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Simulation of ship maneuvering using the plane motion model

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Present study employs a free-body diagram to establish motion equations and uses a Runge-kutta method to solve the differential equations and investigate straight line motion, turning and zigzag motions. During linear motion, a ship's velocity is affected by both resistive and propulsive forces. During turning motion, the turning radius decreases with an increase in the rudder angle, and the water flowing along the hull and into contact with the rudder is accelerated to facilitate turning. During zigzag motion, the initial turning time, the time to check yaw and the overshoot are explored for switching between different positive and negative rudder angles.

[**Keywords**: Ship maneuvering motion, Straight line motion, Turning motion, Zigzag motion, Runge-Kutta method]

Introduction

Ship maneuvering can be represented by six degrees of freedom motion (6DOF) equations, which are extremely complicated. However, maneuvering can be explored by studying the motion equation related to the XY plane and the Z rotation axis. For instance, Yoshimura's and Sakurai's ship maneuvering mathematical model uses three-axis planar motion. Pérez and Clemente have also studied parameters affecting a ship's motion using three-axis planar motion. This sufficiently proves that three-axis planar motion can demonstrate the characteristics of ship motion, such as the relationship between straight line motion, turning motion and zigzag motion[1,2,3].

During ship maneuvers, the literature groups force into three types: propeller propulsion, force by the hull and force by the rudder. Additionally, during planar motion, the relationships between the acceleration, velocity, angular acceleration and angular velocity are a result of the action of the forces. Usually, slight differences exist in studies using motion equations when different factors are taken into account. For example, Pérez and Clemente included added mass and moment of inertia on a ship's motion, whereas Yoshimura did not always address this effect[1,2,3].

The estimation of propeller propulsion is roughly the same throughout the literature. Some researchers adopt hydrodynamic equations to study the force on the hull at the XY plane and the Z-axis[1,2], but some address the X-axis acting force as the hull resistance, such as Pérez and Clemente and Fuwa and Kashiwadani[3,4]. A better estimation for hull resistance was proposed by Holtrop and Mennen in 1982[5] and is adopted in this paper. The rudder force is a function of the ship velocity, propulsive efficiency coefficient and the propeller and wake fractions. Additionally, researchers have also adopted the propeller slip ratio to estimate the velocity of the fluid on the rudder[6].

MATLAB Simulink and fourth-order Runge-kutta methods are usually adopted for simulations because the motion equations are a complicated set of non-linear differential equations[3,7]. Certain parameters do not appear in these papers; therefore, a third-order Runge-kutta method with a better accuracy of numerical result together with the Holtrop and Mennen resistance methodology is adopted for this paper to simulate ship motion and provide a reference for ship building.

Materials and Methods

Mathematical Model of Ship Motion

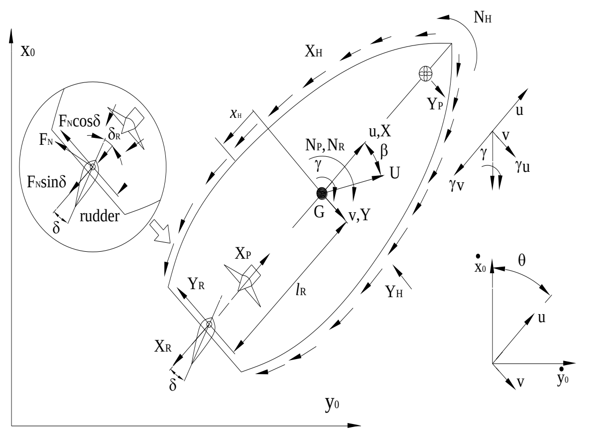

Fig. 1 Free-body diagram of ship motion and coordinate system

Ship motion is represented by six degrees of freedom motion equations. However, for the purposes of this work, three degrees of freedom equations to describe a ship's motion on the water surface can be adopted for a moving situation. In general, the total mass is not uniformly distributed over a ship, and therefore, the added mass and sway affect the XY plane of motion and the rotational motion. Additionally, these masses cannot be easily calculated from the ship's motion. If the rotational velocity and the acceleration are sufficiently small, this effect can be neglected. In this paper, we do not consider the added mass during modeling. We assume that all the mass acts at the center of gravity of the ship, *G*. The acceleration effects is shown in Fig. 1, u is the X-axis velocity, v is the Y axis velocity and γ is the angular velocity about the rotation axis; two acceleration effects can be caused by the angular velocity, $-v\gamma$ is an X-axis acceleration and $u\gamma$ is a Y-axis acceleration; the total X-axis acceleration is expressed $\dot{u}-v\gamma$, $\dot{u}$ is the X-axis acceleration, the total Y-axis acceleration is expressed $\dot{v}+u\gamma$, $\dot{v}$ is the Y-axis acceleration. The mechanical relationship is shown in Fig. 1 and the equations are:

