Climbing control for a flapping foil UUV based on sliding mode variable structure control theory

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In order to obtain accurate and efficient control way of underwater climbing motion, linear model with small disturb in longitudinal plane has been established by analyzing the kinematic and dynamic of the UUV. Sliding mode variable structure control method is selected to design the climbing motion controller, and the simulations of climbing in longitudinal plane respectively by the flapping foil angel and by palmiped angle have been done. Simulation results show that the climbing motion can achieve the desired purpose.

Keywords: Flapping foil UUV; Climbing movement; Kinematic model; Dynamic model; Sliding mode variable structure control

Introduction

The UUV is a kind of intelligent moving platform. The requirement for its maneuverability is higher and higher, especially in complex environments like the complex submarine topography, undercurrent, waves and swells. So, flapping foil UUV is born by studying some flapping foil marine organisms, like sea lions, sea turtles and penguins. It moves more flexibly, quietly and stably. And it can moves exactly in narrow and complex underwater environment. So, flapping foil UUV has a broad developing prospect.

For the advantage of flapping foil UUV, many institute developed various flapping foil UUV. Such as Madeleine1 which developed by VASSAR university of America, Sea Turtle2 which is developed by MIT of America, BAUV3 which is developed by Naval Undersea Warfare Center of America, 3MDMPF4 which is developed by Osaka University of Japan.

The study of the flapping foil UUV focuses on hydrodynamic force of the flapping foil5,6,7. Study of how to control the UUV by flapping foil is not as many as the study of how to control the aircraft by flapping wing8,9,10, it will be explored step by step.

The study of the control system of flapping foil UUV includes two aspects, the attitude control11,12 and the path control. This paper researches how to design a controller to achieve the vertical plane path of a flapping foil UUV which developed by Northwestern Polytechnical University, named Smart Turtle. The model of the controller includes the force and moment model, motion model which deduced by kinematic model and dynamic model. Present study provides a motion model in vertical plane, then control the UUV to realize the vertical path by the angle of foil(angle between the balance point of the flapping foil and the body of UUV) and the angle of horizontal palmiped. Control law is designed by the Sliding mode variable structure control method. And simulations have been made.

In this paper, the mathematical model of climbing control for Smart Turtle is built. By the model, the flapping foil angel, which is the angel between the equilibrium position of flapping and the body of UUV, and palmiped angle in horizontal plane are used for climbing control. And the control law is designed by sliding mode variable structure control theory. Then the comparison of the control based on the two kinds of angels is analyzed.
The rest of the paper is organized as follows. Section 2 gives a brief introduction on Smart Turtle UUV, a dynamic model of the UUV is developed, and the equations of motion for the UUV by the dynamic model and kinematic model is presented. In section 3, the design of the control law in climbing motion of the UUV is presented, and simulations are made. Section 4 draws some useful conclusions of the work.

Materials and Methods
Smart Turtle UUV

Smart Turtle is a flapping foil UUV which is developed by Northwestern Polytechnical University. It has an ellipse rigid body, about 600mm in length, 420mm in width, and 220mm in thickness. It has a pair of two degrees of freedom flapping foil. The chord-length of the foil is 100mm, and the span of the foil is 200mm. The flapping foils propel the UUV and act as surfaces for orientation control. It has a pair of palmipeds also, which is used as an accessory climbing controller (see Fig. 1).

Fig. 1 The appearance and coordinate system definition of Smart Turtle

Dynamic Model of the UUV

Smart Turtle has a compressed body and a pair of 1-DOF flapping drivers and two palmipeds in horizontal plane. One flapping driver is set on the left body of UUV, and another is set on the right. So the climbing movement and the changing of pitching angle can be controlled conveniently. The stability of the UUV movement can be increased. The appearance of the Flapping foil UUV is shown in fig. 1.

Because the kinematic model of this UUV is the same as the model of normal UUV, then the Dynamic model is studied here.

In order to analyzing the movement of flapping foil for the UUV, the position attitude and the force must be expressed. So, some coordinate systems are necessary. Generally, any coordinate system can be chosen. But, the right one can be more helpful to solve problems.

The earth coordinates system and the body coordinate system are selected and shown in fig. 1.

Here are some basic hypothesis:

1. UUV is rigid body. And its shape is symmetric about plane \( \alpha xy, \alpha xz \);
2. UUV is submerged in the fluid medium completely, and in bedew state completely;
3. The rotation and the curvature of the earth are not considered. The earth coordinates system is considered as an inertial system.

