An emerging intuitionistic fuzzy based groundwater level prediction

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Primary objective of this paper is to compare the efficiency of two computational intelligence techniques in groundwater level prediction of a watershed. Techniques under comparison are Artificial Neural Networks (ANNs) and Intuitionistic Fuzzy Logic based Neural Network (IFLNN). Performance of the proposed model is measured against the generalization ability of the two techniques in groundwater level prediction of a watershed.

[Key words: Intuitionistic Fuzzy Logic, Artificial Neural Network, Groundwater level prediction]

Introduction

Artificial Neural Networks and Fuzzy Logic have been increasingly in use in engineering fields since their introduction as mathematical aids\(^1\) and \(^2\) respectively. Being branches of Artificial Intelligence, both emulate the human way of using past experiences, adapting itself accordingly and generalizing. While the former has the capability of learning by means of parallel connected units, called neurons, which process inputs in accordance with their adaptable weights usually in a recursive manner for approximation; the latter can handle imperfect information through linguistic variables, which are arguments of their corresponding membership functions. After the introduction of back-propagation algorithm for training multi-layer networks, Artificial Neural Networks have found many applications in numerous interdisciplinary areas. On the other hand, Fuzzy Logic made a great advance in the mid 1970s with some successful results of laboratory experiments\(^3\). In 1985, Takagi and Sugeno\(^4\) proposed a new rule-based modeling technique using FL. Operating with linguistic expressions; fuzzy logic can use the experiences of a human expert and also compensate for inadequate and uncertain knowledge about the system. On the other hand, ANNs have proven superior learning and generalizing capabilities even on completely unknown systems that can only be described by its input-output characteristics\(^5\). A common nonlinear method for groundwater problems is the artificial neural network (ANN). Many kinds of algorithms for training the network have been developed for groundwater level forecasting. A significant advantage of the ANN approach in system modeling is that one need not have a well-defined physical relationship for systematically converting an input to an output. There have been various papers considering the application of ANN techniques in water resource problems. In the groundwater domain, ANN has been used for groundwater management\(^6\). Several papers have reported about the use of ANN for groundwater level forecasting \(^7\). In 2004, groundwater table change before and after a bridge pier construction was analyzed by ANN\(^8\). Many previous researchers have pointed out that ANN as non-linear model is a powerful tool to estimate a fluctuation of groundwater level by considering hydrological variables as inputs. A detailed theory and application of ANN in hydrology can be found in Govindaraju (2000a, b)\(^9,10\). The application of a more promising soft computing technique, the fuzzy inference system (FIS), has recently been increasing in hydrology. Lu and Lo (2002)\(^11\) used self-organizing maps (SOM) and fuzzy theory for diagnosing reservoir water quality. In 2003 fuzzy logic algorithms were used for estimating sediment loads from bare soil surface\(^12\). In the same year volume of rainfall was predicted using SOM, BPNN (Back propagation neural networks), and fuzzy rule systems\(^13\) and \(^14\) fuzzy based ANN was used to identify the rock parameter. Many researchers predicted water
level using fuzzy logic and ANN\textsuperscript{3,4,13,15,16}. During the past decade, a fuzzy neural network has been found to solve many problems which cannot be solved before\textsuperscript{17}. For instance, the fuzzy neural The IFSs have proved to be more powerful to deal with vagueness and uncertainty than fuzzy sets, combination of IFSs and artificial neural networks is investigated by many experts. Max-min intuitionistic fuzzy Hopfield neural network (IFHNN) is proposed in the year 2007\textsuperscript{20}. In 2012, an intuitionistic fuzzy neural model based on simple intuitionistic inference system was introduced\textsuperscript{21}.

In this present work two different models have been developed using two different soft computing techniques namely Artificial Neural Network and Intuitionistic Fuzzy Inference system based Artificial Neural Network for groundwater level prediction of a watershed.

**Materials and Methods**

Definition (Atanassov,1986)\textsuperscript{22} Let X be a nonempty fixed set and I the closed interval \([0,1]\). An intuitionistic fuzzy set (IFS) A is an object of the following form

\[ A = \{ \langle x, \mu_A(x), \gamma_A(x) \rangle | x \in X \} \]

where the mapping \( \mu_A : X \rightarrow I \) and \( \gamma_A : X \rightarrow I \) denote the degree of membership (namely \( \mu_A(x) \)) and the degree of nonmembership (namely \( \gamma_A(x) \)) for each element \( x \in X \) to the set A, respectively, and \( 0 \leq \mu_A(x) + \gamma_A(x) \leq 1 \) for each \( x \in X \).