$$m[\dot{u}-v\gamma]=X_P-X_R-X_H \quad , \quad (1)$$

$$m[\dot{v}+u\gamma]=Y_P-Y_R-Y_H \quad , \quad (2)$$

$$I_{zz}\dot{\gamma}=N_P+N_R-N_H \quad , \quad (3)$$

where, m is hull mass, I_{zz} is moment of inertia, $\dot{\gamma}$ is the angular acceleration about the rotation axis, X_P is the X-axis propeller propulsion, X_R is the X-axis rudder force, X_H is the X-axis hull resistance, Y_P is the Y-axis side propulsion, Y_R is Y-axis rudder force, Y_H is the Y-axis hull resistance, N_P is the propeller turning moment that is caused by Y_P force of side thrusters, N_R is the rudder turning moment that is caused by rudder force, Y_R, and N_H is the hull resistance.

During ship maneuvers, the motion equation uses a coordinate system based on the center of gravity. The following equations are generated based on the diagram in Fig. 1:

$$\dot{x}_0=u\cos\theta-v\sin\theta \quad , \quad (4)$$

$$\dot{y}_0=u\sin\theta+v\cos\theta \quad . \quad (5)$$

Using Equations (4) and (5), a relationship between the center of gravity and the Earth-fixed coordinate system can be established.

Resistance and Moment of Force on Hull

Resistance on hull

The variable X_H is the resistance to ship navigation and is the total of all the resistances including: frictional resistance, resistance of appendages, wave resistance, additional pressure resistance of the bulbous bow near the water surface, additional pressure resistance from an immersed transom stern, and the model-ship correction resistance. The total resistance X_H can be represented by the following equation[5,8]:

$$X_H=\frac{1}{2}C_T\rho\, S\, u^2 \quad , \quad (6)$$

where S is the wetted surface area of the hull, C_T is the total resistance coefficient, u is the X-axis velocity, and ρ is the mass density of water.

The total resistance for ship navigation was proposed by Holtrop and Mennen in 1982. In this paper, a better estimation is relied on to calculate the X-axis resistance for a ship.

When a ship moves, the force that resists the movement and the force when the ship turns can be calculated from the following two equations:

$$Y_H=\frac{\rho}{2}LdU^2\times\{Y'_\beta\beta+(Y'_r-m'_x)\gamma'+Y'_{\beta\beta\beta}\beta^3+Y'_{\beta\beta r}\beta^2\gamma'$$

$$+Y'_{\beta rr}\beta\gamma'^2 + Y'_{rrr}\gamma'^3\}, \quad (7)$$

$$N_H = \frac{\rho}{2}L^2 dU^2 \times \{N'_\beta \beta + N'_r \gamma' + N'_{\beta\beta\beta}\beta^3 + N'_{\beta\beta r}\beta^2\gamma' + N'_{\beta rr}\beta\gamma'^2 + N'_{rrr}\gamma'^3\}, \quad (8)$$

where $\beta = -\sin^{-1}(v/U)$, $\gamma' = \gamma(L/U)$, and $U = \sqrt{v^2+u^2}$. The variable L is the hull length and d is the mean draught. The variables Y'_β, $Y'_\gamma - m'_x$, $Y'_{\beta\beta\beta}$, $Y'_{\beta\beta\gamma}$, $Y'_{\beta\gamma\gamma}$, $Y'_{\gamma\gamma\gamma}$, N'_β, N'_γ, $N'_{\beta\beta\beta}$, $N'_{\beta\beta\gamma}$, $N'_{\beta\gamma\gamma}$ and $N'_{\gamma\gamma\gamma}$ are all hydrodynamic parameters, which can be obtained from model testing.