The momentum of the UUV \( \vec{Q} \) can be written as:

\[
\vec{Q} = m\vec{v_c}
\]

where \( m \) is the mass of the UUV; \( \vec{v_c} \) is the velocity vector of the UUV center of mass in earth coordinate system, and

\[
\vec{v_c} = \vec{v_0} + \omega \times \vec{r_c}
\]

where \( \vec{v_0} \) is the velocity vector of the UUV center of buoyancy; \( \vec{r_c} \) is the radius vector from the center of mass to the center of buoyancy; \( \omega \) is rotational velocity.

In the same way, the UUV momentum moment \( \vec{K} \) can be expressed as

\[
\vec{K} = J_0 \omega + \vec{r_c} \times m\vec{v_0}
\]

where \( J_0 \) is matrix of moment of inertia relative to the UUV center of buoyancy, and is shown as

\[
J_0 = \begin{bmatrix} J_{xx} & -J_{xy} & -J_{xz} \\ -J_{yx} & J_{yy} & -J_{yz} \\ -J_{zx} & -J_{zy} & J_{zz} \end{bmatrix}
\]

where, \( J_x, J_y, J_z \) are the moment of inertia to body coordinate system, \( J_{xx}, J_{xy}, J_{xz}, J_{yx}, J_{yy}, J_{yz}, J_{zx}, J_{zy} \) are product of inertia to the three axes, and \( J_{xy} = J_{yx}, J_{xz} = J_{zx}, J_{zy} = J_{zy} \).

Now the expressions of the momentum and the moment of momentum (expression (1) and expression (3)) can be turned into the body coordinate system. Then the following equation can be gotten:

\[
\begin{bmatrix} Q_x \\ Q_y \\ Q_z \\ K_x \\ K_y \\ K_z \end{bmatrix} = \begin{bmatrix} m & 0 & 0 & m_zy & -m_yz \\ 0 & m & 0 & -m_zx & 0 \\ 0 & 0 & m & -m_xz & 0 \\ 0 & -m_zx & m_yz & J_{yx} & -J_{zy} \\ 0 & m_yz & -m_xz & J_{xy} & -J_{xz} \\ -m_yz & mx_z & 0 & -J_{yx} & -J_{zy} \end{bmatrix} \begin{bmatrix} v_x \\ v_y \\ v_z \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}
\]
\( Q_x, Q_y, Q_z \) are the momentums of three axes of body coordinate system, \( K_x, K_y, K_z \) are the momentum moments of three axes of body coordinate system, \( \omega_x, \omega_y, \omega_z \) are rotational velocitys of three axes of body coordinate system, \( x, y, z \) are the distances from the center of mass to the center of buoyancy in three axes of body coordinate system.

\( v_{ox}, v_{oy}, v_{oz} \) are the velocity of Smart Turtle in earth coordinate system.

The forces on the UUV are as follows:

The ideal fluid inertia force and the ideal fluid inertia moment on the UUV can be shown as:

\[
\left[
\begin{array}{ccc}
R_x & dc/dt & 0 \\
R_y & dc/dt & 0 \\
R_z & dc/dt & 0 \\
M_x & d\omega_x/dt & 0 \\
M_y & d\omega_y/dt & 0 \\
M_z & d\omega_z/dt & 0 \\
\end{array}
\right] = \lambda \left[
\begin{array}{c}
v_x \\
v_y \\
v_z \\
\omega_x \\
\omega_y \\
\omega_z \\
\end{array}
\right]
\]

\( R_x, R_y, R_z \) are the ideal fluid inertia force of three axes in earth coordinate system, \( M_x, M_y, M_z \) are the ideal fluid inertia moment of three axes in earth coordinate system.

The resistance is opposite to the motion direction of the UUV, along the opposite direction of the OX axis in velocity coordinate system. So the resistance can be shown as:

\[
X = C_{ss} \frac{1}{2} \rho v^2 S
\]

where, \( C_{ss} \) is resistance coefficient;
\( \rho \) is density of water;
\( v \) is the velocity of the UUV;
\( S \) is the maximum cross-section area of UUV.

The lift force can be shown as:

\[
Y = Y_1 + Y(\alpha) + Y(\chi) + Y(\delta)
\]

where,
\( \alpha \) is attack angle;
\( \chi \) is the angle between the balance point of the flapping foil and body of the UUV. In this paper we call it the angle of foil (left angle of foil equals right angle of foil);
\( \delta \) is the angle between the palmiped and body of the UUV.