Obviously, every fuzzy set A on a nonempty set X is an IFS of the following form

\[ A = \{ \langle x, \mu_A(x), \gamma_A(x) \rangle | x \in X \} \]

\[ \pi_A(x) = 1 - \mu_A(x) - \gamma_A(x) \]

In addition the for every Intuitionistic fuzzy index of x in A is defined as \( \Pi_A(x) = 1 - \mu_A(x) - \gamma_A(x) \), where \( \Pi_A(x) \) – is the degree of indeterminacy or uncertainty or hesitancy of the element \( x \) in A where, for every \( x \in X \), \( 0 \leq \pi_A(x) \) \leq 1. Obviously it expresses a lack of knowledge of whether \( x \in X \) or not.

The study area is located between 10° 10’ 00”N to 10° 57’ 20” N latitude and 76° 43’ 20”E to 77° 12’ 30”E longitude covers an area of 23888sqkm. The study area (Figure 1) includes four sub basins namely Sholayar, Aliyar, Palar and Walayar. Among, Sholayar sub basin is covered with structured hills and no groundwater development exists in this area.

This study area consists of eight interconnected reservoirs, in which six reservoirs are located in hilly region and two reservoirs are at plains through which the water is supplied to the command area. Temperature of the study area ranging from 13.78° to 35.65° C and receiving rainfall of minimum 1591mm to maximum 9794mm. This area receives an average annual rainfall from 445.20mm to 4364mm. Study area receives maximum rainfall during southwest monsoon (June to September). The study area has undulating topography with maximum contour elevation in the plain is 300m and maximum spot height in the plain is 385m above MSL. One third of the basin area (822.73sqkm) is covered with hills and dense forest cover. This command area is feed by various canals such as Vettaikkarapudur canal, Pollachi canal, Sethumadai canal, Aliyar feeder canal, Parambikulam main canal, Udumalpet canal and High level canal. These canal systems remain operational nearly 9 months in year. Monthly average of groundwater and rainfall from the year 1972 to 2010, were collected from the Groundwater wing of the State Government. Two wells one from Walayar sub basin and another from Aliyar sub basin have been selected for this study.

In order to develop a suitable neural network for predicting the groundwater levels, appropriate assignment of past values of inputs to input nodes is required. Basically, the crucial process of developing a predictive model is to identify the input variables among the available variables for each output variable. In this study initially cross correlation is performed between the groundwater data of the wells selected and the data of neighboring wells to select the influencing wells. Further cross correlation is performed between the groundwater levels of wells under consideration and rainfall data of the influencing...
rain gauge station to identify the lags. Then auto correlation is performed to identify the lags up to which the influence of groundwater and rainfall data of the given month is influenced by the previous month data. Finally the selected input variables are shown in table-1.

Table-1 Input Neurons

<table>
<thead>
<tr>
<th>Name of the well</th>
<th>Influencing OB Well</th>
<th>Influencing Raingauge station</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pulliam patty (63708)</td>
<td>gw (t-1) gw (t-2)</td>
<td>Pollachi</td>
</tr>
<tr>
<td>Pollachi (63709)</td>
<td>gw t</td>
<td></td>
</tr>
<tr>
<td>Gopala puram (63717)</td>
<td>gw (t-1)</td>
<td></td>
</tr>
<tr>
<td>Pichanur (63614)</td>
<td>gw (t-1)</td>
<td></td>
</tr>
<tr>
<td>M.N. Palayam (63710)</td>
<td>Gw (t)</td>
<td>Anamalai</td>
</tr>
</tbody>
</table>

Intuitionistic Fuzzy Neural Networks

![Intuitionistic Fuzzy System based Artificial Neural Network Prediction Model](image)

The figure 2 shows the prediction model of the proposed work in which the dataset is collected from the Parambikulam Aliyar Basin area. Influencing wells ground water level and the rainfall are used as input and the output is prediction of ground water level of the corresponding well as mentioned in the table 2. Input is normalized for the use of the IF based prediction model the values lies 0 to 1. The membership and nonmembership of each variable is obtained and the intuitionistic fuzzy inference system generates the set of possible rules based on the degree of fulfillment and non-fulfillment of the rule.

