Soret and Dufour effects on MHD flow with heat and mass transfer past a permeable stretching sheet in presence of thermal radiation

G Sreedevi*, D R V Prasada Rao, O D Makinde & G Venkata Ramana Reddy

*Department of Mathematics, K L University, Green Fields, Vaddeswaram, Guntur 522 502, India

bDepartment of Mathematics, S K University, Anantapur 515 003, India

cFaculty of Military Science, Stellenbosch University, Private Bag X2, Saldanha 7395, South Africa

Received 14 June 2016; accepted 3 January 2017

An analysis has been carried out to study the combined effects of the magnetic field, Joule heating, thermal radiation absorption, viscous dissipation, Buoyancy forces, thermal-diffusion and diffusion-thermion the convective heat and mass transfer flow of an electrically conducting fluid over a permeable vertically stretching sheet. The boundary layer equations for the fluid flow, heat and mass flux under consideration have been obtained and reduced into a system of non-linear ordinary differential equations by using appropriate similarity transformation. Using shooting method coupled with the fourth order Runge-Kutta integration scheme, the numerically solutions have been obtained and presented graphically. The effects of various embedded thermo-physical parameters on the fluid velocity, temperature, skin friction, Nusselt number and Sherwood number have been determined and discussed quantitatively. A comparison of a special case of our results with the one previously reported in the literature shows a very good agreement. An increase in values of thermal radiation, viscous dissipation, suction/injection coefficient and chemical reaction results in the increase of velocity, temperature and heat-mass transfer rates. It is further noted that the velocity, temperature and heat-mass transfer rates reduces on the boundary layer of a permeable vertical stretching sheet due to increase in the values of Soret or decrease in values of Dufour. Further, this work leads to study different flows of electrically conducting fluid over a permeable vertical stretching sheet problem that includes the two dimensional non-linear boundary equations.

Keywords: Joule heating, MHD, Thermal radiation, Buoyancy forces, Soret effect, Dufour effect

1 Introduction

Natural convection permeates, either be it a study of natural phenomenon like the structure of stars and planets or be it in an engineering application study say cooling of nuclear reactors. On the other hand, an increase in the temperature of a conductor by the result of resistance to the electrical current flowing through it is referred to as Joule heating. However, at an atomic level, Joule heating is the resultant of moving electrons colliding with atoms in a conductor, whereupon momentum is transferred to the atom, increasing its kinetic energy. Electro migration is when similar collisions results in a permanent structural change, rather than just an elastic response. The rise in the temperature of the conductor is due to the increase in the kinetic energy of the ions which manifests itself as heat. Accordingly, the energy is transferred from electrical power supply to the conductor and any other material that comes in thermal contact.

*Corresponding author (E-mail: sreedevihari2007@gmail.com)
interaction on boundary layer flow along a vertical thin cylinder. The study of natural convection boundary layer flow of a viscous incompressible fluid along an isothermal horizontal plate with the effects of conduction-radiation interaction was done by Hossain and Takhar\textsuperscript{9}. The study of heat transfer over a stretching surface of variable temperature under a steady nonlinear hydromagnetic flow was done by Devi and Thiagarajan\textsuperscript{10}. An analysis was carried out by Mukhopadhyay and Andersson\textsuperscript{11} on the effects of slip and heat transfer analysis of flow over a stretching surface with unsteady condition. Makinde\textsuperscript{12} further analyzed the unsteady MHD natural convection from a heated vertical plate with constant heat flux, chemical reaction and thermal radiation effects. The combined effect of thermal radiation and viscous dissipation on fluid flows on a non-linearly stretched permeable wall was studied by Cortell\textsuperscript{13}. Makinde \textit{et al.}\textsuperscript{14} investigated the problem of MHD flow of a variable viscosity nanofluid over a radially stretching convective surface. The study over non-aligned MHD stagnation point flow of variable viscosity nanofluids past a stretching sheet with radiative heat was studied by Khan \textit{et al.}\textsuperscript{15}. The effects of thermal radiation and non-uniform heat source/sink on mixed convection flow and heat transfer over an unsteady stretching permeable sheet was advanced by Pal\textsuperscript{16}. Sivaraj and Kumar\textsuperscript{17} analyzed the viscoelastic fluid flow over a moving vertical cone and flat plate with variable electric conductivity. A study of different mathematical analytical methods applied on MHD permeable heat flow past stretching/shrinking two-three dimensional objects was done by Turkyilmazoğlu\textsuperscript{18}.