\( Y_1 \) is the forces because of asymmetry of the UUV. The possibility of asymmetry is so little that we can neglected it;

\( Y(\chi) \) is lift force because of angle of foil, it depends on the angle of foil;

\( Y(\delta) \) is lift force because of angle of palmiped, it depends on the angle of palmiped \( \delta \);

The buoyancy force of UUV \( B \) can be shown as:

\[
B = \rho V
\]

where \( V \) is the displacement volume of UUV.

The direction of buoyancy force is up, along the opposite direction of the \( oy_0 \) axis in earth coordinate system. The components in the torpedo body coordinate system are:

\[
\left[
\begin{array}{c}
X_B \\
Y_B \\
Z_B \\
\end{array}
\right] = C_b^0 \left[
\begin{array}{c}
0 \\
0 \\
0 \\
\end{array}
\right] = \left[
\begin{array}{c}
B \sin \theta \\
B \cos \theta \cos \phi \\
-B \cos \theta \sin \phi \\
\end{array}
\right]
\]

where \( C_b^0 \) is transformation matrix from earth coordinate system to torpedo body coordinate system.

Gravity of the UUV is:

\[
G = mg
\]

The direction of gravity is down, along the opposite direction of the \( oy_0 \) axis in earth coordinate system. The components in the torpedo body coordinate system are:

\[
\left[
\begin{array}{c}
X_G \\
Y_G \\
Z_G \\
\end{array}
\right] = C_b^0 \left[
\begin{array}{c}
0 \\
0 \\
0 \\
\end{array}
\right] = \left[
\begin{array}{c}
-G \sin \theta \\
-G \cos \theta \cos \phi \\
G \cos \theta \sin \phi \\
\end{array}
\right]
\]

The direction of thrust force is along the anterior of the flapping foil, the angle of foil is the angle with the x axis in torpedo body coordinate system. The thrust force can be shown as:

\[
T = C (\nu) \frac{1}{2} \rho v^2 S_{mm}
\]

where,
\( C (\nu) \) is thrust coefficient;
\( \rho \) is wet density;
\( \nu \) is approaching velocity, can be equivalent to sailing velocity approximately;
\( S_{ym} \) is the projected area of the flapping foil from up to down.

Thrust force can be disassembled into \( x \) direction and \( y \) direction

\[ F_x = C(v) \frac{1}{2} \rho v^2 S_{ym} \cos(\chi), \quad F_y = C(v) \frac{1}{2} \rho v^2 S_{ym} \sin(\chi) \]

The dynamic model of the UUV is built according to theorems of momentum and momentum moment:

\[
\begin{align*}
\frac{d\bar{Q}}{dt} + \bar{\omega} \times \bar{Q} &= \bar{F} \\
\frac{d\bar{K}}{dt} + \bar{\omega} \times \bar{K} + \bar{\omega}_0 \times \bar{Q} &= \bar{M}
\end{align*}
\] (14)

Then, the motion dynamic model of the UUV in space can be gotten:

\[
A_{ml} \begin{bmatrix} \dot{v}_{ox} \\ \dot{v}_{oy} \\ \dot{v}_{oz} \\ \dot{\omega}_x \\ \dot{\omega}_y \\ \dot{\omega}_z \end{bmatrix} + \frac{dA_{ml}}{dt} \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = -A_{vo} \begin{bmatrix} v_{ox} \\ v_{oy} \\ v_{oz} \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} + A_{FM}
\] (15)

where \( A_{ml} \) is inertia matrix:

\[
A_{ml} = \begin{bmatrix}
m + \lambda_{11} & \lambda_{12} & 0 & 0 & 0 & 0 \\
\lambda_{21} & m + \lambda_{22} & 0 & 0 & 0 & 0 \\
0 & 0 & m + \lambda_{33} & 0 & 0 & 0 \\
0 & -mz_c & my_c + \lambda_{43} & -mz_c & 0 & 0 \\
my_c + \lambda_{51} & mx_c + \lambda_{52} & 0 & \lambda_{53} & 0 & 0 \\
-mz_c & mx_c + \lambda_{52} & \lambda_{53} & 0 & 0 & 0 \\
my_c + \lambda_{34} & mx_c + \lambda_{35} & 0 & 0 & 0 & 0 \\
J_{xx} + \lambda_{44} & -J_{xy} + \lambda_{45} & -J_{xz} & 0 & 0 & 0 \\
-J_{xy} + \lambda_{45} & J_{yy} + \lambda_{55} & -J_{yz} & 0 & 0 & 0 \\
-J_{xz} & -J_{yz} & J_{zz} + \lambda_{66} & 0 & 0 & 0 \\
0 & 0 & 0 & \lambda_{61} & \lambda_{62} & \lambda_{63} \\
0 & 0 & 0 & \lambda_{62} & \lambda_{64} & \lambda_{65} \\
0 & 0 & 0 & \lambda_{63} & \lambda_{64} & \lambda_{65} \\
0 & 0 & 0 & 0 & \lambda_{65} & \lambda_{66} \\
0 & 0 & 0 & 0 & 0 & \lambda_{66} \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\] (16)