Intuitionistic fuzzy Layers functionalities are as follows:
- The first layer is input layer.
- The second layer is membership and non-membership function creating layer.
- The third layer is inference layer. The fourth layer is normalized layer.
- The fifth layer is output layer.
Layer 1: no computation is done in this layer, it just pass inputs in to next layer.
Layer 2: the membership and non-membership are determined for the neurons generation criteria
Layer 3: It is Intuitionistic fuzzy inference layer. Each node represents a fuzzy rule. The degree of fulfillment and non-fulfillment of the rule
Layer 4: It normalized the degree of fulfillment and non-fulfillment of the fuzzy and calculated the hesitation margin index.
Layer 5: The output of the intuitionistic neural network with rules n

In this proposed model, an intuitionistic fuzzy neural network model with a triangular form membership and 1 minus triangular form non-membership functions are proposed. The network structure has six layers, and adopts Intuitionistic fuzzy reasoning. A new fuzzy inference system is applied in the model, which contains hesitation margin as a part. Two steps are used in this dynamic optimal training algorithm. First step involves membership function parameter’s training and the second step involves non-membership function parameter’s training, which can promise the summing of the trained membership and non-membership functions on finite universal set less than 1.

Definition (2):

i) Let the intuitionistic fuzzy IF-THEN rule be interpreted as a relation, in a physical domain with membership function and non-membership function defined as:

\[
Q_{MM}((x, x'), (y, y')) = (Q_x^M(x, y), Q_y^M(x', y'))
\]

\[
\mu_{Q_M} (x, y) = \min[\mu_{IFP1}(x), \mu_{IFP2}(y)]
\]

\[
v_{Q_M} (x', y') = \min[v_{IFP1}(x'), v_{IFP2}(y')]
\]

Then \(Q_{MM}\) is called the intuitionistic fuzzy Mamdani minimum implication.

(ii) Let the intuitionistic fuzzy IF-THEN rule be interpreted as a relation, \(MP\) in the physical domain with membership function and non-membership function defined as:

\[
Q_{MP}((x, x'), (y, y')) = (Q_x^P(x, y), Q_y^P(x', y'))
\]

\[
\mu_{Q_M} (x, y) = \mu_{IFP1}(x), \mu_{IFP2}(y)
\]

\[
v_{Q_M} (x', y') = 1 - (1 - \mu_{IFP1}(x))(1 - \mu_{IFP2}(y'))
\]

Then \(Q_{MP}\) is called the intuitionistic fuzzy Mamdani product implication. Both the IF Mamdani minimum and Mamdani product implications use the following IF-THEN rule:

IF IF 1 THEN IFP 2 OTHERWISE EMPTY

where EMPTY means the given IF-THEN rule is not satisfied for a particular combination of elements x and y. Hence, a membership value of 0 and a non-membership value of 1 are given to the combination.

![Figure 3 Initial Membership and non membership function for the both the wells 63708 and 63710](image)

The figure 3 shows the initial membership and non-membership value of the overall variables used in this model. Intuitionistic Fuzzy logic uses any logical value from the set of real numbers between 0 (completely false) and 1 (completely true) which is known as its membership value and the function that represents such values is called a membership function. Here bell shaped membership function is as follows:

\[
\mu_A(x) = \frac{1}{1 + \left(\frac{x - c_i}{a_i}\right)^{2b}}
\]

where \(\{a_i, b_i, c_i\}\) is the parameter set. As the values of these parameters change, the bell-shaped function varies accordingly, thus exhibiting various forms of membership function for fuzzy set A. Parameters in this layer are referred to as premise parameters.

As the value of intuitionistic membership function lies between 0 to 1 the range is divided into Extremely Low (EL), Very Low (VL), Low (L), Medium (LM), Medium(M), High (H), Very High (VH) and Extremely High (EH).

For predicting groundwater level, the IFSANN model had 2 neurons in the input layer, 12 and 20 neurons in the hidden layers, and 1 neuron in the output layer. Antecedent rainfall and water levels were taken as inputs, and the future water level was the target output. For the number of neurons in the hidden layer, a trial-and-error procedure was used. For training the network, 400 sets of data were used. And for testing 78 data sets were used. The training of IFSANN is done using Kalmann filter.
Algorithm: Groundwater Level Prediction using IFSANN

Input: Influencing Groundwater level and rainfall
Output: Future Groundwater level

- Initialize input layer with given inputs
- The membership and non-membership are determined for the neurons generation criteria

\[
\mu(x) = \exp \left( -\frac{(x-c)^2}{2\sigma^2} \right)
\]

\[
\gamma(x) = \left(1 - \exp \left( -\frac{(x-c)^2}{2\sigma^2} \right) \right)^k, \quad k \geq 1
\]