The study of heat and mass transfer of viscous fluid over a steady stretching sheet with viscous dissipation and internal heat generation was conducted by Vajravelu and Hadjinicolaou\textsuperscript{19}. The combined heat and mass transfer of an electrically conducting fluid in MHD natural convection, in a vertical surface with ohmic heating was studied by Chen\textsuperscript{20}. The effects of ohmic heating and viscous dissipation on steady MHD flow near to stagnation point on an isothermal stretching sheet was studied by Sharma and Singh\textsuperscript{21}. The combined effect of Hall current, Joule heating and thermal diffusion in a steady free convection heat and mass transfer was investigated by Singh and Gorla\textsuperscript{22}. The combined effect of mixed convection with thermal radiation and chemical reaction on MHD flow of viscous and electrically conducting fluid past a vertical permeable surface embedded in a porous medium was analyzed by Pal and Talukdar\textsuperscript{23}. The effects of ohmic heating and viscous dissipation was studied on mass transfer by Babu and Reddy\textsuperscript{24} on MHD mixed convective flow over a vertical surface. Babaélahi \textit{et al.}\textsuperscript{25} analysed the effects of viscous and ohmic dissipation in a visco-elastic MHD fluid of a boundary layer flow over a stretching surface. Raju \textit{et al.}\textsuperscript{26} studied the viscous dissipation and Joule heating in a horizontal channel with insulated and impermeable bottom wall. Employing the Chebyshev finite difference method, Khader and Ahmed\textsuperscript{27} studied the effect of viscous dissipation over a permeable stretching surface embedded in a porous medium with a second order slip. An investigation was carried out by Motsumi and Makinde\textsuperscript{28} on the effects of thermal radiation and viscous dissipation over a permeable moving flat plate on boundary layer flow of nanofluids. Advancing studies on stretching sheet, Pal and Mondal\textsuperscript{29} studied the viscous-ohmic dissipation on MHD convective–radiative boundary layer flow of nanofluids. The study on MHD fluid flow and heat transfer past two-dimensional porous stretching/shrinking bodies was done by Turkyilmazoğlu\textsuperscript{30}.

Soret and Dufour effects (S & D effects) are assumed to be negligible in the above studies. The mass flux created by a temperature gradient and the energy flux resulted by concentration differences is known as thermal-diffusion (Soret) effect and diffusion-thermo (Dufour) effect, respectively. The importance of the S & D effects is visible in many practical applications preferably in areas such as geosciences, chemical engineering etc. The S & D effects past a vertical plate embedded in a porous medium in a MHD mixed convection was studied by Makinde\textsuperscript{31}. The study of S & D effects over a vertical plate in a mixed convection was conducted by Alam and Rahman\textsuperscript{32}. Over a vertical isothermal flat plate Afify\textsuperscript{33} studied the effects of a viscosity/temperature parameter, S & D on free convective heat and mass transfer of a viscous fluid. Investigations were carried out by Makinde and Olanrewaju\textsuperscript{34} on the unsteady mixed convection with S & D effects past a porous plate moving through a binary mixture of chemically reacting fluid. Further studies were conducted by Beg \textit{et al.}\textsuperscript{35} over chemically reacting mixed convective heat and mass transfer along inclined and vertical plates with S & D effects. Combined influence of chemical reaction, heat source, radiation, stretching surface and variable viscosity coupled with
the S & D effects was studied by Tsai and Huang in a porous medium for the Hiemenz flow. El-Arabawy studied the heat and mass transfer by natural convection from vertical surface embedded in a fluid-saturated porous media with S & D effects under variable surface temperature and constant concentration. S & D effects in a MHD free convection boundary layer flow past a semi-infinite moving vertical plate, with suction/injection was investigated by Olanrewaju and Makinde and focusing on viscous dissipation was investigated by Reddy and Reddy. Makinde et al. investigated the S & D effects on MHD boundary layer flow over a convectively heated surface. The study of radiation, heat absorption/generation, heat and mass transfer effects of the unsteady electrically conducting fluid flow past a vertical infinite flat plate with two different types of the thermal boundary conditions was done by Tukyilmazoglu and Pop. Pal and Mondal investigated the influence of thermophoresis and S & D effects on MHD heat and mass transfer over a non-isothermal wedge with thermal radiation and ohmic dissipation. The combined effect of thermophoresis, viscous dissipation and Joule heating on steady MHD flow over an inclined radiative isothermal permeable surface with variable thermal conductivity was investigated by Reddy. The study on unsteady MHD mixed convective flow past an infinite vertical plate with ohmic dissipation and heat source with S & D effects was conducted by Sharma et al.

Influenced by the above studies, we set the objective of this paper to study the effect of Joule heating, thermal radiation, radiation absorption, Soret and Dufour parameters and dissipation on the convective heat and mass transfer of a conducting viscous fluid past a stretching sheet in the presence of a magnetic field. We investigated the coefficient of heat flux, mass flux and shearing stress at the plate and are analyzed with the specific values of Prandtl number (Pr=0.72). The governing equations, which describe the problem, were transformed and solved numerically using the fourth order Runge–Kutta integration scheme featuring a shooting technique.