\( A_{vo} \) is velocity matrix:

\[
A_{vo} = \begin{bmatrix}
0 & -\omega_y & \omega_x & 0 & 0 & 0 \\
\omega_z & 0 & -\omega_x & 0 & 0 & 0 \\
-\omega_y & \omega_z & 0 & 0 & 0 & 0 \\
0 & -v_{ox} & v_{oy} & 0 & -\omega_z & \omega_y \\
v_{oz} & 0 & -v_{ox} & \omega_x & 0 & -\omega_x \\
v_{oy} & v_{ox} & 0 & -\omega_y & \omega_x & 0
\end{bmatrix}
\] (17)

\( A_{FM} \) is force matrix:

\[
A_{FM} = \begin{bmatrix} X_{aq} + X_B + X_G + X_T + X_p \\ Y_{aq} + Y_{ap} + Y_B + Y_G + Y_p \\ Z_{aq} + Z_{ap} + Z_B + Z_G + Z_p \\ M_{aqx} + M_{aqx} + M_{Gx} \\ M_{aqy} + M_{aqy} + M_{Gy} + M_{py} \\ M_{aqz} + M_{aqz} + M_{Gz} + M_{pz} \end{bmatrix}
\] (18)

\[ v_x = (F_x - C_{vz} \frac{1}{2} \rho v^2 S \Delta G \sin \theta) / (m + \lambda_{11}) \]

\[ v_y = (bk - a) / (k_z - k_i k_j) \]

\[ \omega_z = (a) / (k_z - k_i k_j) \]

\[ \dot{\theta} = \omega_z \]

\[ x_0 = v_x \cos \theta - v_y \sin \theta \]

\[ y_0 = v_x \sin \theta + v_y \cos \theta \]

\[ a = -m v_x \omega_z - \Delta G \cos \theta + F_y + \frac{1}{2} \rho v^2 S (C_{vz} \alpha + C_{\alpha \gamma} \chi) + C_{\chi \chi} \dot{\chi} + C_{\gamma \gamma} \dot{\gamma} \]

\[ b = -m x v_x \omega_z + \frac{1}{2} \rho v^2 S L (m x \alpha + m \xi \chi + m \xi \chi + m \xi \xi \chi) + \]

\[ G (y \sin \theta - x \cos \theta) + M \]

\[ k_1 = m + \lambda_{22} \]

\[ k_2 = mx_c + \lambda_{26} \]

\[ k_3 = J_{xx} + \lambda_{66} \]

\[ \nu = \sqrt{v_x^2 + v_y^2} \]

\[ \alpha = -\arctan(v_x / v_y) \]

\[ F_i = C(v) \frac{1}{2} \rho v^2 S_{ym} \cos(\chi) \]

\[ F_i = C(v) \frac{1}{2} \rho v^2 S_{ym} \sin(\chi) \]

\[ M_i = F_i L_1 \]

where, \( L_1 \) is the distance between flapping foil and \( z \) axis; \( \delta \) is the angle of the horizontal palmiped.

**Results and Discussion**

By analyzing the structure and the mathematical model of the UUV, the flapping foil and the horizontal palmiped can be used to control the climbing motion of the UUV. The angle of the flapping foil and the angle of the horizontal palmiped can be served as control input in climbing motion.
When the angel of flapping foil is served as the control input, the motion of the UUV is a kind of vector propulsion motion. In this situation, the motion in vertical plane is realized by resolution of thrust. In this paper the angels of the left and right flapping foils are set as equal, and the control of the fix amplitude and fix frequency of flapping is analyzed.

The climbing motion control by the angel of flapping foil

When the angel of flapping foil is served as the control input, the motion of the UUV is a kind of vector propulsion motion. In this situation, the motion in vertical plane is realized by resolution of thrust. In this paper the angels of the left and right flapping foils are set as equal, and the control of the fix amplitude and fix frequency of flapping is analyzed.

