Eigenvector Based Wideband Spectrum Sensing with Sub-Nyquist Sampling for Cognitive Radio

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In the Cognitive Radio (CR) technology, fast and precise spectrum sensing is essential, so that the Secondary Users (SUs) can quickly adapt their parameters by dynamic monitoring of the spectrum, enabling them to utilize the available spectrum, and more importantly to prevent interference with Primary Users (PUs). To this effect the implementation of the classical spectrum sensing methods in a wideband scenario is a great challenge. This is because the classical methods need sampling rates greater than or equal to the Nyquist rate. Modern Compressive Sensing (CS) techniques exploit the sparseness of a typical wideband spectrum. In this paper a subNyquist sampling based sensing technique is studied. The correlation matrix of a limited number of samples containing noise is constructed and the Eigenvector (EV) estimator is used to discern the functional channels of the spectrum. The performance of this technique is assessed by calculating the probability of detection of the occupied signal as a function of the number of samples and the SNR parameters of random input. Simulation results show that a robust detection is possible, even with less number of samples and at low SNR.

Keywords: Cognitive Radio, Correlation Matrix, Eigenvector, Multicoset Sampling, Noise Subspace, Spectrum Sensing

Introduction

Cognitive Radio (CR) has gained importance due to the efficient use of the available spectrum providing opportunity to the Secondary User (SU) to use the same frequency band used by the licensed holder (Primary User – PU), without interfering the PU1-8. In this paper, the Eigenvector method for wideband spectrum sensing at a subNyquist sampling rate is studied, in which the correlation matrix of a limited number of samples including noise is constructed and used by an Eigenvector (EV) estimator9. The performance of the detection is assessed by computing the probability of detecting the occupied signals versus the number of samples and the SNR parameters of random input signals.

Problem statement

The problem may be stated as ‘to detect the presence/absence of the signals for a wideband signal of 1GHz bandwidth using the Eigenvector method with subNyquist sampling’.

Wideband spectrum sensing

The received analog signal $x(t)$ is sampled at a subNyquist rate using a multicoset sampler. The output of the multicoset sampler is fractionally shifted through a multirate system, consisting of an up sampling stage, interpolation stage, delay stage, and a down sampling stage. Using a finite number of data thus obtained, the sample correlation matrix is computed. This correlation matrix is then analyzed using the celebrated subspace methods, for estimating the number of occupied channels and thereby recovering the occupied channel set. Finally, the eigenvector estimator is used to find the location of the occupied channels using the correlation matrix. Various stages of the wideband spectrum scheme are explained below.

Multicoset sampling

Sampling the received signal $x(t)$ uniformly at the Nyquist rate gives us $x(t) = nT$ samples, which contain all the information regarding $x(t)$. Multicoset sampling is a collection of certain samples from this grid. The grid is partitioned into blocks, each of which are composed of $L$ samples consecutively. Consider a set $C$, of length $p$. The indices of these $p$ samples are placed in each block, and the rest of the elements are set to zero. Thus we may represent the sampling pattern as:
\( C = \{ c_i \}_{i=1}^p \), where \( 0 \leq c_1 < c_2 < \cdots < c_p \leq L - 1 \).

The \( i^{th} \) sampling sequence for \( 1 \leq i \leq p \) may be defined as
\[
x_{c_i[n]} = \begin{cases} 
x(t = nT) = mL + c_i \text{ for } m \in \mathbb{Z} \\
0 \text{ otherwise}
\end{cases} \quad \ldots \ (1)
\]

The sampling stage is implemented by \( p \) uniform sampling sequences with a period \( 1/(LT) \), where the \( i^{th} \) sampling sequence is shifted by \( c_iT \) from the origin. Hence, a multicoset system is uniquely characterized by the parameters \( L, p \) and the sampling pattern \( C \).