Propeller propulsion and moment

If no lateral propulsion system and no transversal propeller force component exist for a ship's motion, both Y_P and N_P are zero. The X-axis propeller propulsion is calculated from[9]:

$$X_p = (1-\eta_s)T, \quad (9)$$

where T is the propeller thrust and η_s is the thrust reduction coefficient for the propeller propulsion, which is estimated from Holtrop-Mennen's methods. For the equation $T = \rho D_p^4 n^2 k_t$, D_P is the diameter of the propeller, n is the number of revolutions per second (rps), k_t is the propeller thrust coefficient, which depends on the Wageningen B systematic series and can be calculated from: $k_t = c_{t1} + c_{t2}J_t + c_{t3}J_t^2$,where $J_t = u\frac{(1-\omega_P)}{nD_P}$, ω_P is the propeller wake fraction, which is defined as, $\omega_P = 0.23 + 1.4(C_b - 0.5)^2$ and C_b is the block coefficient.

Force by rudder

The force by the rudder F_N can be represented as $F_N = 0.5\rho\, f_a A_R U_R^2 \sin\delta_R$, where f_a is the life coefficient of rudder, $f_a = 6.13\Lambda/(2.25+\Lambda)$, Λ is the rudder aspect ratio, δ_R is the inflow angle of rudder, A_R is the rudder area and U_R is the rudder inflow velocity.

The rudder inflow angle δ_R is rather difficult to assess. The fluid is accelerated after flowing through the propeller. The lift and draft flowing velocity through the rudder can be determined from:

$$u_R = \varepsilon(1-\omega_P)u\sqrt{\eta\left\{1+k(\sqrt{1+8\eta_t/(\pi J_t^2)}-1)\right\}^2+(1-\eta)}, \quad (10)$$

where $\varepsilon = (1-\omega_R)/(1-\omega_P)$ and ω_R is the wake fraction, which can be calculated from:

$\varepsilon = -156.2*(C_b B/L)^2 + 41.6*(C_b B/L) - 1.76$,

$k = k_x/\varepsilon$,

where k_x = ratio of propeller stream on rudder, $\eta = D_p/H_R$, D_p is the diameter of propeller and H_R is the height of rudder.

The rudder inflow velocity can be affected by the hull shape, making it necessary to apply some correction factors. Therefore, the lateral rudder inflow velocity can be expressed as:

$$v_R = \gamma_R(v + \gamma l_R), \quad (11)$$

where v, γ_R and γ are the Y-axis velocity, correction for the flow-straightening factor of the hull and the angular velocity about the rotational axis, respectively. The variable l_R is the distance between the ship's center of gravity and the rudder center, as shown in Fig. 1.

The rudder inflow velocity is defined as:

$$U_R = \sqrt{v_R^2 + u_R^2}, \quad (12)$$

$$\delta_R = \delta - \tan^{-1}\left(\frac{-v_R}{u_R}\right). \quad (13)$$

The hull-propeller-rudder combination factor is added to correct the longitudinal force, lateral forces and yawing moment. The longitudinal force is modified by a rudder drag coefficient, t_R and the disturbed flow around the hull composing the lateral hull force is proportional to the lateral rudder force. Therefore, the lateral force consists of the lateral force, $F_N\cos\delta$, at the rudder and $F_N a_H \cos\delta$ on the ship's hull, where a_H is the hull correlation coefficient. These effects are expressed as:

$$X_R = F_N(1-t_R)\cos(\pi/2-\delta) = F_N(1-t_R)\sin\delta, \quad (14)$$

$$Y_R = F_N(1+a_H)\sin(\pi/2-\delta) = F_N(1+a_H)\cos\delta, \quad (15)$$

$$N_R = F_N(x_{RG} + a_H x_H)\sin(\pi/2-\delta) = F_N(x_{RG} + a_H x_H)\cos\delta, \quad (16)$$

where x_{RG} is the distance from the rudder force application point to the midship position, x_H is the distance from the lateral force (defined as $F_N a_H \cos\delta$) application point to the midship position. In this paper, we assume that the midship position is at the center of gravity of the ship.