2 Problem Formulation

A steady incompressible two-dimensional boundary layer of an electrically conducting fluid spreading over a permeable plane surface has been considered in this paper. An applied magnetic field of strength \( B(x) \) is applied transverse to the plate in the \( y \)-direction and varies in strength as a function of \( x \). We further consider a Cartesian rectangular coordinate system \( O(x, y) \) with the plate in the \( y \)-direction and \( x \)-axis normal to the boundary. The sheet is located at \( y=0 \) and its leading edge is the origin of the Cartesian coordinate system. The external electric field assumed to be zero and the magnetic Reynolds number is assumed to be small. Hence the induced magnetic field is small compared with the external magnetic field. Figure 1 shows the sketch of the physical model.

The fluid of density \( \rho \) is at rest and the motion is created by stretching the sheet with a speed proportional to the distance from the fixed origin \( (x=0) \). The dynamic viscosity coefficient \( \mu \) is constant, the pressure gradient and the body forces are negligible in the presence of viscous dissipation and internal heat generation. The effects of uniform mass and heat transfer characteristics in stationary surroundings are investigated. Under these assumptions, the governing equations within the boundary layer are given by:

Continuity equation:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad \cdots (1)
\]

Momentum equation:

\[
u \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) = -\frac{\sigma (\beta x)^2}{\rho} - u + \sigma (T - T_{\infty}) + g \beta (C - C_{\infty}) \quad \cdots (2)
\]

Fig. 1 — Sketch of the physical model.
Energy equation:
\[
\frac{\partial T}{\partial x} + \frac{\partial T}{\partial y} = \frac{k_f}{\rho C_p} \frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho C_p} \frac{\partial u}{\partial y} + \frac{\alpha (B(x))^2}{\rho C_p} u^2 - \frac{1}{\rho C_p} \frac{\partial \rho}{\partial y} + \frac{D_m K T}{C_s p} \frac{\partial^2 C}{\partial y^2} + \frac{Q_1 (C - C_{\infty})}{...} \tag{3}
\]

Concentration equation:
\[
\frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_1 \frac{\partial^2 C}{\partial y^2} - k_f (C - C_{\infty}) + \frac{D_m K T}{T_m} \frac{\partial^2 T}{\partial y^2} ... \tag{4}
\]

where \( u \) and \( v \) are the velocity components along the \( x \) and \( y \) axes, respectively, \( \nu \) is kinematic viscosity, \( T \) and \( C \) are the temperature, concentration in the fluid, \( T_m \) and \( C_{\infty} \) are ambient temperature, concentration, respectively, \( k_f \) is the thermal conductivity, \( C_p \) is the specific heat at constant pressure, \( k_T \) is the coefficient of chemical reaction, \( D_1 \) is the molecular diffusivity and \( q_{ab} \) is the radiant heat flux. The boundary conditions are:

\[
u = d x, v = v_0, \quad -k_j \frac{\partial T}{\partial y} = q_w(x) = b x^m \tag{5}
\]

\[
u = 0, \quad T = T_{\infty}, \quad C = C_{\infty} \quad \text{as} \quad y \to \infty
\]

Following Rosseland approximation (Raptis47) we introduce:

\[
q_{R} = -\frac{4a_{\bullet}}{3\beta_R} \frac{\partial T^4}{\partial y} \quad \text{and} \quad T^4 \approx 4T_{\infty}^4 - 3T_{\infty}^4 \tag{6}
\]

Using Eq. (6), the Eq.(3) reduces to:

\[
\frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \left( \frac{k_f}{\rho C_p} + \frac{16a_{\bullet}^3}{3\beta_R} \right) \frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho C_p} \frac{\partial u}{\partial y} + \frac{\alpha (B(x))^2}{\rho C_p} u^2 + \frac{D_m K T}{C_s p} \frac{\partial^2 C}{\partial y^2} + \frac{Q_1 (C - C_{\infty})}{...} \tag{7}
\]

Using Eq. (8), equation of momentum (Eq. (2)) and equation of energy (Eq. (7)) reduces to:

\[
\frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \left( \frac{k_f}{\rho C_p} + \frac{16a_{\bullet}^3}{3\beta_R} \right) \frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho C_p} \frac{\partial u}{\partial y} + \frac{\alpha (B(x))^2}{\rho C_p} u^2 + \frac{D_m K T}{C_s p} \frac{\partial^2 C}{\partial y^2} + \frac{Q_1 (C - C_{\infty})}{...} \tag{9}
\]

From Eq. (8) and Eq. (2) the momentum equation can be rewritten as:

\[
u \frac{\partial \nu}{\partial y} = -\frac{\sigma}{\rho C_p} \frac{\partial T}{\partial y} + \frac{\nu}{\rho C_p} \frac{\partial u}{\partial y} \quad \text{as} \quad y \to \infty
\]