Given \( L \) the number of occupied cells are formulated using the relation \( \left\lceil \frac{NBL}{\eta_{\text{max}}} \right\rceil \leq q \leq N + \left\lfloor \frac{BL}{\eta_{\text{max}}} \right\rfloor \); \( q_{\text{min}} \leq q \leq q_{\text{max}} \). The number of occupied cells which depends upon the band location is chosen between the above two bounds. We have chosen \( q = q_{\text{min}} \).

Choosing \( T = 1/B_{\text{max}} \), the average sample rate of this theory is \( f_{\text{avg}} = \gamma \cdot B_{\text{max}}, \) where, \( \gamma = (p/L) \) is called the subNyquist factor. The factor \( p \) should be greater than \( q_{\text{max}} \). We can choose \( p = q_{\text{max}} + 1 \). Though the contents or the information in the bands may remain constant, the carriers may deviate, so that a maximum number of occupied channels are filled.

**Correlation matrix**

To correlate the task of spectrum sensing with that of parameter estimation, the correlation matrix of a special configuration of sampled data is calculated. Each non-uniformly sampled signal sequence \( x_i(m) \) is oversampled by a factor \( L \), each of which contain \( p \) number of samples, and \( C = \{ c_1, c_2, \ldots, c_p \} \) is the sample pattern. Then it is further filtered using an interpolation filter. Next, the output filtered sequence is delayed with \( c_i \) samples. Thus the sequences are down sampled by a factor \( L \). The cumulative process of oversampling, filtering, delaying and downsampling from \( x_i(m) \) to \( x_{di}(m) \) is considered as a partial shifting of the sequence \( x_{di}(m) \). Defining the snapshot vector \( x_d(m) \) as \( X_d(m) = \{ x_{d1}, x_{d2}, \ldots, x_{dp} \} \). The sample correlation matrix from \( M \) samples of the partially shifted sequence is computer using the relation.
\[
\hat{R} = \frac{1}{M} \sum_{m=1}^{M} X_d(m) \cdot X_d^*(m) \quad \ldots \ (2)
\]
Assuming that \( \hat{R} \rightarrow R \) as \( M \rightarrow \infty \).

**Subspace analysis**

In each of the spectral band the signal is assumed to be uncorrelated and distinct with other bands. Therefore, the correlation matrix will have full-rank. The subspace methods used are as described.

**Estimation of the number of occupied channels**

The eigenvectors of \( R \), with respective eigenvalues \( \sigma^2 \), geometrically is orthogonal to the modulation matrix \( A(k) \), as given by the relation \( R = E[y(f), y^*(f)] \). All the other eigenvectors are in the range space of \( A(k) \) and thus they are called as signal eigenvectors. Decomposing \( R \) into signal and noise subspaces, we obtain \( R = E_s \Lambda_s E_s^* + E_n \Lambda_n E_n^* \), where, \( \Lambda_s \) is the diagonal matrix of the signal eigenvalue, \( \Lambda_n \) is the diagonal matrix of the noise eigenvalue, and \( E_s \) is the matrix of the signal eigenvector and \( E_n \) is the matrix of the noise eigenvectors. The eigenvector matrix \( E_s \), span the range space of \( A(k) \), which denotes the signal subspace and the noise eigenvector is \( E_n \perp A(k) \). Using the orthogonality property we estimate the signal parameters by evaluating the dimension of the noise subspace. The ordered eigenvalues of \( \hat{R} \) are \( \lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_q \geq \cdots \geq \lambda_p \), where, \( q \) eigenvalues are significant, and the remaining are all equal to \( \sigma^2 \). Thus, for a large \( M \), the dimension of signal vector can be evaluated from the multiplicity of the least eigenvalues of \( \hat{R} \). In practice the number of samples is a function of the sensing period. Thus, ideally the number of samples must be least possible, more so in case of time-varying channels. Therefore, the sample matrix \( \hat{R} \neq R \). Further, the separation between the signal and noise eigenvalues needs a threshold level. This threshold level depends on the number of samples and the corresponding noise power. For the present work the noise power is presumed to be a constant factor and the threshold is kept fixed throughout the analysis for the detection of the input frequencies. The upper half of the Figure 1 shows the ordered eigenvalues that are bifurcated by a horizontal line representing the threshold level. It is observed that there are four eigenvalues greater than the threshold. This indicates the number of occupied channels. To overcome the difficulties of setting the threshold, theoretical information criteria, such as minimum description length procedures detailed below have been used. The lower section of Figure 1 shows the calculated values of probability of detection of occupied channel as a function number of samples for different values of
SNR. In minimum description length, the number of occupied channels satisfying the criterion within the range \(q_{\min} \leq r \leq q_{\max}\) can be evaluated using
\[
\hat{q} = \arg \min_r -M(p-r) \log \frac{g(r)}{a(r)} + \frac{1}{2} r (2p-r) \log M
\]  
(3)

