to the maximum load was noted was equal to 2% while for 1.78% the response was comparable to the lower amounts of fibers (1.2%). However, in these investigations the three-point loading tests on notched beams of much bigger size (150×150×600 mm) were performed. Khaloo and Afshari\(^\text{11}\) admittedly investigated SCC with short hooked fibers with rectangular cross-section in three-point bending tests, however, on slabs (100×140×1200 mm) with a span equal to 1100 mm. In the analyzed range of the amounts of fibers (0.5%-2.0%) only brittle flexural response was observed.

The explanation of the behavior described in this paper and in ref.\(^\text{12}\) can be found in the Naaman’s classification\(^\text{8}\) of steel fiber-reinforced concrete (FRC). According to the shape of the load-deflection curve, the flexural response of the fiber-reinforced cement-based materials can be deflection-softening or deflection-hardening (Fig. 7). Analyzing the test results with KE steel fibers, the strengthening phase of the load-deflection curve after achieving the first crack load was noted in case of 1.5% of the fibers (Fig. 5a). It is worth to point out that in all cases a single failure crack was noted. Thus, perhaps the critical volume fraction (\(V_{f_{\text{cri}}}\)) of fibers needed to achieve deflection-hardening behavior in bending was found in case of KE fibers.

Flexural tests, where corrugated fibers with non-circular cross-sectional shape were applied, were also reported in refs.\(^\text{13-15,18}\). The details about the amount of fibers and their geometrical parameters were described earlier. Flexural tests were performed on beams with dimensions of 100×100×500 mm in four-point bending tests\(^\text{14,15}\), and in three-point bending test where the beams were notched\(^\text{18}\). The researchers noted a pronounced increase in the flexural tensile strength which was proportional to the increase in the amount
Randomly distributed short steel fibers prevent or delay initiation and propagation of matrix cracking which was affirmed in all the referred papers where a pronounced enhancement of toughness with an increase in $f_{ts}$ was noted. The fibers attract the cracks according to their length. The micro fibers are applied to prevent the micro-cracks formation, while the longer ones arrests macro-cracks. On one side, the increase of the length of the fibers should cause the increase of the post-peak parameters. On the other hand, the total number of short fibers in the cross-section is higher than in case of the longer ones for the same volume ratio of fibers. Additionally, the short fibers are usually more homogenously dispersed in the matrix comparing to the longer ones. However, even though the number of the long fibers was lower in the cross-sectional area, that kind of fibers was pronouncedly more effective in increasing the flexural parameters of SFRC than the shorter ones. The increase of these properties was in most cases proportional to the amount of long fibers with a high aspect ratio.

In case of the matrix reinforced with non-circular short and hooked fibers the flexural behavior was rather unpredictable. The fibers with circular cross-section caused a proportional increase of flexural parameters of concrete even for short fibers. Their effectiveness in the matrix depended mainly on their longitudinal shape, which is responsible for the anchorage of the fibers in the matrix. This is the main difference observed between the behavior of the fibers with circular and non-circular cross-section.

**Formulas to predict the flexural response**

The formula used to calculate the strength at first cracking of the composite in tension proposed in ref. was adopted in this paper and previous works of authors for bending behavior:

$$\sigma_{cr} = \sigma_{mc} \left(1 - V_f \right) + \alpha \tau_f \frac{f_{ct}}{d}$$

... (9)
where $\sigma_{mu}$ is the strength of the matrix; $V_f$ is the volume fraction of steel fibers; $\alpha$ is a product of several coefficients; $\tau$ is the average bond strength at the fiber-matrix interface; $L$ and $d$ are length and diameter of steel fibers, respectively.

The coefficient $\alpha$ evolves over time, however, it mainly depends on fiber distribution, orientation and bond efficiency.

The Eq. (9) was complemented by Naaman to account for the shape of the cross-section of the fiber in the $\sigma_{cc}$, which gave a more general equation (10).

$$\sigma_{cc} = \sigma_{mu} (1 - V_f) + \alpha \alpha_f \frac{FIER}{4}$$  \hspace{1cm} \ldots (10)

In the present paper, the formula (10) was simplified to the form discussed in ref.\textsuperscript{22}, where the coefficients $k_1$ and $k_2$ were introduced:

$$\sigma_{cc} = k_1 \sigma_{mu} + k_2 \gamma_f \frac{FIER}{4}$$ \hspace{1cm} \ldots (11)

Applying Eqs (4) and (6) to (11), the formula to predict the flexural tensile strength of the composite can be defined as:

$$\sigma_{cc} = k_1 \sigma_{mu} + k_2 \gamma_f \frac{l}{d_{FIER}}$$ \hspace{1cm} \ldots (12)

The Eq. (12) was adopted to describe a variability of the flexural tensile strength of tested composites according to $V_f$ (Fig. 8). The coefficient $k_1$ was assumed to be equal to 0, as the flexural tensile strength of unreinforced beams should be equal to the strength of the matrix. The obtained values of $k_2$ were 2.68 and 4.45 for KE and SW fibers, respectively. The equation represents well the test results for SW fibers. For KE fibers this formula does not meet the expectations. The problem is connected with the shape of the load--deflection curve presented in Fig. 5a.

As other parameters ($k_2$) describing the fibers efficiency in Eq. (12) are hard to determine, application of the correct value of diameter seems not to be crucial. In case of accurate calculations this factor should not be omitted.

**Conclusions**

(i) The main conclusions based on the flexural and compressive tests on self-compacting concrete reinforced with steel fibers with non-circular cross-sections can be drawn as follows:

(ii) The investigated steel fibers had a minor effect on the compressive strength of SFR-SCC. However, the failure pattern was pronouncedly affected by the addition of the fibers.

(iii) The tests performed with the use of short fibers with rectangular cross-section indicated that this reinforcement was less effective in increasing the flexural tensile strength, toughness and other flexural properties than the corrugated fibers. However, 1.5% of these fibers was highly effective in increasing the toughness and the deflection corresponding to the maximum load than other tested volumes of these fibers. Generally, the influence of these fibers on the mechanical parameters of the matrix was hard to predict and differed from the one observed on circular fibers. Meanwhile, the corrugated fibers with non-circular cross-sectional shape behaved as any other fibers with circular cross-section. The increase in flexural parameters was proportional to the applied amount of fibers. It seems that the effort put in preparing such short fibers with the hooked shape, which was possible thanks to by the rectangular cross-sectional shape of the fiber, did not give the expected effect of increasing the flexural properties.

(iv) The shape of the cross-section of a fiber influences mechanical properties of FRC. Taking account of the real cross-section is the basis for proper determination of the aspect ratio, which is further used to compare the effectiveness of the fibers in the concrete matrix. It was proven that the approach presented in the current recommendations can lead to underestimation in determination of the aspect ratio, thus the alternative formula to calculate the equivalent diameter was quoted.
It was further applied in formula to compute the flexural tensile strength of SFR-SCC. For the fibers with a section of a circle cross-section it was satisfactorily matched, in contrary to fibers with rectangular cross-section. However, considering the aspect ratio calculated for the actual cross-sectional shape in description of the flexural parameters seems not to be crucial because other parameters are still unknown.

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