In view of the continuity Eq. (1) we define a stream function \( \psi(x, y) \) as:

\[
u = \frac{\partial \psi}{\partial y} \quad \text{as} \quad y \to \infty
\]

Using Eq. (11) we find that:

\[
u = d x f'(\eta), \quad v = -\frac{\partial \psi}{\partial x} \quad \text{as} \quad y \to \infty
\]

where prime denote differentiation with respect to \( \eta \). The following transformations are introduced:

\[
\eta = \frac{x}{L}, \quad \psi = \frac{d x f(\eta)}{L} \quad \text{as} \quad y \to \infty
\]

\[
C - C_{\infty} = \frac{D_m}{D_{\eta}} x^2 \phi(\eta) \tag{13}
\]

\[
\text{The momentum, energy and concentration equations can be written in similarity variables as:}
\]

\[
f'''' + (f')^2 + f f''' - M^2 f^2 + \frac{G}{R_{ex}^2} (0 + N f) = 0 \tag{14}
\]

\[
\frac{1}{R^2} \left( 1 + \frac{4N}{3} \right) f'' + Ec(f^{'})^2 - 20f' + 4E cM^2 (f')^2 + Q_{1} \phi + Du \phi'' = 0 \tag{15}
\]

\[
\phi'' + (f f' - 2 f'') - (k_T) \phi + S \phi' = 0 \tag{16}
\]

Following Nachtsheim and Swigert48, transformed boundary conditions may be written as:

\[
f(0) = f'(0) = f''(0) = 0, \quad \phi(0) = 0, \quad a = -1, \quad b = 0 \tag{17}
\]

But as \( \eta'(0), \) \( \phi'(0) \) must be equal to -1, this implies (as could be be concluded from Eq. (17)) that \( m = 2, n = 2, a = 1, b = 1 \).
where \( f_w = -\frac{\nu}{(dx)^{1/2}} \) is the dimensionless wall mass transfer coefficient such that \( f_w > 0 \) indicates wall suction and \( f_w < 0 \) indicates wall injection. The following parameters are used in the Eqs (14)-(16):

\[
R_{ex} = \frac{dx^2}{\nu} \quad \text{(Local Reynolds number)}
\]

\[
S_t = \frac{D_m f K_m}{D_T m} \quad \text{(Soret parameter)}
\]

\[
G = \frac{\beta q L}{v^2} \quad \text{(Grashof number)}
\]

\[
Du = \frac{D_m dK}{D_T C}' \quad \text{(Dufour parameter)}
\]

\[
N = \frac{4 \beta q^3}{\beta R K f} \quad \text{(Buoyancy number)}
\]

\[
N_r = \frac{4 \beta q^3}{\beta R K f} \quad \text{(Thermal radiation parameter)}
\]

\[
M^2 = \frac{\alpha T g^2}{\beta R} \quad \text{(Magnetic parameter)}
\]

\[
k_f = \frac{k}{d} \quad \text{(Chemical reaction parameter)}
\]

\[
Ec = \frac{k_f L^2 d^3}{D \mu c_p} \quad \text{(Eckert number)}
\]

\[
Pr = \frac{\mu c_p}{k_f} \quad \text{(Prandtl number)}
\]

Using Ahammad and Mollah\(^{49}\) the other physical quantity of interest is the local skin friction \((C_f)\), local Nusselt number \((Nu)\) and the local Sherwood number \((Sh)\) may be written as:

**Local skin friction:**

\[
\tau_w = \left[\mu \frac{\partial \theta}{\partial y}\right]_{y=0} = \mu u \frac{d}{\partial y} f''(0)
\]

It is clear that the wall shear stress will increase with increasing \( x \). The skin friction coefficient \( C_f \) takes the form:

\[
C_f = \frac{\tau_w}{0.5 \rho (dx)^{1/2}} = \frac{2}{(R_{ex})^{1/2}} f''(0)
\]

\[
\frac{1}{2} C_f R_{ex}^{1/2} f''(0) = \frac{1}{\theta(0)} \quad \text{(20)}
\]

**Local Nusselt number:**

\[
Nu_x(R_{ex})^{1/2} = \frac{1}{\theta(0)} \quad \text{(20)}
\]

**Local Sherwood number:**

\[
Sh_x(R_{ex})^{1/2} = \frac{1}{\phi(0)} \quad \text{(21)}
\]

### 3 Numerical Procedures

The system of non-linear ordinary differential Eqs. (14-16) together with the boundary conditions (Eq. 17) has been numerically solved by using Nachtsheim-Swigert shooting iteration technique (guessing the missing value) along with sixth order Runge-Kutta initial value solver (Alam and Ahammad\(^{50}\)). In a shooting method, the missing (viz., unspecified) initial condition at the initial point of the interval is assumed, and the differential equation is then integrated numerically as an initial problem to the terminal point. The accuracy of the assumed missing initial condition is then checked by comparing the calculated value of the dependent variable at the terminal point with its given value there. If a difference exists, another value of the missing initial condition must be assumed and the process is repeated. This process is continued until the agreement between the calculated and the given condition at the terminal point is within the specified degree of accuracy. In this type of iterative approach, it is pertinent to know whether there is a symmetric way of finding each succeeding value (assumed value) of the initial condition.