where \(M\) is the number of samples; \(g(r)\) is the geometric mean of the \((p-r)\) eigenvalues of the correlation matrix and is given by \(g(r) = \prod_{i=r+1}^{p} \lambda_i^{1/p-r}\); \(a(r)\) is the arithmetic mean of the \((p-r)\) eigenvalues of the correlation matrix and is given by \(a(r) = \frac{1}{p-r} \sum_{i=r+1}^{p} \lambda_i\). The probability of detection of the number of occupied channels, [i.e., \(P_d = \text{Prob}[\hat{q} = q]\)], is related to noise distribution, SNR and the number of samples, and is always not equal to unity. However, this can be improved by using the method of locating peaks, as done in EV algorithm.

**Active channel set recovery**

Let \(\hat{E}_{n}\) denote the \(p \times (p - \hat{q})\) matrix consisting of all noise eigenvectors in descending order. After calculating the number of occupied channels, we denote the noise eigenvectors by \(v_{r+1}, v_{r+2}, \ldots, v_{p}\) corresponding to the noise eigenvalues \(\lambda_{r+1}, \lambda_{r+2}, \ldots, \lambda_{p}\). The location of the occupied channels can be recovered using the eigenspectrum of the eigenvector algorithm.

\[
P_{EV}(k) = \frac{1}{\sum_{i=r+1}^{p} |a_k v_i|^2}, 0 \leq k \leq L - 1
\]  
(4)

where \(\lambda_i\) is the eigenvalue associated with the eigenvector \(v_i\), \(k\) is the channel index and \(a_k\) is a column of \(A(k)\), given by
\[
a_k = \frac{1}{LT} \left[ e^{j2\pi k c_1 L}, e^{j2\pi k c_2 L}, \ldots, e^{j2\pi k c_p L} \right].
\]

The algorithm generates \(L\) values corresponding to the \(L\) channels. If \(k\) is an occupied channel’s label, \(P_{EV}(k)\) is significant at that point, else it will have a smaller value. The location of occupied channels is obtained by comparing the \(P_{EV}\) values with a threshold value as given by \(\hat{k} = \{k_i | P_{EV}(k_i) > \text{threshold}\}\). It can be seen that the significant values corresponding to the occupied channels are indicated and the other channels are considered to be vacant channels, which are available for the CR.

**Results and Analysis**

The process of blind spectrum sampling and reconstruction using eigenvector algorithm is implemented in LabVIEW™. The performance analysis is carried out through Monte-Carlo simulations. The signal input to the CR is given by:
\[
x(t) = \sum_{i=1}^{N} A_i \sin(c(B_i(t - t_i))) \exp(j2\pi f_i t)
\]  
(5)

Where \(\sin(c(x)) = \frac{\sin(\pi x)}{(\pi x)}\) and \(N\) is the number of bands. The \(i^{th}\) band of \(x(t)\) has an amplitude \(A_i\), bandwidth \(B_i\), time offset \(t_i\) and carrier frequency \(f_i\).

In the experiment, it is assumed that there are \(N = 4\) sub-bands, in the frequency range \(F \in [0, B_{max}] = (0,1000)\) MHz. Hence the Nyquist rate for this signal is \(B_{max} = \frac{1}{T} = 1000\) MHz.