The Runge-kutta method for motion model

Because the Runge-kutta method adopted for the numerical simulation is modified, the final equations are shown below[10,11]:

$$f(t,u,v,\gamma,\theta,x_0,y_0) = \begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{\gamma} \\ \dot{\theta} \\ \dot{x}_0 \\ \dot{y}_0 \end{bmatrix} = \begin{bmatrix} \frac{X_P}{m} + v\gamma - \frac{X_R}{m} - \frac{X_H}{m} \\ -u\gamma - \frac{Y_R}{m} - \frac{Y_H}{m} \\ \frac{1}{I_{zz}}(N_R - N_H) \\ \gamma \\ u\cos\theta - v\sin\theta \\ u\sin\theta + v\cos\theta \end{bmatrix}. \quad (17)$$

For a third-order Runge-kutta method:

$$f_{n+1}(t,u,v,\gamma,\theta,x_0,y_0) = f_n(t,u,v,\gamma,\theta,x_0,y_0) + \frac{1}{6}(k_1 + 4k_2 + k_3),$$

where:

$$k_1(k_{u1},k_{v1},k_{\gamma1},k_{\theta1},k_{x01},k_{y01}) = hf(t,u_n,v_n,\gamma_n,\theta_n,x_{0n},y_{0n}),$$

$$k_2(k_{u2},k_{v2},k_{\gamma2},k_{\theta2},k_{x02},k_{y02}) = hf(t+\frac{h}{2},u_n+\frac{k_{u1}}{2},v_n+\frac{k_{v1}}{2},$$
$$\gamma_n+\frac{k_{\gamma1}}{2},\theta_n+\frac{k_{\theta1}}{2},x_{0n}+\frac{k_{x01}}{2},y_{0n}+\frac{k_{y01}}{2}),$$

$$k_3(k_{u3},k_{v3},k_{\gamma3},k_{\theta3},k_{x03},k_{y03}) = hf(t+h,u_n-k_{u1}+2k_{u2},$$
$$v_n-k_{v1}+2k_{v2},\gamma_n-k_{\gamma1}+2k_{\gamma2},\theta_n-k_{\theta1}+2k_{\theta2},$$
$$x_{0n}-k_{x01}+2k_{x02},y_{0n}-k_{y01}+2k_{y02}).$$

Using these equations, the Runge-kutta method can be used to simulate the relationship between the ship velocity, angular velocity when the hull turns and angular displacement.

Results and Discussion

Ship Motion Simulation

To mathematically simulate a ship's motion, large quantities of parameters need to be assessed because no correct values can be obtained. The parameters related to the size of common ships are given in Table 1. The density of sea water is ρ =1025 kg/m^3. The moment of inertia when the hull turns can be obtained from the equations $I_{xx} = k_{xx} * B^3 * L * \rho$, $I_{yy} = k_{yy} * B * L^3 * \rho$ and $I_{zz} = I_{xx} + I_{yy}$, where k_{xx} and k_{yy} are functions of C_{wL}. The moment of inertia when the hull turns is I_{zz} = 325 $kg\text{-}m^2$. The hydrodynamic parameters given in Table 2 are obtained from Yoshimura's ship data[1]. Various hydrodynamic parameters can also be assessed by the estimation based on the work of Matsunaga[11].

Both the ship resistance and the rudder force are affected by the velocity, requiring numerical simulations to use a third-order Runge-kutta method. The simulation consists of the Runge-kutta main program, the ship resistance subprogram and the rudder force subprogram.

For $u(0)$=0.514 m/sec, $v(0)$=0, $\gamma(0)$=0, $\theta(0)$=0, $x_0(0)$=0 and $y_0(0)$=0 are the initial conditions used for the numerical simulation. The time interval is h =0.01 sec. The time simulated determined the required conditions.

Table 1 – Major parameters required for the ship-model

Hull	Symbol	
Length	L (m)	2.480
Beam	B (m)	0.496
Draft	T (m)	0.197
C_p		0.623
C_m		0.98
$C_b = C_p * C_m$		0.611
Cwp		0.75
C_{wL}	$C_b/(0.471+0.551C_b)$	0.7565
Displacement ∇	$\nabla = L*B*T*C_b$ (m^3)	0.148
m	$m = \rho\nabla$	151.763
k_{xx}		0.048
k_{yy}		0.04
Rudder		
Area ratio	Ar/Ld	1/37.5
Aspect ratio	Λ	1.801
γ_R		0.467
l_R		-0.996
Propeller		
Diameter	Dp (m)	0.135
Pitch ratio	P	0.775
Rudder height	H (m)	0.147
Propeller speed		8rps

The total resistance is calculated using the method described by Holtrop and Mennen. Therefore, in addition to the relevant information listed in Table 1, the following information is also required. The center of the bulb area above the keel line is hb = 0 m, the transverse bulb area is

Abt = 0 m^2, the transom area is At = 0.16 m^2, the wetted area of the appendages is Sapp =0 m^2, the stern shape parameter is Cstern =10, and the longitudinal position of the center of buoyancy forward of 0.5 L as a percentage of L is lcb=-0.75%.