The method of numerically integrating a two-point asymptotic boundary value problem of the boundary-layer type, the initial value method is similar to an initial value problem. Thus it is necessary to estimate as many boundary conditions at the surface as were given at infinity. The governing differential equations are then integrated with these assumed surface boundary conditions. If the required outer boundary condition is satisfied, a solution has been achieved. However, this is not generally the case. Hence, a method must be devised to estimate logically the new surface boundary conditions for the next trial integration. Asymptotic boundary value problems such as those governing the boundary-layer equations are further complicated by the fact that the outer boundary condition is specified at infinity. In the trial integration, infinity is numerically approximated by...
some large value of the independent variable. There is
no a priori genera method of estimating these values.
Selecting too small maximum value for the
independent variable may not allow the solution to
asymptotically converge to the required accuracy. In
Eq. (17) there are three asymptotic boundary
conditions and hence three unknown surface
conditions $f''(0), \theta'(0)$ and $\phi'(0)$. The
selected as the step size that satisfied a convergence
criterion of $10^{-6}$ for the calculations of parameters in
different phases.

4 Comparison
In the absence of viscous dissipation $Ec=0$ and
thermal radiation $Nr=0$, the results obtained herein are
compared with Alam and Ahammad. To assess the
accuracy of the present numerical method, we have
compared the results of our rate of heat transfer
$(1/\theta(0))$ with those of Elbashbeshy and Chen for
their Newtonian fluid case, in the absence of $M, Q_1,
Ec, Nr, Sr, Du$ for different values of $Pr$ in Table 1
and Table 2 and the results are found to be in
excellent agreement.

5 Results and Discussion
In order to get a clear insight of the physical
problem, numerical results are displayed with the help of
graphical illustrations. The results are given
through a parametric study showing the influence of
several non-dimensional parameters, Hartmann
number $M$, Eckert number $Ec$, radiation absorption
parameter $Q_1$, thermal radiation parameter $Nr$,
Soret parameter $Sr$, Dufour parameter $Du$, Prandtl

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|}
\hline
Sr & Du & $C_f$ & $Nu$ & $Sh$ \\
\hline
2 & 0.03 & 3.3565689 & 1.4361584 & 0.6767115 \\
1 & 0.06 & 3.1145821 & 1.4024461 & 0.7542388 \\
0.5 & 0.12 & 3.0271220 & 1.3662225 & 0.8023145 \\
\hline
\end{tabular}
\caption{Comparison of skin friction ($C_f$), heat flux ($Nu$) and mass flux ($Sh$) for our $\alpha=90, f_w=0.2, Sr=1.3, Nr=0, Ec=0$ case with different Sr and Du with Alam et al.}
\end{table}

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|}
\hline
$Pr$ & Literature & Literature & Present values \\
\hline
0.72 & 0.7711 & 0.76217 & 0.7622077 \\
1.00 & 1.00060 & 1.00616 & 1.0062479 \\
10.00 & 7.0921 & 7.09295 & 7.092874 \\
\hline
\end{tabular}
\caption{Comparison of heat transfer $(1/\theta(0))$ with Elbashbeshy and Chen of their Newtonian fluid case for $f_w=0.6, Sc=0.22$ and $M=Q_1=Ec=Nr=Sr=Du=0$ for our different values of $Pr$.}
\end{table}

number $Pr$, suction parameter $f_w$, buoyancy ratio $N$,
chemical reaction parameter $\gamma$ and heat source parameter $Q$. The non-linear equations governing the
flow, heat and mass transfer have been solved
numerically by employing sixth order Runge–Kutta
shooting method. The sheet is assumed to be non-
isothermal with prescribed heat flux varying with
length.

The velocity ($f'$), temperature ($\theta$) and concentration
($\phi$) are plotted versus $\eta$ for the values $M=1, Ec=0.01,
Q_1=0.5, Nr=1, Sr=2, Du=0.01, Pr=0.71, f_w=2, N=0.5,
$\gamma=0.5$, $Q=0.5$ and specific variations of physical
parametric values are varies unless remain same. We
follow the convention that the non-dimensional
temperature or concentration is positive or negative
according to the actual temperature or concentration
that is greater or lesser than $T_\infty/C_\infty$.