Here, the state vector and the control input vector of the system is chosen as follows:

\[
x = (vy, \omega, y, \theta)^T
\]

\[
u = u
\]

By linearization of the motion equation of the UUV at the equilibrium, the minimum disturbance model can be gotten:

\[
\Delta x = A\Delta x + B\Delta u
\]

where

\[
A = \begin{bmatrix}
\frac{\partial F}{\partial x}
\end{bmatrix}_{x_d, \mu_d}
\quad B = \begin{bmatrix}
\frac{\partial F}{\partial u}
\end{bmatrix}_{x_d, \mu_d}
\]

\[
\Delta x, \Delta u \text{ is minimum disturbance state and input.}
\]

Because reference motion (the motion without disturbance) is steady motion generally, that is, motion parameters are constants and their derivatives are zero, and the uniform linear horizontal motion is chosen as reference here, then:

\[
v = \alpha = \omega = \theta = 0
\]

\[
sin(\alpha) \approx 0, cos(\alpha) \approx 1, \theta \approx \alpha
\]

The sliding mode variable structure control theory is used to design control law. By the Ackerman formula, C in sliding mode variable structure control theory is known. So the slide plane is:

\[
S = Cx
\]

The equation of control law is:

\[
u = (CB)^{-1}(\varepsilon sgn(s) - ks - CAx)
\]

The climbing motion control by the angel of palmiped

The process described here is the motion from one depth to another in water under a little angle of pitch.

And the climbing control of the UUV with fixed amplitude and frequency is analyzed. One point can be known from the ordinary UUV is that the horizontal palmiped has a limiting angel.

The linearization method and the state variable are the same as above section.

Simulation example

(1) The undetermined parameters in model

By taking the real motion of the UUV into consideration, the flapping amplitude is fixed at 30°, and the frequency is 6Hz.

The thrust of the UUV is forwards, and it has angle \( \chi \) with the x axes of body coordinate system, which is the angle of flapping foil. Thrust coefficient can be shown as the function of flow velocity. That is

\[
c = c(v)
\]

In this paper, the flapping amplitude of foil is chosen as 30°. By the help of FLUENT software and wind tunnel experiment the thrust coefficient between 0.5m/s to 1.6m/s can be gotten (this will be discussed in another paper):

\[
f = \frac{f^2}{248.2v^5 - 854.9v^4 + 1139.7v^3 - 704.1v^2 + 201.5v - 19.7}
\]

Then the thrust can be shown as follows when the flapping amplitude is 30°:

\[
T = C(v) \frac{1}{2} \rho v^2 S_m \cos(\chi)
\]

where: \( C(v) \) can be gotten with equation (27) in the velocity 0.5m/s to 1.6m/s.

When the angle of flapping foil is input, the ultimate angle of flapping foil is set as 70° in control simulation study. When the angle of palmiped is input, the ultimate angle of palmiped is set as 15°.

(2) The simulation of climbing motion with angle of flapping foil

Datas can be put into linearization models(equation (20) to (23))and the following equations can be gotten:
\[
A = \begin{bmatrix}
-0.0014 & -0.4546 & 0 & -0.0149 \\
-0.0123 & -0.3176 & 0 & 0.3000 \\
1.0000 & 0 & 0 & 1.5800 \\
0 & 1.0000 & 0 & 0
\end{bmatrix}, \quad B = \begin{bmatrix}
1.5527 \\
-0.3106 \\
0 \\
0
\end{bmatrix}
\]

That is

\[
\begin{bmatrix}
\dot{v} \\
\dot{\omega} \\
\dot{y} \\
\dot{\theta}
\end{bmatrix} = \begin{bmatrix}
-0.0014 & -0.4546 & 0 & -0.0149 \\
-0.0123 & -0.3176 & 0 & 0.3000 \\
1.0000 & 0 & 0 & 1.5800 \\
0 & 1.0000 & 0 & 0
\end{bmatrix} \begin{bmatrix}
v \\
\omega \\
y \\
\theta
\end{bmatrix} + \begin{bmatrix}
1.5527 \\
-0.3106 \\
0 \\
0
\end{bmatrix} u
\]

Controllable criterion is\(^{17}\):

\[
\text{rank}[B \ AB \ \cdots \ A^3B] = 4
\]

Considerable criterion is\(^{17}\):

\[
\text{rank}[C \ CA \ \cdots \ C A^3] = 4
\]

so the system is controllable and considerable.