Table 2 - Hydrodynamic parameters required for the ship-model

Y_H		N_H	
Y'_β	0.3979	N'_β	-0.0992
$Y'_\gamma - m'_x$	0.0918	N'_γ	-0.0579
$Y'_{\beta\beta\beta}$	1.6016	$N'_{\beta\beta\beta}$	0.1439
$Y'_{\beta\beta\gamma}$	-0.2953	$N'_{\beta\beta\gamma}$	-0.3574
$Y'_{\beta\gamma\gamma}$	0.4140	$N'_{\beta\gamma\gamma}$	0.0183
$Y'_{\gamma\gamma\gamma}$	-0.0496	$N'_{\gamma\gamma\gamma}$	-0.0207
Propeller			
$c_{t1} = 0.356$, $c_{t2} = -0.293$, $c_{t3} = -0.1675$			
Rudder			
$1 - t_R = 0.28 * C_b + 0.55$ =0.7211		$a_H = 0.633 * C_b - 0.153$ =0.2338	
$x_{RG} = -L/2$ = -1.24		$x_H = -0.377L$ = -0.935	
γ_R = 0.412		l_R = -0.996	
k_x = 0.642			

The steps of the numerical simulation are as follows:

(1) The rudder angle, δ , is set by motion situtation.

(2) The variable X_H is calculated by introducing the velocity into the resistance subprogram, and Y_H and N_H are calculated by equations (7) and (8).

(3) The X_P is calculated by equations (9).

(4) Equations (10) through (13) and other relation equations are used to calculate F_N

(5) The modified equations (14) through (16) describe the relationship between the force and position to calculate, X_R, Y_R and N_R.

(6) Using equation (17), we can find $u, v, \gamma, \theta, x_0$ and y_0.

(7) Repeat steps 1 and 6 to build the ship motion data.

The numerical simulation of a ship's motion starts with a linear motion by setting the rudder angle to δ =0° to observe the changes in the ship's resistance and propeller propulsion. As shown in Fig. 2, when the rudder angle is δ =0°, the rudder force is 0 N. Because the initial ship velocity is high, the ship resistance is high, the propeller propulsion is low and the ship fails to maintain its original velocity. When the ship slows down, the ship resistance decreases from 5.503 N to 4.222 N and the propeller propulsion increases from 3.903 N to 4.222 N and a final balance is achieved. Because the rudder angle is δ =0°, the Y-axis force and moment are 0 N and 0 N-m, respectively. Fig. 3 shows that there is displacement only on the X-axis and that the Y-axis displacement is 0 m (because the movement is linear). The ship starts moving stably after the X-axis velocity decreases from the initial 0.514 m/s to 0.447 m/s. The Y-axis velocity is 0 m/s, the initial rudder inflow velocity is 0.514 m/s and the velocity reaches u_R =0.698 m/s after being accelerated by propeller. The value for u_R stabilizes after it reaches and maintains a velocity of 0.688 m/s. The turning velocity is 0 rad/s and the Z-axis turning angle is 0 rad.