The bandwidths of the subbands are \(B_1 = B_2 = B_3 = B_4 = 10\) MHz. The central frequencies of the subbands are \(f_1 = 250\) MHz, \(f_2 = 500\) MHz, \(f_3 = 750\) MHz and \(f_4 = 850\) MHz. A multicore sampler is used to sample the signal at an average sampling rate of 90 MHz, which corresponds to a \(\gamma = 0.09\) of the Nyquist rate. Using sequential forward selection algorithm nine \(c_i\) numbers are selected from the set \(L\). At first the efficiency of estimating \(\hat{q}\) is examined. Next, using 250 Monte-Carlo simulations, the number of occupied channels from the eigenvalues of the correlation matrix are estimated, for various values of \(M\) and SNR, as in equation (4). Lastly, we calculate the empirical values of the probability of detecting the four occupied channels, for a chosen SNR and different numbers of samples. Using equation (4) the number of occupied cells are estimated to be \(\hat{q} = 4\). The estimated spectral index from the equation \(\hat{k} = \{k_i | P_{EV}(k) \text{ > threshold}\}\) is found to be \(\hat{k} = 250, 500, 750, 850\). The probability of detecting the four occupied channels is computed at low values of SNR and at different numbers of samples. For larger values of SNR and number of samples, it is expected that the estimator will detect the exact magnitude with high probability. From the lower section of Figure 1 we can deduce the sub Nyquist samples required for detecting the four occupied channels for different SNR values. The number of samples required for detecting the occupied channels with high probability, increases from 44 to 62 samples as the SNR decreases form +4 dB to -4 dB. It is suggested that at lower SNR values the number of samples required to sense the occupied channels with higher probability increases.
with a decrease in the SNR. Next, the probability of detecting the signal occupancy and the probability of false alarm are expressed as:

$$P_d = \frac{1}{N} \sum_{i=1}^{N} P_r(k_i e | k_i e)$$

$$P_f = \frac{1}{1-P_d} \sum_{i=1}^{N} P_r(k_i e | k_i e)$$

… (6)

Where $k^C = L - k$ is the complement set of $k$. The $P_d$ and $P_f$ are calculated from equation (6) for different number of samples and SNR. The results shown in the upper section of Figure 2 show a good detection performance even at lower values of SNR, and a fewer number of samples. It is noticed that at an SNR equal to +4 dB and for number of samples greater than 72, the probability of detecting the occupied channels, of the eigenvector algorithm is close to unity. The values of $P_f$, shown in the bottom section of Figure 2, decrease significantly with an increase in number of samples, such that for values of $M$ greater than 30, it approaches zero for all values of SNR. Comparing Figure 2 reveals that with correct estimation of a perfect detection of the channel occupancy is possible. For comparison with the other methods, we observed that there is no compression capability in Wavelet detection method, Multiband Joint detection method and Filter Band detection method compared to compression capability in Music like algorithm and the EV method. Also the former uses Nyquist sampling where as the later uses subNyquist sampling. Further, the implementation complexity is low in the EV method when compared to the Music like algorithm.

**Conclusion**

The simulation results demonstrate that the proposed methodology of wideband spectrum sensing can reliably detect the occupied channels, at low sampling rates, and in a noisy environment. The setup is not too complex and thereby is economical and viable. The multicose sampling scheme adopted here requires lower sampling rates, nearly equal to the channel occupancy. With the current approach, the problem of spectrum sensing can be transformed into a problem of estimating suitable parameters, which can be solved by the standard subspace methods. The multicose samples are fractionally shifted and used to calculate the correlation matrix of the signal. For sparse spectrums, the computational time varies linearly with the amount of data, and thereby results in substantial savings. For a typical wideband system, with SNR of +4 dB, with just 72 samples, at a subNyquist rate of 0.09, the probability of detection is 99.9% and the probability of false alarm is 0.1 %.
References