5.1 Velocity profiles
Figure 2 represents the axial velocity ($f'$) with
Hartmann number $M$. It can be seen from the profiles
that the axial velocity reduces with increase in $M \leq 1.5$
and enhances with higher $M \geq 3$. This is due to the fact
that the magnetic field provides a resisting type of
force known as the Lorentz force. This force tends to
lesser the motion of the fluid and as a consequence
the velocity reduces. The variation of $f'$ with $Ec$ shows
that the axial velocity enhances with increasing values
of $Ec$ owing to the energy release which increases the
momentum. Figure 3 represents \( f' \) with radiation absorption parameter, \( Q_1 \) and thermal radiation parameter, \( Nr \). It is found that there is a significant depreciation in \( f' \), in the presence of the thermal radiation throughout the boundary layer. The radiation parameter is found to reduce the hydrodynamic boundary layer along the \( x \)-direction. An increase in \( Q_1 \) reduces the velocity \( f' \). This is due to the fact that the thickness of the momentum boundary layer reduces with increasing \( Q_1 \). Figure 4 shows the variation of \( f' \) with Soret parameter (\( Sr \)) or Dufour parameter (\( Du \)). It can be seen from the profiles that the increasing the Soret parameter (\( Sr \)) (or decreasing Dufour parameter (\( Du \))) reduces the velocity \( f' \) with reference to Prandtl number (\( Pr \)). We find that the velocity \( f' \) shows a reduction for increasing values of Prandtl number and hence the thickness of the boundary layer also reduces. Figure 5 represents \( f' \) with suction parameter \( f_w \) and buoyancy ratio (\( N \)). It is found that when the molecular buoyancy force dominates over the thermal buoyancy force, the velocity \( f' \) enhances when the buoyancy forces are in the same direction and for the forces acting in opposite direction \( f' \) reduces in the boundary layer. An increase in the suction/injection parameter \( f_w \) enhances the velocity \( f' \). This is due to the fact the thickness of the momentum boundary layer increases with increasing \( f_w \). Figure 6 represents \( f' \) with chemical reaction parameter \( \gamma \). It can be seen from the profiles that \( f' \) enhances in both degenerating/generating chemical reaction cases. Figure 7 shows \( f' \) with heat source parameter \( Q \). The velocity \( f' \) enhances with increasing values of heat generation/absorption. This is due to the fact that the presence of the heat source/sink generates
energy in the thermal boundary layer and as a consequence the temperature rises.

5.2 Temperature profiles

Figure 8 illustrates the temperature ($\theta$) with Hartmann number, $M$. The temperature enhances with $M$. This is due to the fact an increase in $M$ results in the reduction of velocity and thereby enhances the temperature in the thermal boundary layer. From Fig. 8 we find that an increase in Eckert number, $Ec$. Figure 9 illustrates temperature ($\theta$) $Q_1$ and $Nr$. The presence of thermal radiation is very significant on the variation of temperature. It is seen that the temperature enhances gradually in the presence of thermal radiation parameter $Nr$ throughout the boundary layer. This may be attribute the fact that as the Rosseland radiative absorption parameter diminishes the corresponding heat flux diverse and thus reduces the rate of radiative heat transfer to the fluid causing a rise in the temperature of the fluid. The thickness of the boundary layer also reduces in the presence of $Nr$. An increase in $Q_1$ leads to a rise in the temperature in the fluid.

Figure 10 shows the variation of temperature ($\theta$) with $Sr$, $Du$ and $Pr$. It can be seen from the profiles that the temperature reduces with increasing Soret parameter $Sr$ (or decreasing Dufour parameter $Du$). As Prandtl number increases there is a significant increase in the thermal boundary layer with a rise in the temperature throughout the boundary layer, since enhancement of Prandtl number amounts to enhancement of thermal-diffusion. Figure 11 represents temperature ($\theta$) with suction parameter $f_w$. An increase in suction/injection parameter $f_w$ reduces the temperature. Figure 12 illustrate temperature ($\theta$) with chemical reaction parameter $\gamma$. It can be observed from the profiles that the temperature enhances with increase in generating chemical reaction case whereas temperature reduces in the degenerating chemical reaction case. Figure 13 represents temperature ($\theta$) with heat source parameter $Q$. The presence of heat source generates, absorbs in the thermal boundary layer and as a consequence the temperature rises while it reduces with increase in the heat sink. This is due to the fact that the

![Fig. 8 — Effect of Hartmann number ($M$) and Eckert number ($Ec$) on temperature profiles.](image1)

![Fig. 9 — Effect of radiation absorption parameter ($Q_1$) and thermal radiation parameter ($Nr$) on temperature profiles.](image2)

![Fig. 10 — Effect of Soret ($Sr$), Dufour parameter ($Du$) and Prandtl number ($Pr$) on temperature profiles.](image3)

![Fig. 11 — Effect of Suction parameter ($f_w$) on temperature profiles.](image4)
energy is absorbed in the boundary layer. Figure 14 represents the temperature ($\theta$) with buoyancy ratio ($N$). It is found that when the molecular buoyancy force dominates over the thermal buoyancy force the temperature enhances in the boundary layer irrespective of the directions of the buoyancy forces.