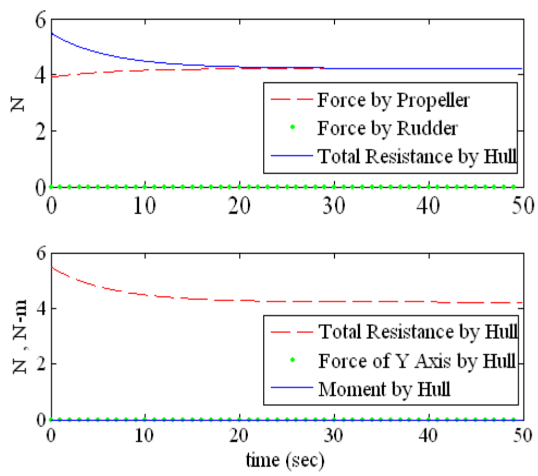

Fig. 2 Relationship between the various forces during straight-line motion

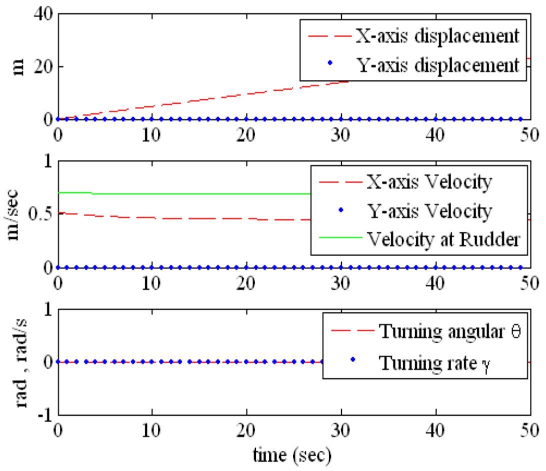

Fig. 3 Relationship between the displacement, velocity, angular displacement and angular velocity for straight-line motion

A turning circle simulation was conducted by setting the rudder angle δ to 15°, 25° and 35°, respectively, and ensuring the other conditions are similar to those for linear motion. The various forces acting on the rudder for an angle of 35° are shown in Fig. 4. When the ship slows down, the rudder force decreases from 5.084 N to 3.973 N and the propeller propulsion increases from 3.903 N to 4.793 N before a final balance is achieved. The ship resistance X_H stabilizes after the ship resistance decreases from 5.503 N to 2.246 N and the moment of N_H decreases from 0 N-m to -4.747 N-m. The value of the variable Y_H increases from 0 N to 6.665 N before stabilizing. As shown in Fig. 5, the X-axis and Y-axis show sinusoidal fluctuations within the range of -1.203 ~ 11.397 m and -0.230~13.004 m, respectively. The X-axis velocity decreases from 0.514 m/s to 0.322 m/s and then stabilizes. The Y-axis velocity changes from 0 m/s to -0.109 m/s and the initial propeller inflow velocity is 0.514 m/s before increasing to u_R =0.698 m/s after being accelerated by the propeller, u_R stabilizes after reaching 0.666 m/s, the turning angular velocity increases from 0 rad/s to 0.054 rad/s before stabilizing and the turning angular displacement about the Z-axis is 0~26.8 rad, or approximately 4.265 circles. As shown in Fig. 6, the turning radius is 20.927 m when the rudder angle is 15°, the "advance at 90° change of heading" is 85.21 sec, the "transfer at 180° change of heading" is 161.86 sec and 304 sec is required for a single turn. The turning radius is 10.634 m when the rudder angle is 25°, the "advance at 90° change of heading" is 52.21 sec, the "transfer at 180° change of heading" is 94.67 sec and 180 sec is required for a single turn. The turning radius is 6.300 m when the rudder angle is 35°, the "advance at 90° change of heading" is 40.24 sec, the "transfer at 180° change of heading" is 69.39 sec and 117 sec is needed for a single turn. The turning radius decreases, and less time is required for a single turn when the rudder angle increases.