Sliding mode variable structure control is used to design controller, and Ackerman formula is used to design the C of sliding mode variable structure controller\(^{19}\). Then

\[
C = [-22.6027, \ -116.2113, \ -12.1940, \ -119.1289]
\]

The climbing motion control is studied here\(^{21}\), then the sliding surface is:

\[
s = Cx - 22.6027vy - 116.2113\omega - 12.1940(y - yd) - 119.1289\theta
\]

The controller is\(^{22,23}\):

\[
u = (CB)^{-1}(-\varepsilon \text{sgn}(s) - ks - CAx)
\]

Here the climbing motion is carried out by angle of flapping foil\(^{25}\), then

If \(|u| > 70^\circ\), then \(u = 70^\circ \cdot \text{sign}(u)\)

The simulation condition is set be: initial depth is 0, climbing 4m then steadily uniform motion in a straight line. \(\varepsilon\) is 0.01, k is 1, the Simulink software is used to simulation\(^{25}\).

The result is shown in fig. 2 to fig. 5.

The process of the simulation is: the initial velocity of the UUV is \(v=1\text{m/s}\), initial pitching angle is 0, initial depth is 0m, the starting time is 0s, the UUV moves to 4m depth with the drive of flapping foil which amplitude and frequency is fixed.

The result indicates that:

With the angle of flapping foil, the UUV can climb up to an appointed depth in a short time. And the pitching angle acceleration reaches 0 in the end. So the UUV can achieve uniform motion in a straight line.

From the figs we can see that the UUV will shake in initial movement with the controller, but the shake can deduce rapidly. And in almost 5s, the dithering of the control input and the pitching angle acceleration can disappear. Then they will keep steady.
The simulation of climbing motion with angel of palmiped

Dates can be put into linearization models (20) to (23) and the following equations can be gotten:

\[
A = \begin{bmatrix}
-0.0005773 & -0.4546 & 0 & -0.0162 \\
0.0017 & -0.3176 & 0 & 0.2925 \\
1.0000 & 0 & 0 & 1.5800 \\
0 & 1.0000 & 0 & 0
\end{bmatrix}, \quad B = \begin{bmatrix}
0.2374 \\
-1.7170 \\
0 \\
0
\end{bmatrix}
\]

That is

\[
\begin{bmatrix}
\ddot{y} \\
\omega \\
\dot{y} \\
\dot{\theta}
\end{bmatrix} = \begin{bmatrix}
-0.0005773 & -0.4546 & 0 & -0.0162 \\
0.0017 & -0.3176 & 0 & 0.2925 \\
1.0000 & 0 & 0 & 1.5800 \\
0 & 1.0000 & 0 & 0
\end{bmatrix} \begin{bmatrix}
y \\ \omega \\ y \\ \theta
\end{bmatrix} + \begin{bmatrix}
0.2374 \\
-1.7170 \\
0 \\
0
\end{bmatrix} u
\]

Controllable criterion is\(^{17}\):

\[
\text{rank}[B \ AB \ \cdots \ A^3B] = 4
\]

Considerable criterion is\(^{17}\):

\[
\text{rank}[C \ CA \ \cdots \ CA^3]^T = 4
\]

So the system is Controllable and considerable.

Here the climbing motion is controlled with angel of palmiped, and then the following enactment is set:

when \(|u| > 15^\circ\), \(u = 15^\circ \cdot \text{sign}(u)\)

The result can be gotten as fig. 6 to fig. 9

The process of the simulation is: the initial velocity of the UUV is \(v=1.58\) m/s, initial pitching angle is 0, initial depth is 0m, the starting time is 0s, the flapping foil UUV moves to 4m depth with the drive of flapping foil which amplitude and frequency is fixed.

The result indicates that:

With the angle of palmiped, the UUV can climb up to an appointed depth in 80s. And the pitching angle acceleration reaches 0 in the end. So the UUV can achieve uniform rectilinear motion.

From the figs we can see that the UUV will shake in initial movement with the controller, but the shake can deduce rapidly. And in almost 10s, the dithering of the control input and the pitching angle acceleration can disappear. Then they will keep steady.

(4) The contrast of two control method of climbing motion

The contrast of climbing motion with angle of flapping foil and angel of palmiped is show as fig. 10.

In fig. 10, real line show the climbing motion with angle of flapping foil, broken line show the climbing motion with angle of palmiped. From fig. 10, it can be gotten that UUV can climbing to 4m depth in about 30s with the control of angle of flapping-foil, and need about 80s with the control of angle of palmiped.
Experiments show that the controller is efficient and accurate.

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