5.3 Concentration profiles

Figure 15 illustrates the concentration distribution, $\phi$ with Hartmann number, $M$. It can be seen from the profiles that the concentration that enhances with increase in $M$. The variation of concentration $\phi$ with Eckert number $Ec$ shows that the concentration increases with increase in $Ec$. It is due to the fact that the thermal energy is reserved in fluid on account of friction heating. Hence the concentration distribution rises in the entire solute boundary layer. Figure 16 shows the variation of concentration distribution $\phi$ with $Q_1$ and $Nr$. We find that higher the radiative heat flux larger the concentration. An increase in $Q_1$ results in a decrease in concentration. It is due to the fact that the thickness of the solutal boundary layer increases with $Q_1$. Figure 17 illustrates the concentration distribution $\phi$ with $Sr$ (decrease in Dufour parameter $Du$) and $Pr$. Increasing the Soret parameter $Sr$ (or decreasing Dufour parameter $Du$) leads to a decrease in the concentration. It is due to the fact that the thickness of the boundary layer reduces with increase in the $Sr$. The mass concentration increases with $Pr$. As a Prandtl...
number increases there is a significance increase in the solutal boundary layer thickness with a rise in the concentration throughout the boundary layer, since enhancement of Prandtl number enhances of concentration diffusion. Figure 18 represent concentration distribution $\phi$ with suction parameter $f_w$. Increase in suction parameter $f_w>0$ enhances the concentration and reduces with injection parameter $f_w<0$. With respect to chemical reaction parameter $\gamma$, we notice an enhancement in concentration in both degenerating and generating chemical reaction cases throughout the boundary layer is illustrated in Fig. 19.

Figure 20 illustrates concentration distribution $\phi$ with heat source parameter $Q$. In the presence of heat source energy is generated in the boundary layer which leads to a rise in the concentration whole the concentration reduces with decrease in the heat sink parameter $Q<0$. This is due to the fact that heat is absorbed in the boundary layer. Figure 21 illustrates concentration distribution ($\phi$) with buoyancy ration ($N$). When the molecular buoyancy force dominates the thermal buoyancy force the concentration reduces in boundary layer, when the buoyancy forces are in the same direction and for the forces acting in opposite directions, it enhances in the boundary layer.

5.4 Skin friction, Nusselt number and Sherwood number

The dimensionless wall values of velocity, temperature and concentrations are graphically represented for various parameters $Nr$, $Ec$, $f_w$, $Q_1$, $Sr$ and $Du$ are illustrated through the graphical notation. The skin friction components $C_f$, Nusselt number $Nu$ and Sherwood number $Sh$ have been plotted in Figs 22-30.

Fig. 17 — Effect of Soret ($Sr$), Dufour parameter ($Du$) and Prandtl number ($Pr$) on concentration profiles.

Fig. 18 — Effect of suction parameter ($f_w$) on concentration profiles.

Fig. 19 — Effect of chemical reaction parameter ($\gamma$) on concentration profiles.

Fig. 20 — Effect of heat source parameter ($Q$) on concentration profiles.

Fig. 21 — Effect of Buoyancy ratio ($N$) on concentration profiles.
Fig. 22 — Effects of thermal radiation parameter ($N_r$) and radiation and absorption parameter ($Q_1$) on skin friction coefficient ($C_f$) for $Ec=0.01$, $Sr=2$, $Du=0.01$, $Pr=0.71$, $f_w=2$, $N=0.5$, $\gamma=0.5$, $Q=0.5$.

Fig. 23 — Effects of thermal radiation parameter ($N_r$) and radiation and absorption parameter ($Q_1$) on Nusselt number ($Nu$), for $Ec=0.01$, $Sr=2$, $Du=0.01$, $Pr=0.71$, $f_w=2$, $N=0.5$, $\gamma=0.5$, $Q=0.5$.

Fig. 24 — Effects of thermal radiation parameter ($N_r$) and radiation and absorption parameter ($Q_1$) on Sherwood number for $Ec=0.01$, $Sr=2$, $Du=0.01$, $Pr=0.71$, $f_w=2$, $N=0.5$, $\gamma=0.5$, $Q=0.5$.

Fig. 25 — Effects of suction parameter ($f_w$) and Eckert number ($Ec$) on skin friction coefficient ($C_f$) for $N_r=0.5$, $Q_1=0.5$, $f_w=2$, $Ec=0.01$, $Pr=0.71$, $N=0.5$, $\gamma=0.5$, $Q=0.5$.