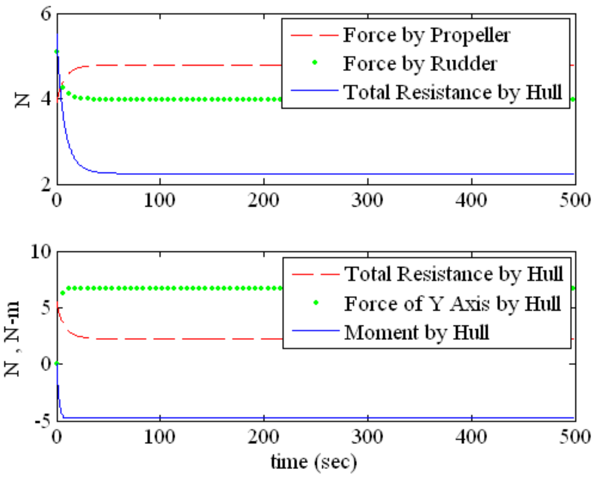

Fig. 4 Relationship between the various forces for a rudder angle of 35°

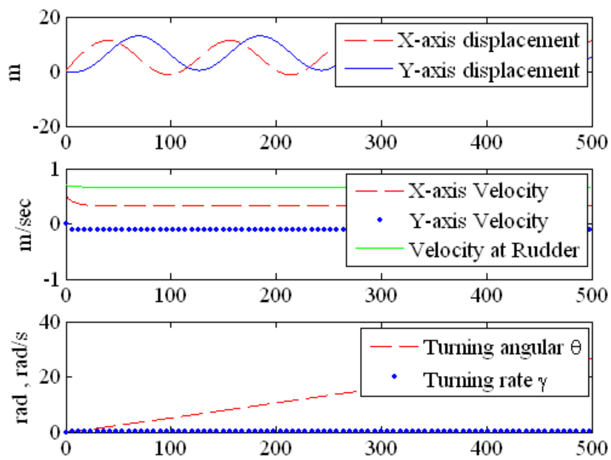

Fig. 5 Relationship between the displacement, velocity, angular displacement and angular velocity for a rudder angle of 35°

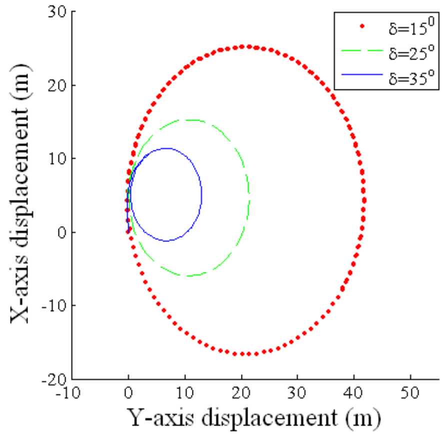

Fig. 6 Plane displacement for various rudder angles

Fig. 7 shows a zigzag simulation by assuming $10^o/10^o$ and $20^o/20^o$. During zigzag operations, the rudder angles change from port to starboard in 5 sec. The initial time is defined as the time required for the ship's heading to turn to the angle specified by the zigzag operation, whereas the time to check yaw is defined as the time for the ship's heading to turn to the first maximum overshoot, less the initial turning time. During the $10^o/10^o$ zigzag operation, the first overshoot is 1.932^o, the second overshoot is -1.858^o, the initial turning time is 15.17 sec and the time to check yaw is 3.93 sec. During the $20^o/20^o$ zigzag operation, the first overshoot is 4.281^o, the second overshoot is -4.401^o, the initial turning time is 15.43 sec and the time to check yaw is 4.17 sec.

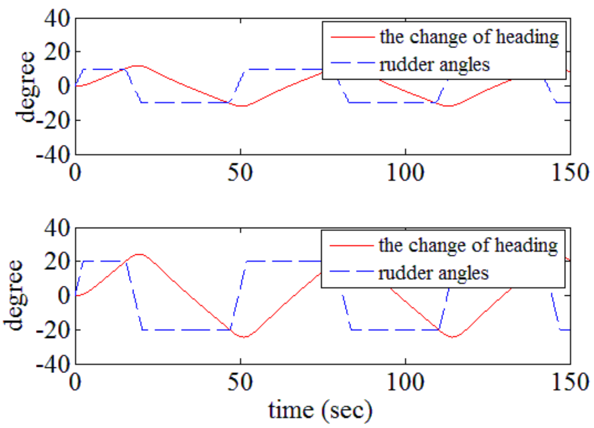

Fig. 7 Ship zigzag operations

Conclusions

During ship motion analysis, the most difficult task is determining the motion parameters, as a large proportion are estimated. As a result, it is difficult to estimate the actual motion trail. This paper establishes a better set of equations and uses a third-order Runge-kutta method to simulate ship motion.

For straight line motion, if a ship's velocity is high and the propeller propulsion is insufficient, the ship will slow down gradually before stabilizing at a certain speed after balance between propulsion and resistance is achieved. During turning motion, the turning radius decreases as the rudder angle increases and the ship moves approximately linearly for a while prior to turning. The inertia of the ship causes the time of the "advance at 90^o change of heading" to be greater than the "advance at 90^o change of heading" to the "transfer at 180^o change of heading" in the same 90^o change of heading.

During the simulation process, the fluid flows along the hull and interacts with the rudder and is accelerated by the propeller to facilitate ship turning. The fluid may have a different velocity when flowing into the rudder. The zigzag motion overshoot is related to both the positive and negative rudder angles and the overshoot increases with the positive or negative rudder angle. Finally, the time to check yaw also increases with the positive or negative rudder angle.

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