Where \( c', \sigma' \) and \( k \) are coefficient, which need to be designed

- It is Intuitionistic fuzzy inference layer. Each node represents a Intuitionistic fuzzy rule. The degree of fulfillment and non-fulfillment of the \( i^{th} \) rule is represented by the following equations

\[
\mu_i = \mu_{i1} \mu_{i2} \cdots \mu_{in} = \prod_{j=1}^{n} \mu_{ij}
\]

\[
\gamma_i = \gamma_{i1} \gamma_{i2} \cdots \gamma_{in} = \prod_{j=1}^{n} \gamma_{ij}
\]

- It normalized the degree of fulfillment and non-fulfillment of the Intuitionistic fuzzy and calculated the hesitation margin index.

\[
\varphi_j = \frac{\mu_j}{\sum_{j=1}^{m} \mu_j}
\]

\[
\phi_j = \frac{\gamma_j}{\sum_{j=1}^{m} \gamma_j}
\]

\[
\pi_j = 1 - \varphi_j - \phi_j
\]

- The output of the intuitionistic neural network with rules can be calculated as:

\[
\bar{y} = n \sum_{i=1}^{n} (y_i - x_i)^2
\]

RMSE is frequently used measure of differences between values predicted by a model or estimator and the values actually observed from the thing being modeled or estimated. It is just the square root of the mean square error as shown in equation given below:

\[
RMSE = \sqrt{\frac{\sum_{i=1}^{n} (y_i - x_i)^2}{n}}
\]

\( R^2 \) assesses the goodness of fit by indicating the deviation of the estimates values from the line of the best fit or the regression line. The value of \( R^2 \) is between zero and unity. A value close to unity indicates a satisfactory result, while a low value implies an inadequate result

\[
R^2 = 1 - \frac{\sum (y_i - x_i)^2}{\sum y_i^2}
\]

Assuming that the actual output is \( x_i \), expected output is \( y_i \), n the number of observations.

**Results and Discussions**

The same training and testing data sets were used to train and test both models to extract more solid conclusions from the comparison results. Accuracy of the two models was
evaluated using $R^2$ and RMSE between the measured and predicted values are shown in Table-2.

Table-2 Performance Indices of the Models

<table>
<thead>
<tr>
<th>Well no</th>
<th>Method</th>
<th>RMSE</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>63708 (Pulliampatty)</td>
<td>ANN</td>
<td>0.79</td>
<td>0.83</td>
</tr>
<tr>
<td></td>
<td>FANN</td>
<td>0.67</td>
<td>0.87</td>
</tr>
<tr>
<td></td>
<td>IFSANN</td>
<td>0.45</td>
<td>0.9</td>
</tr>
<tr>
<td>63710 (M.N.Palayam)</td>
<td>ANN</td>
<td>0.86</td>
<td>0.76</td>
</tr>
<tr>
<td></td>
<td>FANN</td>
<td>0.71</td>
<td>0.9</td>
</tr>
<tr>
<td></td>
<td>IFSANN</td>
<td>0.23</td>
<td>0.96</td>
</tr>
</tbody>
</table>

Analysis of data in randomized sets clearly showed that IFSANN model is best fit for predicting the groundwater level in terms of statistical significance.

Further, the data were analyzed separately for each independent well point to have a clear comparison of the mean observed and estimated water levels for the two wells 63708 and 63710. The scatter plot of the measured against predicted water level for the two models is given in Fig. 5 and 6. As this figure showed that IFSANN model predicted with high accuracy, which this point demonstrate applicability and performance of IFSANN for prediction of groundwater level.

**Conclusion**

In this paper, a better forecasting model using IFANN has been developed for predicting monthly groundwater level fluctuations in the
Parambikulam Aliyar Basin area, Tamilnadu, India. Proposed IFANN method presented in this paper shows a good potential to model predicted water level data as shown in Fig. 6, from which it is seen that the IFANN model satisfactorily predicted the measured data than Mamdani fuzzy model and conventional. Considering the complexity of the relationship between the input and the output, results obtained are very accurate and encouraging. High-quality architecture of inference system based on artificial neural networks is usually formed by trial and error. The best function and architecture of neural networks is enhanced by IFSANN and it is proved by the experimental result which revealed the lowest value of RMSE and the highest value of R² compared to other existing approaches used in this paper. Lower RMSE obtained by the IFSANN method suggests its good generalization capability. The result of IFANN experiments in this study proves that neural networks give better forecasting results than the fuzzy ANN and conventional method.

References