Fig. 26 — Effects of suction parameter ($f_w$) and Eckert number ($Ec$) on Nusselt number for $N_r=0.5$, $Q_1=0.5$, $Sr=2$, $Du=0.01$, $Pr=0.71$, $N=0.5$, $\gamma=0.5$, $Q=0.5$.

Fig. 27 — Effects of suction parameter ($f_w$) and Eckert number ($Ec$) on Sherwood number for $N_r=0.5$, $Q_1=0.5$, $Sr=2$, $Du=0.01$, $Pr=0.71$, $N=0.5$, $\gamma=0.5$, $Q=0.5$.

Fig. 28 — Effects of Soret ($Sr$) and Dufour ($Du$) parameter and Prandtl number ($Pr$) on skin friction coefficient ($C_f$) or $N_r=0.5$, $Q_1=0.5$, $f_w=2$, $Ec=0.01$, $N=0.5$, $\gamma=0.5$, $Q=0.5$.

Fig. 29 — Effects of Soret ($Sr$) and Dufour ($Du$) parameter and Prandtl number ($Pr$) on Nusselt number ($Nu$) for $N_r=0.5$, $Q_1=0.5$, $f_w=2$, $Ec=0.01$, $N=0.5$, $\gamma=0.5$, $Q=0.5$. 

Skin friction parameter $C_f$ enhances with increase in $Nr$, $Ec$ and $Q_1$ and reduces with $f_w$. An increase in Prandtl number $Pr$, Soret parameter $Sr$ (or decrease in Dufour parameter $Du$) results in depreciation in $C_f$ at the wall.

The rate of heat transfer (local Nusselt number $Nu$) enhances with increase in $f_w$. Higher the radiative heat flux or radiation absorption or heat dissipation larger $|Nu|$ at the wall. The variation of $Nu$ with $Pr$, $Sr$ and $Du$ shows that the rate of heat transfer reduces with increase in $Sr$ (or decrease in Dufour parameter $Du$). The rate of mass transfer (local Sherwood Number $Sh$) shows that the rate of mass transfer enhances with $f_w$. Also higher the radiative heat flux or radiative absorption or heat dissipation larger the rate of mass transfer at the wall. Increase in the, Prandtl number $Pr$, Soret parameter $Sr$ (or decrease in the Dufour parameter, $Du$) results in decrease in the rate of mass transfer at the wall.

6 Conclusions

We investigated the coefficient of heat flux, mass flux and shearing stress at the plate and are analyzed with the specific values of Prandtl number ($Pr=0.71, 1.71, 3.71, 5$). Graphical results for various parametric conditions were presented and discussed for different values. The main findings are summarized as follows:

The velocity $f'$ increases with the increase in thermal radiation $Nr$, Eckert number $Ec$, suction parameter $f_w$ in the entire flow region, while the radiation absorption parameter $Q_1$ and Soret $Sr$ (decreasing in Dufour $Du$) parameters reduces the thermal boundary layer in the entire flow region.

The temperature $\theta$ falls with rise in Soret/Dufour parameter, Eckert number $Ec$. Whereas temperature increases with rise in radiation absorption parameter $Q_1$, Prandtl number $Pr$ and thermal radiation $Nr$ in the flow region. The temperature $\theta$ increases in both suction $f_w>0$ and injection parameter $f_w<0$ in the entire flow field.

The generating chemical reaction parameter and degenerating chemical reaction parameter increases with increase in the temperature. The molecular buoyancy forces dominates the thermal buoyancy force, the temperature enhances in the entire boundary layer.

The influence in radiation absorption parameter $Q_1$, suction parameter $f_w$ and Soret or Dufour parameters causes depreciation in the concentration in the entire flow field. Whereas, the effect of concentration experiences an enhancement with increasing $f_w>0$ and depreciation with $f_w<0$. With increase in Eckert number $Ec$, thermal radiation parameter $Nr$, Soret parameter $Sr$ (or decrease in Dufour parameter) the concentration $\phi$ experiences an enhancement in the entire flow region.

Skin friction $C_f$ enhances with increase in thermal radiation parameter $Nr$, Eckert number $Ec$, radiation absorption parameter $Q_1$, Prandtl number $Pr$ and Soret parameter $Sr$ (or decrease in Dufour parameter $Du$) at the wall.

The rate of heat transfer enhances with increase in suction parameter $f_w$. With increase in radiative heat flux or radiation absorption or heat dissipation $|Nu|$ increases at the wall. The rate of heat transfer reduces with increase in Soret parameter $Sr$ (or decrease in Dufour parameter $Du$).

The rate of mass transfer enhances with increase in suction parameter $f_w$, radiative heat flux, radiative absorption, heat dissipation and Prandtl number at the wall. An increase in Soret parameter $Sr$ (or decrease in Dufour parameter $Du$) results a depreciation in the rate of mass transfer at the wall.

References